Conduction and Convection Heat Transfer
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 36
Review of Fluid Mechanics – IV

Now we will see that what are the different terms? What are the different terms meaning here? So, to understand that let us simplify the left-hand side of the equation.

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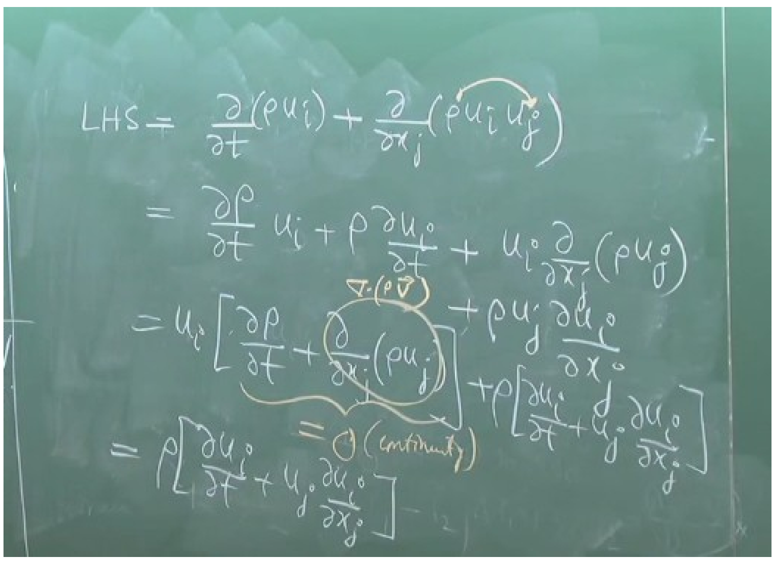

So, what we have done is we have clubbed this rho u j and u i separately and have used the product rule for differentiation for the second term. Rho and u j together and u i separately. So now you can write this as—what is this? By continuity this is equal to zero. This is delta dot rho v. In index notation, this is divergent of the velocity vector. So, this is zero by continuity. So, the left-hand side becomes rho. So, what is there inside?

So, this is like if it was x component it is like delta u/ delta t plus u, delta u/ delta x plus b delta y plus w delta z that thing is written in an index notation. So, what is this? This is the total derivative of the velocity or the acceleration along i. So, the left-hand side is what? Rho into acceleration along i. So, the left-hand side is rho into acceleration along i that means mass into acceleration per unit volume.

So, the left-hand side is mass into acceleration per unit volume. So, what is right hand side, right hand side is the force per unit volume. This is the surface force per unit volume. This is body force per unit volume. So, this is like Newton second law of hot fluid. So, this is mass into acceleration per unit volume right hand side force per unit volume. Now let us look into the situation that how many equations and how many independent equations.

And how many unknown, do we have? Do you have sufficient numbers of equations to solve the velocity? See in fluid mechanics one of the primary matters of importance is to solve the velocity to obtain the velocity field. So how many equations and unknowns do we have? Let us see.

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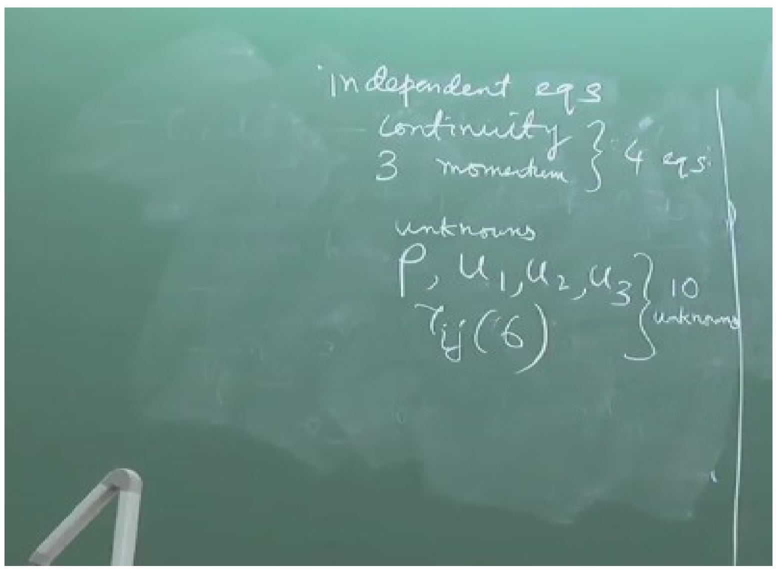

Independent equations, how many equations one is continuity equation. The second is you have this equation for three components so three momentum equations. So total four equations. Unknowns, what are the unknowns? In general, you may have rho as an unknown, density may be unknown then u 1, u 2, u 3 then six components of tau i, j body forces usually prescribed it is know because you are interested about the motion given a particular body force.

So how many are there? One, two, three, four plus six, ten unknown so you can see that whatever equations we have derived that is not a closed system of equations. Because you have four independent equations, with ten unknowns so you require further simplification and for further simplification you need to treat the fluid as a special type of fluid. So far, we have treated the fluid as a general fluid so it can be used for any type of fluid.

But to make further simplifications we have to consider fluid by fluid separately. So, that means when you consider fluid by fluid separately of course that means that may be some particular type of oil or water or plant whatever different types of fluids have to be given due consideration for their constitution and this is what is called as constitutive behavior of the fluid. So that means how does the fluid respond to the given stress that is unique for different types of fluids that needs to be recognized.

We will come into that but one final point which I intend to mention here is that it is an illusion that this rho when it comes out of the derivative it gives an elusive understanding as if rho is a constant. This rho has come out of the derivative not because rho is a constant but because of simplification using the continuity equation so this rho can still be variable. So, rho outside the derivative does not mean that rho is constant.

It has come out of the derivative by virtue of simplification using the continuity equation. Normally, this form we use for computational implementation of the problem or for CFD solution. The computational fluid dynamic solution this is called as conservative form and this form we commonly used for analytical solution of the problem and this is called as non-conservative form. Why this is conservative form?

Because this form is derived directly from the conservation equation and this form is a simplification of that using the continuity equation. So now we will go to the constitutive behavior that is;

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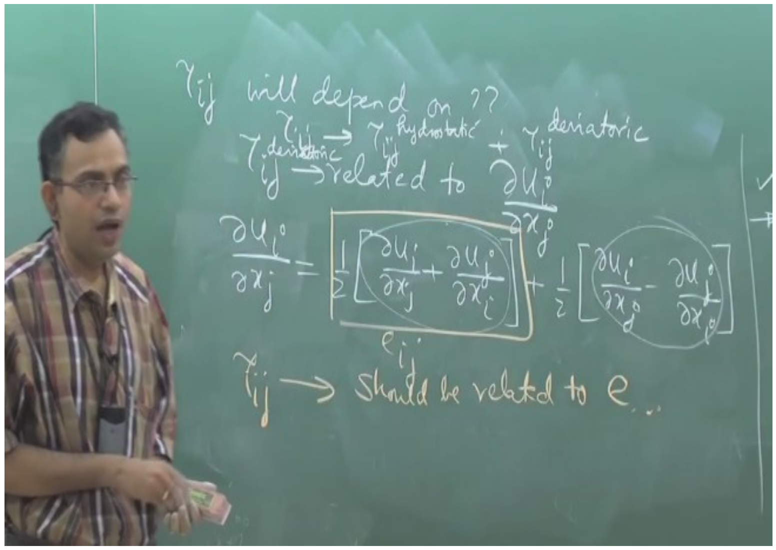

That is tau i j will depend on what? In fluid mechanics, stress is related to rate of strain or rate of deformation and rate of deformation by using your knowledge in fluid kinematics you know that rate of deformation is related to the gradients of velocity. So, tau i j is related to –now you can write –but you have understand one thing very carefully that if a fluid is at rest then there is no gradient in velocity then there no stress.

There is always some stress even when the fluid is at rest. What is the difference between the fluid at rest and fluid at motion? Fluid at motion will be subjected to both shear stress and normal stress whereas fluid at rest it cannot sustain any (()) (39:18) It will be subjected to only normal stress and that is called as hydrostatic stress. So, we will divide tau i j into two parts, one is tau i j hydrostatic and the remaining we call as tau i j deviatoric.

Deviatoric component is a component other than the hydrostatic component. Hydrostatic component is the limiting case when the fluid is at rest so then the deviatoric component is zero. But when the fluid is under motion you have both hydrostatic and deviatoric component. So, tau i j deviatoric is related to delta u i/ delta x j. Now you can write –this is nothing but a equal to half of a plus b plus half of a minus b and it is no new equation.

So why we have written this because there is physics that is associated with this equation. What does it represent? This is shear deformation like delta u delta y plus delta b delta x, right. Shear deformation and what does it represent? Rotation. So out of shear deformation and rotation which will give you stress. You have to keep in mind that this is (()) (41:47) shear deformation but when i equal to j this gives linear deformation.

So linear stroke shear deformation and this is angular sorry this is rotation. So out of linear deformation, shear deformation and rotation which will not give you stress. Rotation will not give you stress. It is like a rigid body type of behavior. So, you can say that you give this a name e i j then tau i j should be related to this e. This is also a second order tensor; this is called as rate of deformation tensor because it uses two indices for its specification.

Now the question the big question is how is tau related to e? How is tau related to e it depends on what type of fluid you are considering if you are considering Newtonian fluid then tau is linearly related to e.

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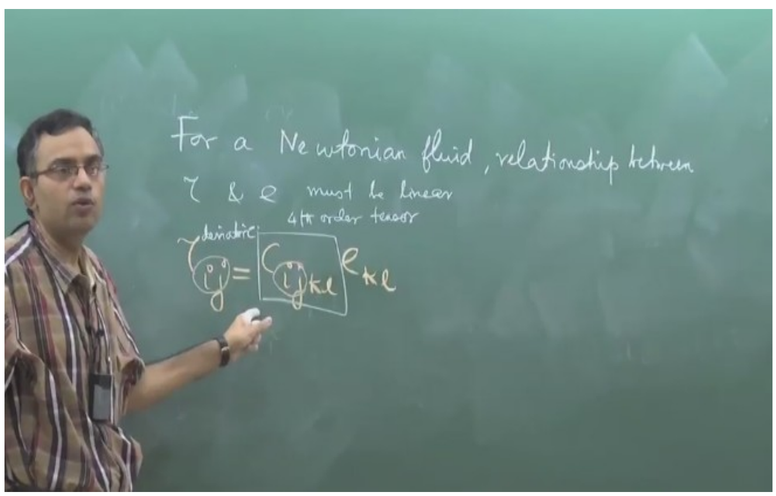

For a Newtonian fluid, the definition is that the tau is linearly related to e. So, for a Newtonian fluid relationship, between tau and e must be linear. So, what we mean by a linear relationship? Tau, what is this tau is a second order tensor. e is also a second order tensor so you must find out what is a linear mapping that maps the second order tensor on to a second order tensor. For example, when we discuss about the Cauchy theorem, we had the normal vector and the traction vector.

The normal vector was mapped to the traction vector by means of the second order tensor. So, a second order tensor map a vector on to a vector similarly there is something which will map a second order tensor e to the second order tensor tau and that is a forth order tensor. So, you can write –this. So, fourth order tensor, this is a forth order tensor. So, a forth order tensor will require four indices for its specification.

You can see in the left-hand side you are free indices i j so in the right-hand side also you have free indices i j there is summation over here and summation over l. Now what does it mean? It means that you can write the rate of stress in terms of the, this is tau i j deviatory if you must remember. This is not the total tau. This is tau i j deviatory. The deviatory component of the stress is related to the rate of deformation through the fourth order tensor.

That means if you know this tensor then given the velocity gradient you know what is the stress? But how many components of C i j k l you require? See each of this i j k l vary from one to three. So, three into three into three into three so 81 constants are in general required to describe the property of the fluid but I mean this will quickly come down not to 81 but to 36 because you have tau i j equal to tau j i.

So that will simplify it a little bit but still when we are describing properties of fluids like water air etcetera we do not use 36 material properties. We use possibly the most important property which is a viscosity. So, the number of properties as larger this is not used for solving a practical problem. So, to solve a practical problem we can come down to less number of properties necessary and that requires the consideration of a special type of fluid known as homogeneous and isotropic fluids.

So, we will discuss about the consequence of what is a homogeneous fluid? What is an isotropic fluid and how can be write C i j k l for homogeneous? isotropic and Newtonian fluid.we were discussing about the constitutive behavior of a fluid and we tried to generalize the concept. But at the same time, we tried to invoke a special consideration of linear relationship between the stress tensor and rate of deformation tensor and that kind of linear relationship is valid for special types of fluids know as Newtonian fluids. So, we will start with that.

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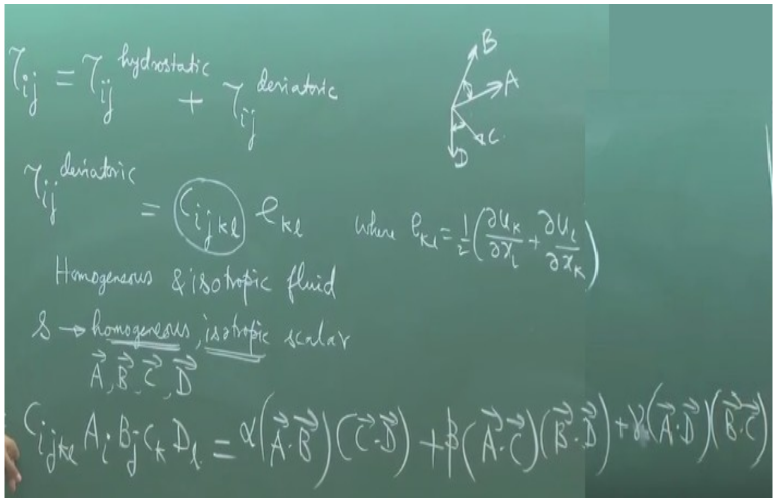

So, we had tau i j equal to tau i j hydrostatic plus. And tau i j deviatoric, we express as C i j k l which is a (()) (1:24) tensor times e k l where e k l is half delta u k delta x l plus delta u l delta k. Now, as we discussed that if we want to describe the behavior of a fluid in this way it will require three into, three into, three into, three into total 81 constant to specify the property of the liquid or fluid. But in general, we do not require that much number of properties.

One possible way by which we reduce the number of propertied required is by utilizing the symmetry of the stress tensor that is tau i j is equal to tau j i. In addition, we also make use of homogeneous and isotropic fluid. So, homogeneous and isotropic fluid. What is a homogeneous fluid? A homogeneous fluid is such a fluid whose properties are especially uniform, right. So, homogeneous means it does not depend on position.

So that means, if you have a property of the fluid that is same at all locations and isotropic means it is independent of direction. That means if you have a co-ordinate system and described by some co-ordinate axis. Let us say you rotate the co-ordinate system. For example, rotation, I am just talking about one type of transformation. It could also be translation, reflection these type of transformations.

So, if you rotate the co-ordinate axis the property description with respect to the rotated co-ordinate axis will also be the same because it is not dependent on direction. So, if that be the case then the number of properties can be reduced further. So, let us try to form a scalar s which is homogeneous and isotropic. Let us say, that our objective is to make a scalar out this fourth order tensor.

And what we have in our hand to convert the tensor to scalar is some vectors. So, how many vectors you require to covert this into a scalar that is the first question. How many vectors you require to convert this fourth order tensor into a scalar? You require four vectors. Why? Because let us say you have the vectors A, B, C, D. Now a vector is specified by one index. So, if you now write C i j k l into Ai Bj Ck Dl. This stands for vector A. This stand for the vector B.

This stand for the vector C and this stand for the vector D. Now, if you write in this way it means that there is an invisible summation over i because it is repeated. Invisible summation over j from 1, 2, 3 summation k and summation over l. Finally, it will have no index so it will be scalar. So, this is a scalar but we want it to be not just any other scalar but homogeneous and isotropic scalar.

So, first of all we will look into isotropic. So, isotropic scalar means what is important. Let us say you have vectors say A, B, C, D. When you have isotropic behavior that means if you now rotate this vector system by some angle then there will be no change in the property description that means that the property will depend on that angle between the respective vector which is preserved during rotation but not the absolute orientation of the vectors A, B, C, D.

But their relative angular position because rotation preserves the relative angular position. Whatever was the angle between A and B before rotation the same will be the angle between A and B after rotation. So that means that this will depend on only the angle between A and B. So, angle between A and B is given by the dot product of A and B. So, you can write this as a combination of –if you pair A with B then you will pair C with D.

How many such combinations are possible? You can pair A with C and then B will pair with D you can pair A with D and then you can pair B with C. I have not written the full expression I have just written what are the terms which will be governing the behavior. So, these are the only combinations possible see A dot B, and B dot A are the same. So, it’s not six but three combinations which are possible.

Now, it will depend on this combination so we can say that because of the linear behavior it is proportional to this times a coefficient alpha which you have to determine. This times beta and this times gamma. Where alpha, beta and gamma are some coefficients which because of homogeneous nature do not change with position. They are not position dependent. Now, we will be write this expression in index notion.

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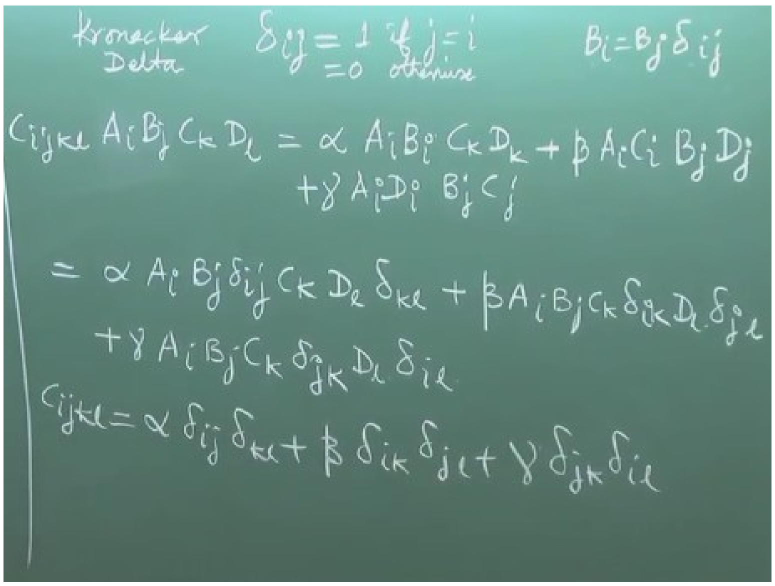

So, C i j k l is equal to, oh sorry, C i j k l into Ai Bj Ck Dl equal to alpha Ai Bi, A dot B you can write Ai into Bi, right then Ck Dk last beta Ai Ci into Bj Dj plus gamma Ai Di into Bj Cj. Now, I will show you a very interesting way of switching the indices. This I j k l you can beautifully switch. You can switch from i to j, j to k, k to l you can play with the indices. How you do that you use an operator delta i j which is called as chronical delta.

So, what it does delta i j is equal to one if j is equal to i and is equal to zero otherwise. This is called as Kronecker Delta. So, you can write Bi is equal to Bj into delta i j. Why? Because delta i j is equal to one if j is equal to i when j becomes i dj will become Bi and this will become one. So, you can see this is how you switch index from one to the other. So, using this, we will write alpha Ai Bj delta i j Ck Dl delta kl.

We are trying to write for all terms Ai Bj Ck Dl because that is what is there is in the left-hand side. So that we will get a expression for C i j k l. Then plus beta Ai Bj Ck delta what will be this? i k then Dl delta j l plus gamma Ai Dj Ck delta what will this? jk. Then Dl delta il. So, from here what we can write? We can write C i j k a l equal to alpha delta i j delta k l plus beta delta i k, delta j l plus gamma delta j k delta i l. Now, how many properties have come?

Magically from 81 properties, you are having three properties alpha, beta and gamma.