Conduction and Convection Heat Transfer
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Lecture – 40
Energy Conservation Equation – III

We continue with, what we were discussing in the previous lecture. So, in the previous lecture, we were attempting to derive the energy equation. So, what are the steps that we followed? We first derive, the total energy conservation then we subtracted that the mechanical energy conservation and then we got the thermal energy conservation equation.

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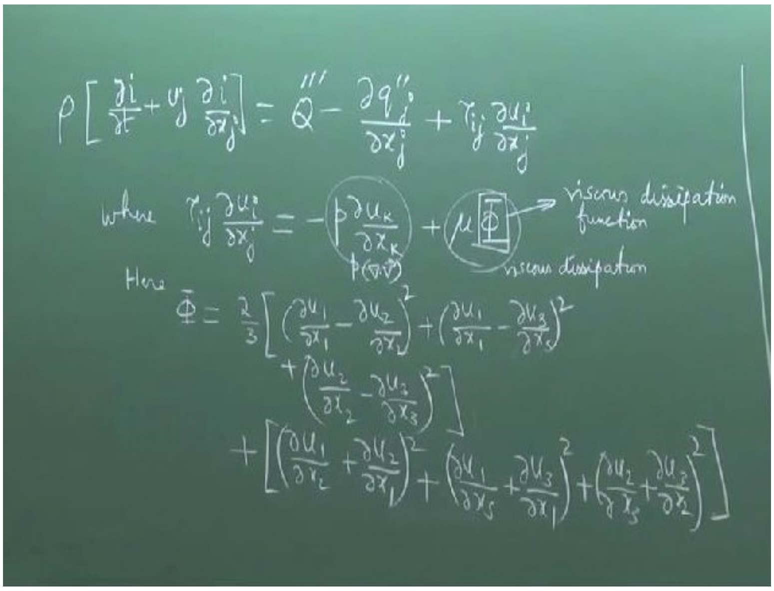

$$\rho\left[\frac{\partial i}{\partial t}+v_j\frac{\partial i}{\partial x_j}\right]=\dot{Q}'''-\frac{\partial q''_j}{\partial x_j}+\tau_{ij}\frac{\partial u_i}{\partial x_j}$$

where $\tau_{ij}\frac{\partial u_i}{\partial x_j}=-\underset{p(\nabla\cdot\vec{v})}{p\frac{\partial u_k}{\partial x_k}}+\underset{\text{viscous dissipation}}{\mu\boxed{\Phi}}$ → viscous dissipation function

Here

$$\Phi=\frac{2}{3}\left[\left(\frac{\partial u_1}{\partial x_1}-\frac{\partial u_2}{\partial x_2}\right)^2+\left(\frac{\partial u_1}{\partial x_1}-\frac{\partial u_3}{\partial x_3}\right)^2+\left(\frac{\partial u_2}{\partial x_2}-\frac{\partial u_3}{\partial x_3}\right)^2\right]+\left[\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right)^2+\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)^2+\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)^2\right]$$

That equation is written here with i as the internal energy so various terms this is like the total rate of change of internal energy. This can be attributed to volumetric heating, surface heat flux and viscous dissipation. So, one part is viscous dissipation and this is pressure work or PDVP, P dot delta v, sorry p into delta dot v. So, p into Delta dot v that term is there which is just like the PDV work in thermodynamic. So, this is due to volumetric change whatever is the work done.

So that is p into delta dot p and this is called as viscous dissipation. This is due to irreversible conversion of the work to overcome the viscous interaction into intermolecular form of energy. And we have shown that this viscous dissipation for a Newtonian and Stokesian fluid is always a positive contribution that is it always gives rise to heating and not cooling. So, the expression that we drive towards the end of the previous lecture that summarizes to this expression.

You can see very well that this phi is called as viscous dissipation functions. So, this viscous dissipation function is a function of basically the sum squares of the velocity gradients or rates of deformation. So, if you know the different rates of deformation then it is some of the squares of rates of deformation plus some of squares of other terms involving the velocity gradients. So, velocity gradients are responsible for this term along with the viscosity of the fluid.

Now, we will complete the derivation of the energy of the energy equation by expressing the internal energy in terms of enthalpy and then enthalpy in terms of temperature. Because temperature is the measurable parameter so we should express the governing equation in terms of temperature. Now, to do that the left-hand side

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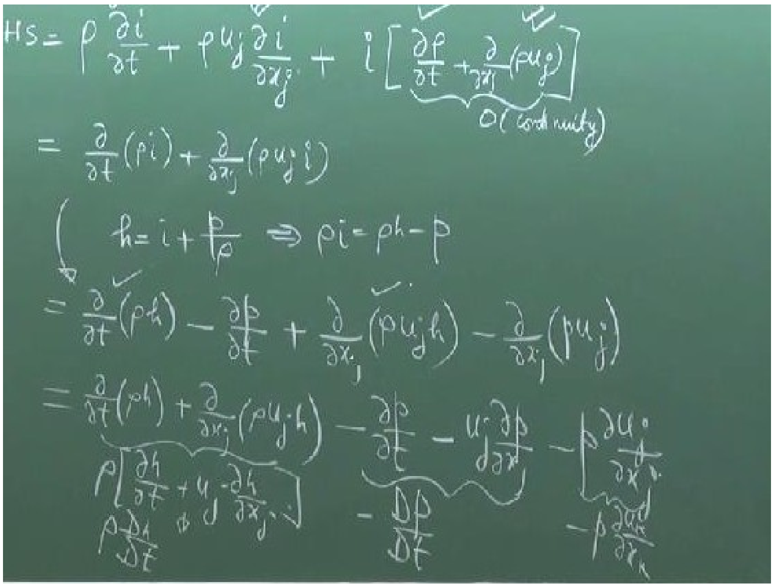

$$HS = \rho\frac{\partial i}{\partial t} + \rho u_j\frac{\partial i}{\partial x_j} + i\underbrace{\left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j)\right]}_{0\text{(continuity)}}$$

$$= \frac{\partial}{\partial t}(\rho i) + \frac{\partial}{\partial x_j}(\rho u_j i)$$

$$h = i + \frac{p}{\rho} \Rightarrow \rho i = \rho h - p$$

$$= \frac{\partial}{\partial t}(\rho h) - \frac{\partial p}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j h) - \frac{\partial}{\partial x_j}(p u_j)$$

$$= \underbrace{\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_j}(\rho u_j h)}_{\rho\left[\frac{\partial h}{\partial t} + u_j\frac{\partial h}{\partial x_j}\right]} \underbrace{- \frac{\partial p}{\partial t} - u_j\frac{\partial p}{\partial x_j}}_{-\frac{Dp}{Dt}} - p\frac{\partial u_j}{\partial x_j}$$

$$\rho\frac{Dh}{Dt} \qquad -p\frac{\partial u_k}{\partial x_k}$$

You can write –what we have done is see this trick, we will often play. We will convert as per our wish from none conservative form to a conservative form and then from a conservative form to a nonconservative form. So, in any of these cases we have to use the continuity equation. So, this is the continuity equation. This is actually zero by continuity equation.

Now, you can combine these terms and write and combine these terms. So, you can see that in one step we can convert the none conservative form to the conservative form. Then what we will do, we will write the internal energy in terms of enthalpy. So, h equal to i plus p/ rho that is the definition of enthalpy h equal to u plus pv that is written in different symbols. So, rho i equal to rho h minus p. So, we can club these two terms. This two terms again I will not repeat.

But what you can do is you can use the continuity equation to write it in none conservative form. These two terms correspond to the conservative form with enthalpy plus you have some correction terms for converting internal energy to enthalpy. So, these two terms together can be converted to a none conservative form how you do that? You simply use, the product rule for differentiating these two terms and use the continuity equation.

So, if you do that you will get rho. The remaining terms will be zero because of continuity equation. So, this in a short hand notation is rho, total derivative of enthalpy. You are familiar with the capital Dh/Dt that is the total rate of change is the change due to change with respect to time at a given location plus due to advection to a different location where you get a new velocity field and the new scalar field.

If this is rho Dh/Dt, what these two terms together are? Dp/Dt, right. And this terms you can write as minus p delta u k/ delta x k with an understanding that delta u j/ delta x j is equal to delta u k/ delta x k, whatever. Delta u j/ delta x j is what? Delta u 1/ delta x 1 plus delta u 2/ delta x 2 plus delta u 3/ delta x 3 so it does not matter whether you use j k l m whatever? So, I have written it in this way is because this term gets cancelled with the corresponding terms in the right-hand side.

So, when you convert the internal energy expression into enthalpy expression the first observation is that the PDV work gets cancelled from both sides. So, you are left with

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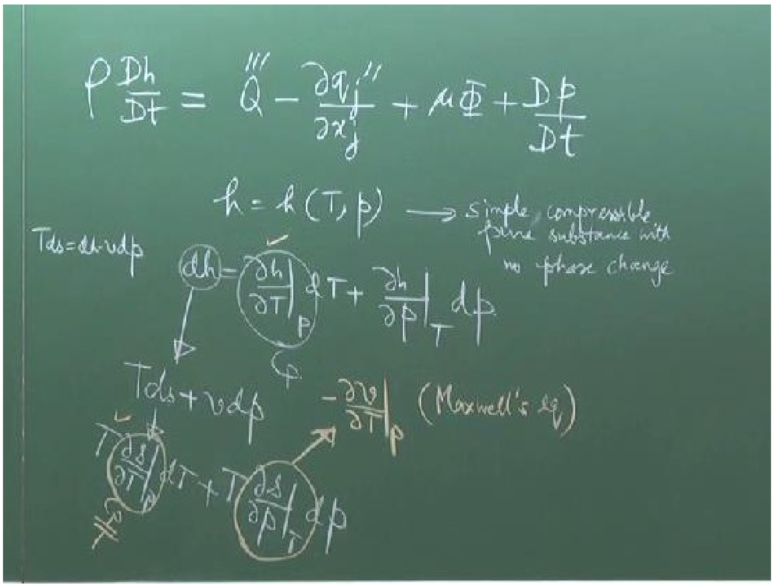

$$\rho \frac{Dh}{Dt} = \dot{Q}''' - \frac{\partial q_j''}{\partial x_j} + \mu \Phi + \frac{Dp}{Dt}$$

$h = h(T, p) \longrightarrow$ simple, compressible pure substance with no phase change

$Tds = dh - vdp$

$$dh = \underbrace{\frac{\partial h}{\partial T}\Big|_p}_{C_p} dT + \frac{\partial h}{\partial p}\Big|_T dp$$

$Tds + vdp$ $\quad -\frac{\partial v}{\partial T}\Big|_p$ (Maxwell's eq)

$$\underbrace{T\frac{\partial s}{\partial T}\Big|_p}_{C_p} dT + T\frac{\partial s}{\partial p}\Big|_T dp$$

The minus DP/ Dt terms from left hand side we have brought to the right-hand side it has become plus Dp/Dt. Now, what we will do is we will write this enthalpy in terms of temperature. So, in general, what are the assumptions under which this is valid? We discussed it earlier, what are the assumptions under which this is valid? Simple, compressible pure substance with no change in phase. So, you can write dh. So, what is this? This is Cp.

By definition of Cp this is what is Cp. Now we will write this using one of the Tdh relationship, that Tdh is equal to dh minus udp. So, in place of dh we write Tds plus vdp. Next enthalpy is itself a function of temperature and pressure. So, you can write this as T. So, if you compare both sides then these terms and these terms they have the same coefficient. That means this will be what? Cp/ T.

And this term you can write express by changing from enthalpy to a measurable quantity by using one of the four Maxwell's equations. So, if you compare both sides now you can write

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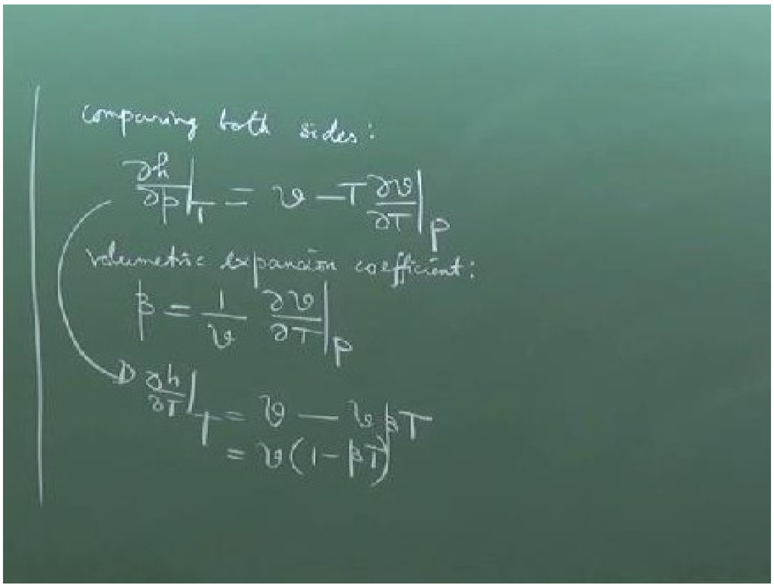

Comparing both sides:

$$\left.\frac{\partial h}{\partial p}\right|_T = v - T\left.\frac{\partial v}{\partial T}\right|_p$$

volumetric expansion coefficient:

$$\beta = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right|_p$$

$$\Rightarrow \left.\frac{\partial h}{\partial T}\right|_T = v - v\beta T = v(1-\beta T)$$

This is basically coefficient of Dp in both sides. Now, we know by the definition of volumetric expansion coefficient, beta is defined as this. So, volumetric expansion coefficient in qualitative in qualitative form is what? Change in volume per unit volume for each degree change in temperature and that has to be evaluated at some parametric conditions. So, the condition is here constant pressure.

So, you write dh equal to Cp dt in our expression what we require is capital Dh/ Dt of h. So, capital Dh/ Dt behaves mathematically in the same manner as small dh/dt. So, the same expression can be used so you can write

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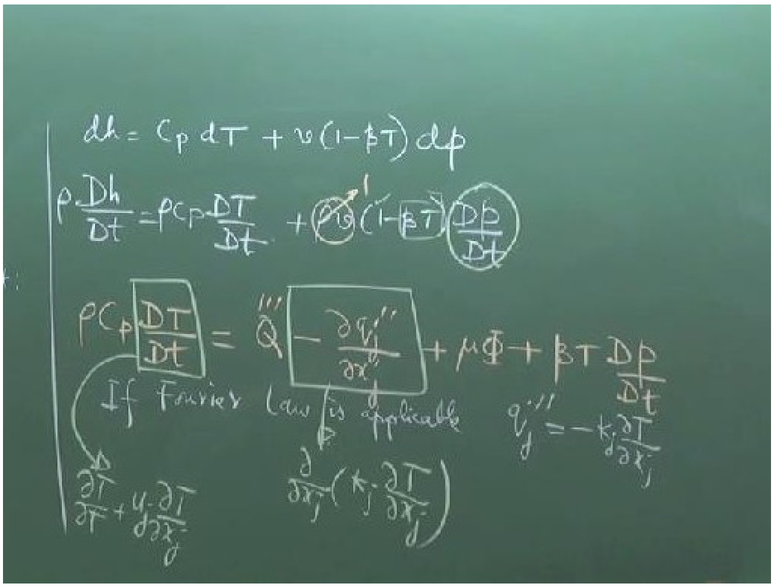

$$dh = C_p\,dT + v(1-\beta T)\,dp$$

$$\rho\frac{Dh}{Dt} = \rho C_p\frac{DT}{Dt} + \overset{1}{\rho v}(1-\boxed{\beta T})\frac{Dp}{Dt}$$

$$\rho C_p\boxed{\frac{DT}{Dt}} = \dot{Q}''' \boxed{-\frac{\partial q_j''}{\partial x_j}} + \mu\Phi + \beta T\frac{Dp}{Dt}$$

If Fourier law is applicable $q_j'' = -k_j\frac{\partial T}{\partial x_j}$

$$\frac{\partial T}{\partial t} + u_j\frac{\partial T}{\partial x_j} \qquad \frac{\partial}{\partial x_j}\left(k_j\frac{\partial T}{\partial x_j}\right)$$

Dh/Dt equal to Cp DT/Dt plus. So, in the left-hand side what is required is rho, you multiply by rho all the terms. So, when you multiply all the terms rho into specific volume is equal one. Now you compare the left-hand side and right-hand side of this energy equation and we can observe one very interesting thing that this Dp/Dt and this Dp/Dt they get cancelled from both sides, right.

This is in the right-hand side and only this part not this part this part is still there only this part with Dp/Dt that gets cancelled. So, you are left with rho Cp. This equation still is not mathematically closed. Why it is not mathematically closed? Because it has an unknown temperature, it has also an unknown heat flux. So just like in Newton law of viscosity we expressed the stress in terms of the rate of deformation.

Here also we will express the heat flux in terms of temperature gradient. That is the constitutive behavior for heat transfer. Now, there are various mechanism by which the heat conduction can take place so there is no single constitutive behavior for the heat flux. So, for the heat flux there can be different constitutive behaviors but we will take as an example the constitutive behavior dictated by the Fourier law of heat conduction.

So, if Fourier law is applicable this is a very interesting thing and sometimes beginners have confusion on this. See we are driving an energy equation where fluid flow is also a part of the consideration. So, it is an equation, governing equation the equation that we have written there is an equation for convection. However, for heat flux we are still using the Fourier law of heat conduction. We are not using any other law that means that actually convection is not a fundamental mode of heat transfer.

The fundamental mode of heat transfer is still conduction but in convection what at best you can say what is happening is advection assisted conduction or advection assisted by conduction. whatever. So, the basic heat transfer mode still remain conduction because if you think of there is a solid boundary and from the solid boundary heat has to reach the fluid before it is advection that heat transfer from the solid boundary to the fluid is predominantly by, in fact is solely by conduction.

Because the fluid molecules adjacent to the solid boundary as stationary. So, conduction is the only mechanism by which heat get transferred from the solid boundary to the fluid and then from the fluid one layer to the other by combination of conduction and advection. So, conduction is present in all occasions and to calculate the heat flux we have to use law of conduction. So, then this term becomes.

This terms you can expand first in the non-conservative from even if you want to use the conservative from you can use the continuity equation to convert it to the conservative form just in the same we did for enthalpy and internal energy you can do it for temperature. So, this equation is the so-called energy equation that is the basic governing equation for convection. So, the left-hand side is the total change in the temperature again.

Rho into Cp comes out of the derivative because of mathematical simplification but not because of rho and Cp are constants. So, even if rho and Cp are not constants they will still come out of the derivative but they themselves may be variables. Right hand side this is the volumetric heat generation. This is the heat transfer due to the surface heat flux. This is the viscous dissipation and this is the pressure work.

So, in this term you can see that for flows with negligible compressibility effect this term is not important. Since in this particular course we will deal with incompressible flow, we will not consider that term to be any more important for the discussion and for the problems that we are solving subsequently. But wherever compressibility effects are important that last term is important.