Conduction and Convection Heat Transfer
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Lecture 4

So far as Prof. Som has discussed on some of the introductory issues related to various modes of heat transfer and we will next proceed towards mathematically describing the equations for conduction heat transfer that is the agenda of today. Now why we require all these, so let us say, I mean let us think about a practical problem.

Let us say that there is a piece of metal, which is acted upon by a heat source and there is some change in temperature within the metal, because of the change in temperature, there will be a temperature gradient and the temperature gradient will in turn dictate the microstructure and the microstructure will dictate the mechanical properties of the material. So, it is very important to analyse that how the temperature varies with position and time within the material.

So, this is one example, I mean we can give many examples. I will just give another example. Let us say that there is some medical doctor, who is doing surgery with a laser, so when the medical doctor is doing surgery with a laser what the medical doctor is essentially doing, the medical doctor is trying to say kill or destroy some tumours. So, this is an example of course laser can be used for several medical purposes, this is just one example.

So when this is happening then one of the big objective is to make sure that only the diseased cells are killed but the normal cells are not damaged, because it is a very common side effect that after a treatment when the disease cells are killed, the normal cells are also destroyed because of the heating effect in a surrounding tissues and then what happens is that basically because the normal cells are also damaged it can give rise to weak immune system of the patient and the patient may suffer from serious side effects of the treatment.

So now how do you know that what will be the extent up to which that effect of the laser treatment is failed. So, then you have a laser source then you have to find out the temperature within the tissue as a function of position and time and then figure out that which are the damaged tissues or which are the damaged cells which are cells not yet damaged and one can essentially figure out a suitable parameter of the laser.

For example, laser power, what should be the radial laser power and all these things, so that the correct treatment can be made. So, there are several situations in which conduction is a dominating mode of heat transfer, number one and number two it is important to find out how the temperature within the material, the material may be a physical material. It may be also a biological material whatever.

The temperature within the material varies as a function of position and time and to solve this problem one has to look into the problem from a mathematical perspective. One can of course do experiments but when you do experiments it is not possible to figure out at each and every point how temperature varies at a function of position and time. But if you numerical stimulations or if you analytical solutions, if analytical solutions are possible then you can figure out that at each and every point how the temperature varies with position and time.

Now before going for the analytical and numerical treatment, you have to understand that what equation you should solve and we will derive that equation today that is the agenda of today's lecture. So, when we are thinking of the equation that governs the temperature as a function of position and time. So, the kind of equation that we are looking for is the differential equation, because essentially a differential equation can only give up point to point variation.

Now if you are interested about a global balance, global picture you can use some equations which are also known as integral equations. In this particular course of conduction and convection heat transfer, we will also discuss about many situations where we will be using the integral equations but today we will start with a differential form of equation for heat conduction.

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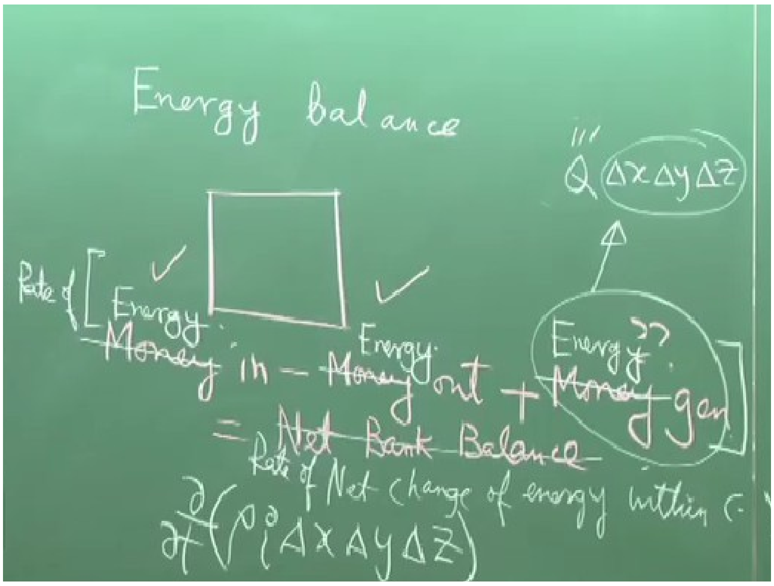

So, to understand that so the agenda is basically to write and expression, a mathematical expression that talks about energy balance. Now energy balance is something which is very obvious, and we will start with a description of energy balance, but I know that right we are having a lecture when it is quite late in the evening, raining outside, there are like compelling situations, which can make you feel distracted from the subject.

So, if we start with energy balance principle and all those things, you may feel oh this is not interesting. So, I will start with something, which is not energy balance, but which is money balance and money balance can make you feel excited at this end of the day. So, we will start with balance of money rather than balance of energy. So, I will start with the story of a good old day when people had to go to a bank to open a bank account.

Say you have got a new, nowadays you do not have to go to a bank to open a bank account. I mean you may have several online resources to do that. The bank person can individually come to you and help you to open the bank account. What is the good old day when we were very young or our fathers were even young in those days people had to go to the bank to open bank account, so let us say that somebody has got a new job.

He has gone to the bank to open a bank account. So, once you get a job and you open a bank account of course you have to deposit some money to the bank. So, when the money is deposited to the bank, your account gets activated. Now every month your salary is being deposited in the bank account. Now someday you have got into the bank and you realise that you have to withdraw some money from the bank.

This is the good old day time so no ATM business. You have to physically go the bank and withdraw the money. But before withdrawing the money, you must know that you are having the correct amount of money in your bank or you have the bank balance to withdraw that money. So, you can check your passbook and then what you do is you are withdrawing some money from that. Now when you are withdrawing some money from that you mentally try to do a balance.

What balance, so some money has flown into your bank account this black box is like a bank account. So, money in how was money in input to the bank account. You first deposited some money and then your employer has deposited your salary in the bank account so money in - money out. Money out is what you are we have withdrawn from the bank is it your net bank balance. This is not your net bank balance.

You see your bank balance is little bit more than this. It is more than this because bank is giving you interest. So, money generated. Money cannot be generated right. It is very difficult to generate money. I have never been successful in generating money in my life. But I mean money is generated because of some issues in economics which is not the agenda of heat transfer. So, we will not get into that.

So, money generated, this is what is net bank balance right. This is the summary of the story of money balance in a bank account. Now think of instead of a bank account you think of a control volume and energy transfer is taking place across the control volume.

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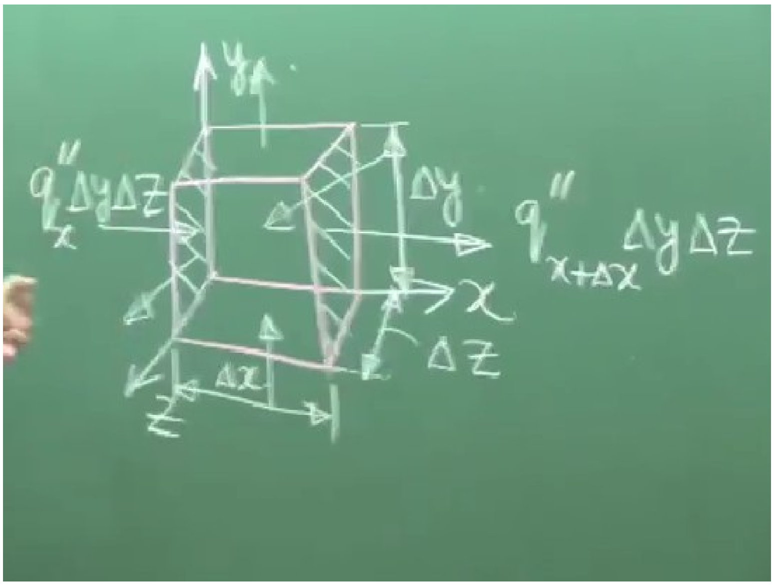

So, this is the control volume and energy transfer is taking place across this. So, when energy transfer is taking place across this you have some energy coming in so this is a control volume. I believe all of you understand what is a control volume right. A control volume is a fixed region in space across which mass and energy can flow.

So as if you are sitting with a camera, focussing your camera over this region and across this region there is some transport of energy and mass. So, when you do that so instead of money in, now you can write energy in - energy out + energy generated. How can energy be generated again like it is questionable term, it is a very loose way of stating what is happening because as we all know from our school days that energy cannot be created or it cannot be destroyed.

So, it is basically some mode of transfer so what is happening. Let us say that in a system there is electrical wire and current is flowing through the electrical wire so when current is flowing through the electrical war there is a heating effect because of dual heating right, I 2 R. So that is something what by an example of what we mean by energy generation or in a combustion reaction there are reactants and the reactants combine together to form a product and then there is a release of thermal energy.

So that is another example okay or there may be nuclear reaction so there are various possibilities by which you can have energy generation. So instead of bank balance now you have net change of energy within the control volume. So, energy in - energy out + energy generated is net change of energy within the control volume. Just it is like net change of money within the bank account instead of money it is energy.

The bank account is similar to a control volume okay. So now because in heat transfer all things are more or less expressed in terms of a rate process so it is often written in terms of rate of energy in - energy out + energy generated is equal to rate of net change of energy within the control volume right. This is a statement of basic energy balance and believe me or not if you have understood it correctly.

And if you know how to represent it mathematically you know 50 percent of heat transfer. So, we will see that how we can write this expression of balance. This is a physically intuitive expression right. Even if we do not know much of a science this is something which comes from physical intuition in - out + generated equal to change. So, we want to write this in terms of mathematical expressions. So, this is a qualitative physical expression.

Next our objective will be to write it mathematically. So, we will proceed towards that. So, to do that, let us say that we have a control volume like this okay. So, this control volume, let us say this is x axis, this is y axis and this is z axis. Now we have chosen such a control volume is because we want to derive the equation in terms of the Cartesian coordinate system but you could choose any other shape control volume.

There is no restriction on the shape of the control volume just for simplicity of using the Cartesian system we have taken such a control volume. Next, what we will do is we will account for the rate of heat transfer over here. In heat transfer, just like in mechanics you have two types of forces one is the surface force another is a volumetric force or a body force.

Similarly, in heat transfer you have either surface heat transfer or volumetric heat transfer. So, this rectangular shape body or control volume which has dimension say delta x along x, delta y along y and delta z along z okay. Now what we do is we will consider all the phases, how many phases are there, you have six phases in the volume. So over each we are interested to write a description of the heat transfer and the description of the heat transfer.

For example, if we consider this phase we will represent it by a quantity which is called as heat flux right. So, this heat flux, this is what rate of heat transfer per unit area normal to the direction of heat transfer. Rate means time rate. That is what is expressed per unit area. So, heat transfer per unit area normal to the direction of heat transfer per unit time, so that is this, this is a typically notation by which we write that.

So, this is the heat flux, what is the total rate of heat transfer through this, this heat flux times the area of the shaded phase. So, what is the area of the shaded phase, Delta y into delta z. Just consider the opposite phase, this phase. We will first investigate what is happening along x and similar things will happen along y and z.

So, we have considered two phases which have normal along x and similar two phases can be considered which have normal along y and other two phases which are normal along z. So, what is heat flux here, so if it is q double prime x, this is q double prime at x + delta x right. Same heat flux, this is at x, if this is x equal to x, this is x equal to x + delta x. This times, see this is energy in, this is energy out right.

So similar thing from the bottom and from the top and from the back and from the front, I am not writing all those clumsy. So, what I will do is I will just write one of the terms which is governing the behaviour along x similar terms will come along y and z.

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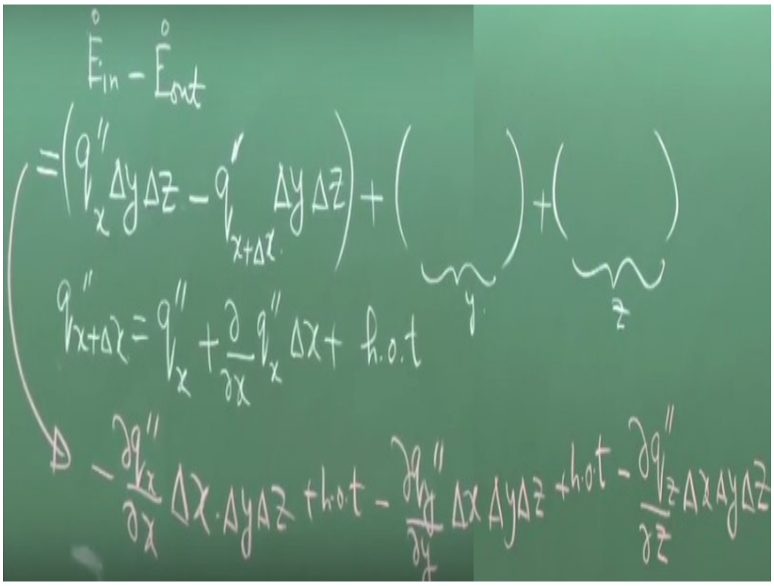

So, rate of energy in - rate of energy out, this dot means rate. So, what is this, this is along x + similar terms along y and similar terms along z. We will write those terms very soon and we will write those terms by looking into the analogy within this. Now you see here that basically we have to write the difference between q double prime x and q double prime x +

delta x.

So, what is q double prime, x + delta x. You can write it in terms of q double prime x by using a Taylor series expansion right. So, q double prime at x + delta x is equal to q double prime x + some higher order terms which involved delta x 2, delta x 3 like that. So, you can simplify this term and write q double prime x - q double prime x + delta x that is equal to this times delta y into delta z + higher order terms.

So, this delta x multiplied with delta y, delta z becomes delta x, delta y, delta z. So, if this is the term along x what will be the term along y. Similar thing and along z also similar term + higher order terms of course are there. So we have written energy in - energy out that means we have written these this term and this term. Then we will write energy generated.

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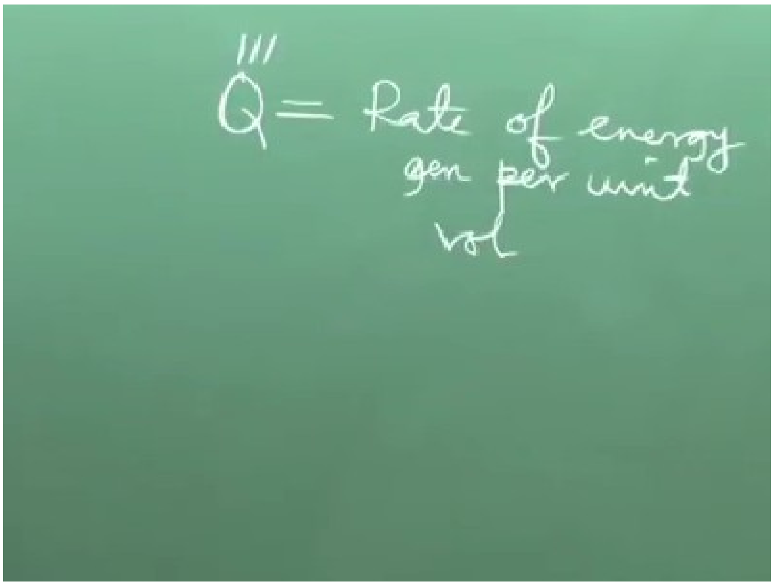

So, for energy generated, we typically consider this as rate of energy generation per unit volume. So, what is the rate of energy generation this term it is simply Q triple prime into delta x, delta y, delta z. This is per unit volume. This is the total volume of the control volume. What is the net rate of change of energy within the control volume. How do you write it, we have written the left-hand side of this. Now we are interested about the right-hand side.

So how do you write the net rate of change of energy within the control volume yes. So that is the in terms of thermodynamics, what is the energy within the control volume which form of energy is representative of the energy within the control volume. It is the internal energy.

Internal energy is not necessarily CP into T or CV into T whatever. We will discuss about that clearly.

So, we call internal energy. What is internal energy, how it should be described and how it should be brought in the context of this equation, we will discuss about that. So, let us say in thermodynamics we have used a symbol U for internal energy. In heat transfer we will avoid that because we will preserve the symbol U for velocity because we have a whole length of discussion on convection where the fluid mechanics will interface with heat transfer.

So there the velocity field will come. So, we will not use the symbol U for internal energy better we will use a symbol I for internal energy. Small i is the small u that you have learnt in thermodynamics for internal energy. So, this is what, this is a property per unit mass. What is the mass of the control volume, the density Rho into delta x into delta y into delta z, okay?

So, it is not this, it is the rate of change of this. So, rate of change of this means this one. Very important, this is not ordinary derivative. This is partial derivative. The reason is pretty clear that when you are considering the net rate of change of energy within the control volume you are fixing up the position and at a give position you are finding out how the internal energy is changing with time.

So, it is a rate of change with respect to time keeping the position fixed. That is why partial derivative and not ordinary derivative.

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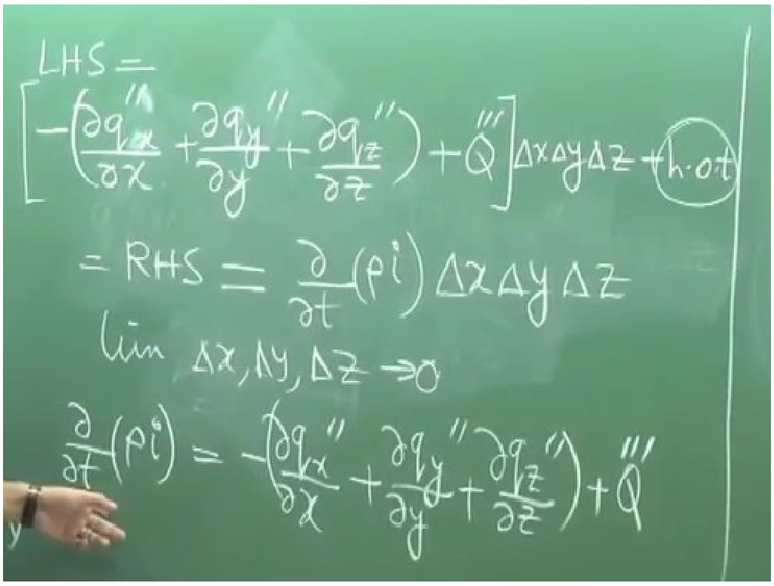

The next job will be to now combine the left-hand side and the right-hand side. So, the left-hand side, because the delta x, delta y, delta z all are constant so we have taken that out of the time derivative okay. So now see, our objective is to derive a differential equation and differential equation means the equation which is valid at a point. At a point in space as well as at a given instant of time okay.

So, you have to basically shrink this rectangular volume to a point. How do you shrink the rectangular volume to a point. You take the limit as delta x, delta y, delta z all tend to zero. So, take the limit as delta x, delta y, delta z tends to zero because these are all tending to zero these are individually not equal to zero so these get cancelled from both sides and the higher order term will tend to zero.

Because the higher order term will involve what either delta x 2, 3 like that or delta y 2 3 like that or delta z 2 3 like that. So, when you take this limit that term will tend to zero. So, then you have left with this equation.