Conduction and Convection Heat Transfer

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Lecture - 38
Energy Conservation Equation - I

So, so far we have discussed about the momentum equation and its applications. Today we will discuss about the energy equation.

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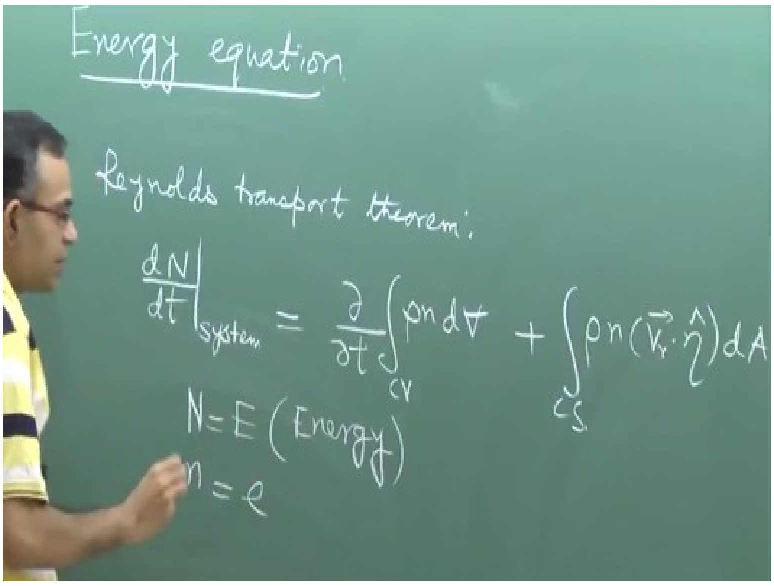

By Energy equation what we mean is that we want to conserve the total energy of the system. So, if we write the Reynolds Transport Theorem, if we write this, then for the total energy conservation N should be equal to E. That is the energy of the system. Remember this is the total energy as per the purview of the loss of thermodynamics. So, that means this is the sum of internal energy, kinetic energy and potential energy.

That is what we are talking about as the total energy. Now when N is equal to the total energy, n is the specific energy. That is the total energy parting with mass.

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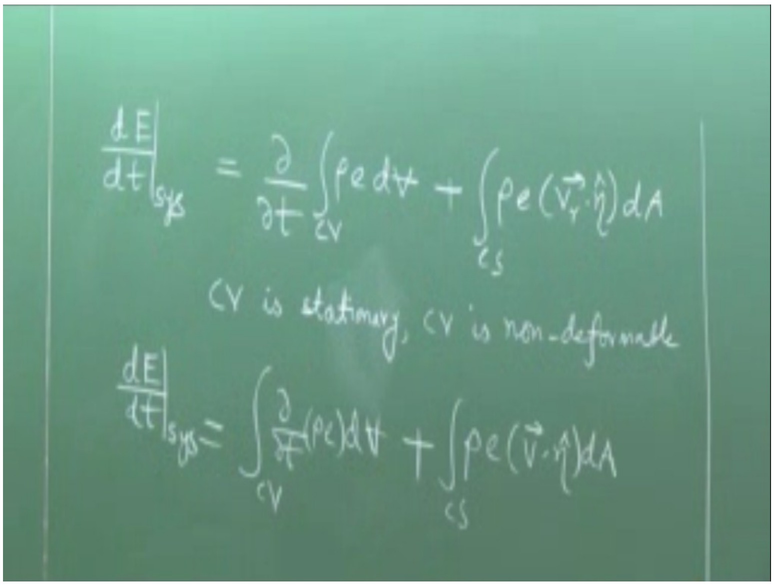

So, you can write. We will further assume that the control volume is steady and control volume is- sorry, stationary, not steady, and control volume is non-deformable. So, the stationary control volume implies that the relative velocity is same as the absolute velocity and non-deformable control volume means you can take this time derivate within the integral. So, that means, you can write

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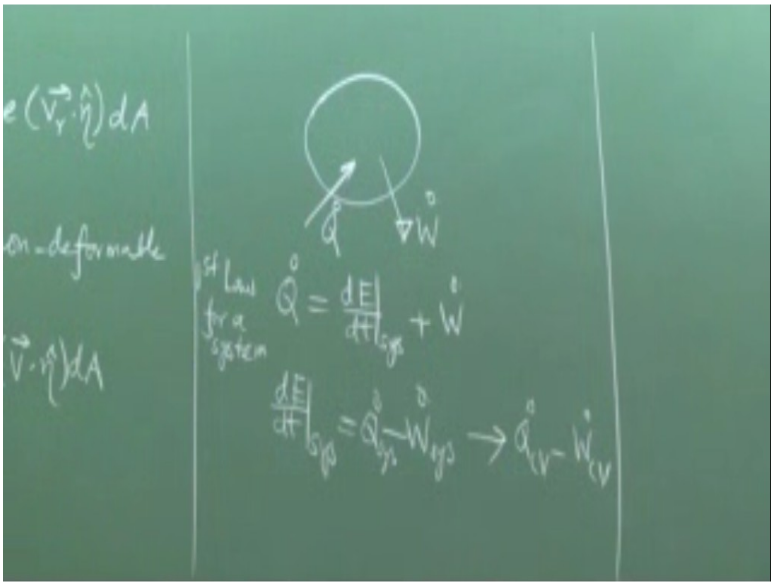

Now what we will do is. We will express this in terms of the first law of thermodynamics. So, think of a system on a controlled mass. The first law of thermodynamics in a rate expression can be written as, if you say that the heat transfer to the system is Q dot and the work done is W dot, then Q dot, this is the first law. First law for a system. Therefore, you can write, delta E delta t is equal to Q dot minus W dot.

And this is Q dot system minus W dot system, in the limit as delta t tends to zero, the system tends to the control volume. That is how we derive the Reynolds Transport theorem. So, therefore this will tend to Q dot cv minus W dot cv, where cv stands for the control volume. So, here in the left-hand side, we have expressed it in terms of the heat transfer and the work done.

Next our objective will be to write the expressions for heat transfer and work done. First let us write the expression for heat transfer.

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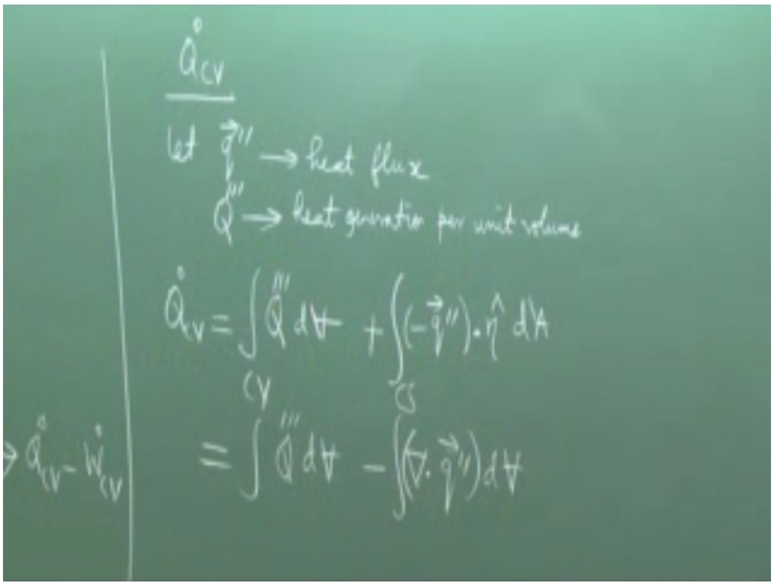

Just like in continuum mechanics you have two types of forces, surface force and body force, similarly heat transfer you have two types. One is surface heat transfer and another is volumetric heat transfer. Solet us say, let this be the heat flux, and - So, Q dot c. This is heat generation part unit volumes, so you multiply it with the elemental volume and integrate it over the control volume. So, this is for the volumetric part.

For the surface heat transfer you have to keep in mind that, if we have a heat flux like this, this is actually a negative heat transfer because heat transfer to the system or to the control volume is positive as per the first law of sign convention. I mean, we could also make a different sign convention, but by writing this we have made this sign convention only. So, this will be adjusted with a negative sign, we can write this as -

By using the divergence theorem, we are converting the area integral into the volume integral, so this is for the heat transfer. Next let us find out what is the work done.

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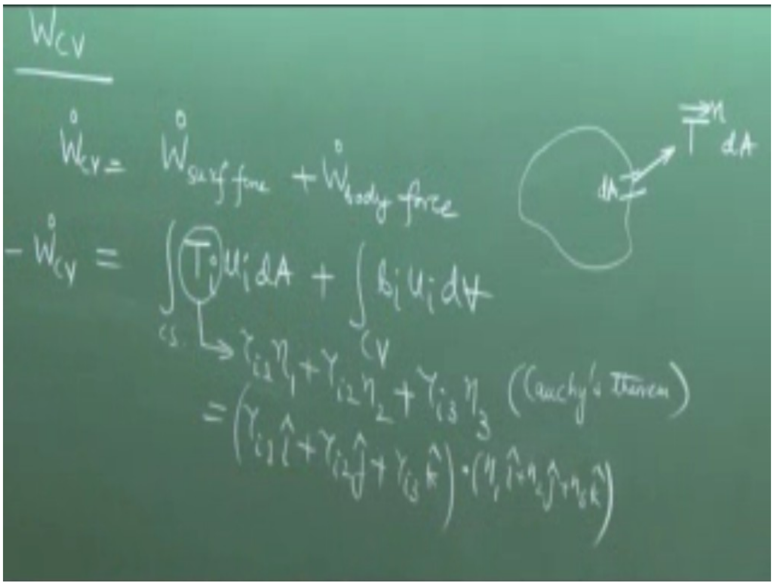

Now in mechanics when we write work done, we write work done by the force as positive and in thermodynamics you have work done by a system against a resistance is actually a positive work. So, that means again we have a conflicting sign convention. To adjust to that, we will write, in terms of mechanics, whatever is the expression for work done when we substitute it in the loss of thermodynamics same expression, but with a negative sign.

So, now work done is because of what, because of surface force and body force, right? Work done is because of presence of forces and there are two types of forces, surface force and body force. So, work done is due to what? Work done by work done due to surface force plus work done multiplied by body force. The surface force, work done due surface force. Now let us say that this is a controlled surface and let us say that you have an area dA.

What is the force per unit area on this? Recall, that is nothing but the traction vector. This is force per unit area. So, this time day is the total force and work done is the, what? The dot product of force and velocity, rate of work done, right? W dot. That is the dot product of force and velocity. So, this will be integral of this vector dot with velocity vector. That means Ti, Ui, dA.

Dot product of two vectors means basically multiplying their individual component and summing up. So, if there are two vectors are A and B that dot product is Ai, Vi, right? So, you have Ti plus rate of work done due to body force. Body force per unit volume when we derive Navier-Stokes equation we assume body force per unit volume is B, so bi, Ui, dV. This is b dot velocity.

So, basically, we will write this as minus of w dot cv to adjust for the sign or w dot cv equal to minus of these whatever. Now what is Ti? Ti could be written by using the Cauchy's theorem. So, Ti is this. Tau ij eta j, right? Cauchy's theorem. So, this you can write as this, just another way of writing this.

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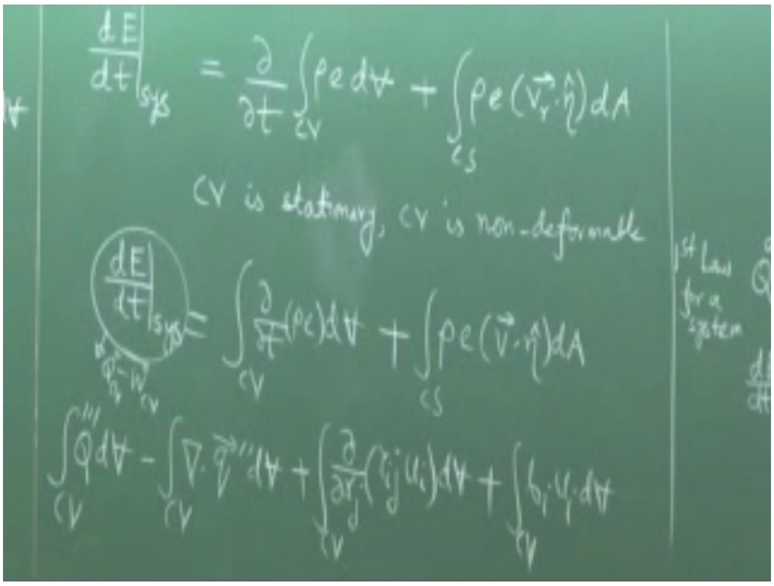

Now, Ti, Ui, dA, so let us give this a name tau i. So, how can we write this? Del dot tau i, u i. That means in index notation del del x j of… This is just another way of writing del dot tau i u i. So, now you can write this. So, we will now substitute that expression here. In the dE dt system it is Q dot minus W dot. So, we will substitute the Q dot and minus W dot. So, Q dot cv minus W dot cv. Q dot cv is integral of this.

I am deliberately trying to mix up notations and symbols so that you get familiar with different types of symbols. Of course, this could be written as del del xj of qj, right? Same thing. This is heat transfer minus W dot cv. Now let us come to the right-hand side. So, left hand side all terms are expressed as volume integrals. Let us express right hand side all the terms also as volume integrals.

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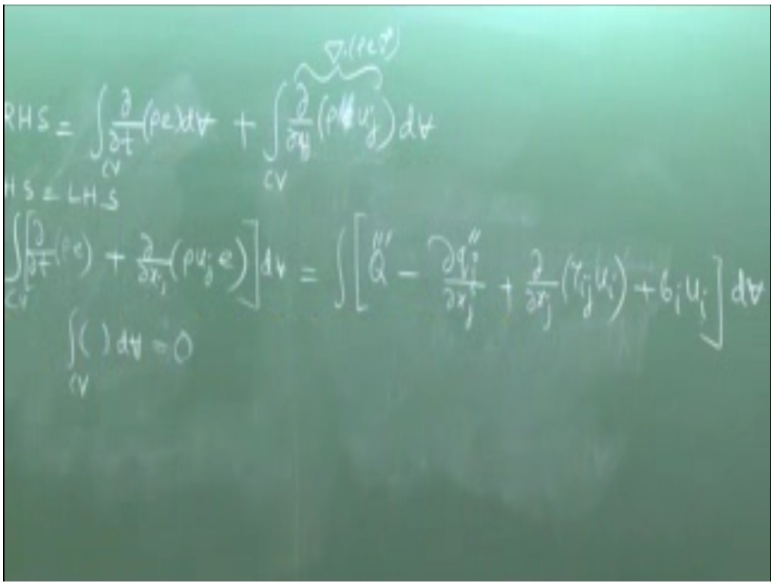

So, right hand side, what is this term? Del.rho e v or del del x j of rho e u j. So, this is del dot rho e v. So, now we can write, right hand side is equal to left hand side. Okay? So, all terms expressed as volume integrals. So, this is as good as writing integral of something into dV equal to zero, since the control volume is arbitrary, this is as good as something is equal to zero. So, that means you can write this.

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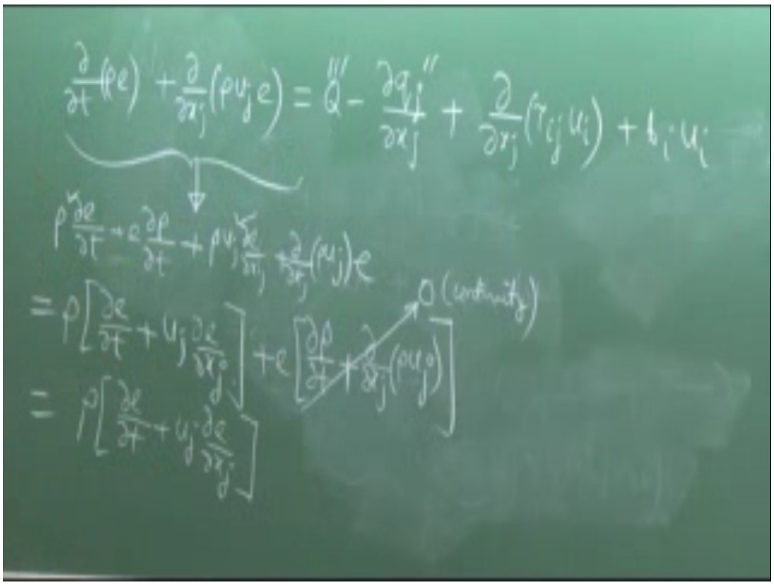

Now, the left-hand side what form is this? Is it a conservative form or a non-conservative form? This is a conservative form because we started with a basic conservation equation. We can convert this into a non-conservative form by using the continuity equation. So, let us do that. So, what we have done is, we have used the product rule of derivate. Now we can write this is equal to rho by combining these terms and plus e.

So, this is zero y the continuity equation. So, this becomes rho, so if it is written in this way, then that is the non-conservative form of the equation. We have to keep in mind, again I am repeating. Although rho is coming out of the derivate that doesn’t mean that rho is a constant. Rho is coming out of the derivate because of simplification using the continuity equation. Now this equation, physically what does it represent? It represents the conservation of total energy in a controlled volume perspective.