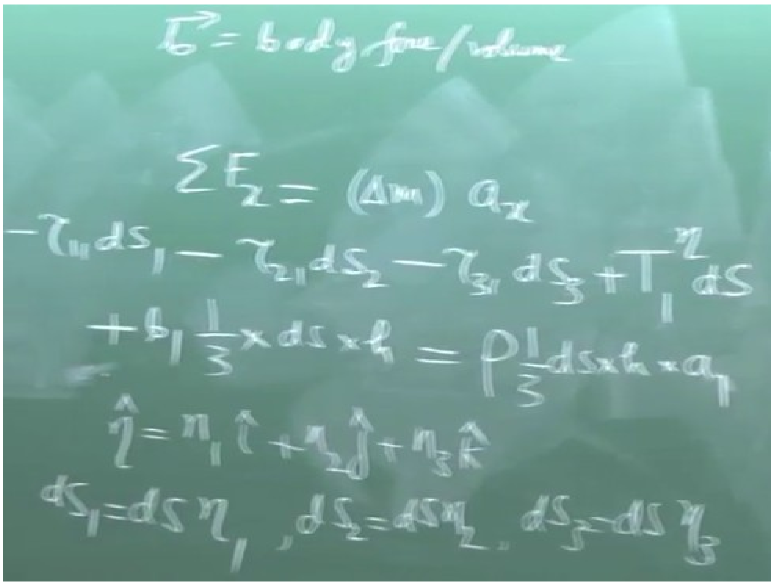
Conduction and Convection Heat Transfer
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 35
Review of Fluid Mechanics – III

(Refer Slide Time: 47:10)

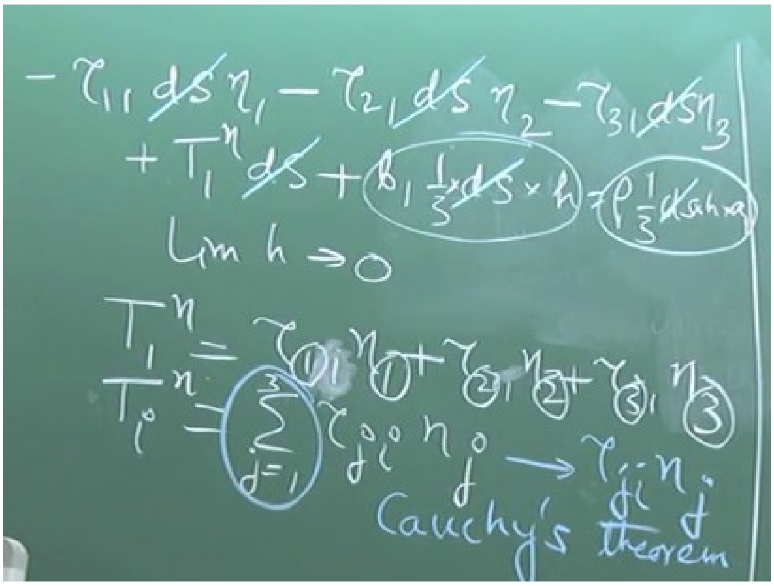


There is also a force, which is a volumetric force on a body force. We have discussed little bit about that, but usually body force is represented by typically either force per unit mass or force per unit volume. So, let us say that B1 or B is a vector which is body force per unit volume. So, B with subscript 1 is the component of the body force per unit volume along x. What is the volume of this element? Let us say that h is equal to perpendicular distance from o to ABC.

So, one-third into ds into h that is the volume, right, h is a perpendicular distance from o to ABC. This is the resultant force. This is equal to what? The mass, rho into acceleration along x, a1. Now let us write this eta as eta1i plus eta2j plus eta3k. eta1, eta2, eta3 are the direction cosines of the unit vector, which is normal to ABC. Then, you can easily see that what is the surface ds1? Ds1 is nothing but the projection of the surface ABC on the x2, x3 plane.

ds1 is the projection of the surface ABC on the x2, x3 plane, right. So, their normal components are such that you can say that ds1 is equal to ds into eta1, right. If you take a dot product of this with i, you will get eta1. Dot product of this with i means basically component of eta in the direction of i. Similarly, ds2 is equal to ds into eta2 and ds3 is equal to ds into eta3.

(Refer Slide Time: 51:31)



So, you can write minus tau11 ds eta1 minus tau21 ds eta2 minus tau31 ds eta3 plus T1ds plus B1(1/3) ds into h is equal to rho into 1/3 ds h into a1, right. Now you can cancel ds from all terms, so we cancel ds. So, you can then, what is our interest? Our interest is at this point o, we are trying to represent the force on the arbitrarily oriented surface in terms of the force on the other surfaces, at the point o.

So, at the point o, if we consider, which would take the limit as h tends to 0. So, take the limit as h tends to 0. So, if you take the limit as h tends to 0 what will happen, the entire volume will shrink to a point. The entire volume will shrink to the point o. So, if you take the limit as h tends to 0, this term will be 0 and this term will be 0. So that means even if there is a body force, even if the fluid element is accelerating.

It will not matter at the end for what we are deriving. It is true even if the fluid is accelerating, it is true even if it is not accelerating. It is true even if there is a body force, it is true even if there is no body force, because eventually these terms are getting cancelled. So, you can write T1 eta is equal to tau11 eta1 plus tau21 eta2 plus tau31 eta3. See this can be written as summation of tau-ji eta j, tau-ji eta j where if you use this index i instead of 1.

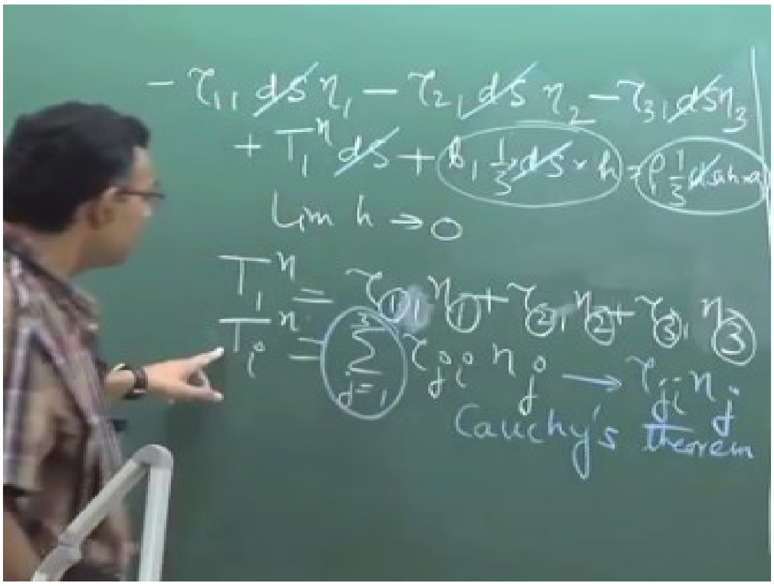
So, 1 is force balance along x. Instead of that you can make force balance in any arbitrary direction i, just replace 1 with i, so instead of force balance along x, you just write force balance along i. So, replace 1 with i and the remaining term summation over j, for j is equal to 1, 2, 3. This is j = 1, sorry this is i = 1. This is j = 1, this is j = 2, this is j = 3, okay. So, this is j – 1, 2, 3.

Einstein introduced a notation, when he said that when there is a repeated index like this, we will not use this summation, we will just write it as tau-ji eta j, with an invisible summation. Summation is not written, because there is a repeated index, the notation is that we will always consider as summation over the repeated index. This is Einstein's index notation and that is what is commonly used in the literature for describing these terms.

So, whenever there is any repeated term and there is no sigma, you have to keep in mind that there is actually a sigma. It is not represented. That is what Einstein introduced. So, this equation is a remarkable equation, which relates the traction vector on any arbitrarily oriented plane with the traction vector or stress tensor components on surfaces, which are oriented either along x or along y or along z.

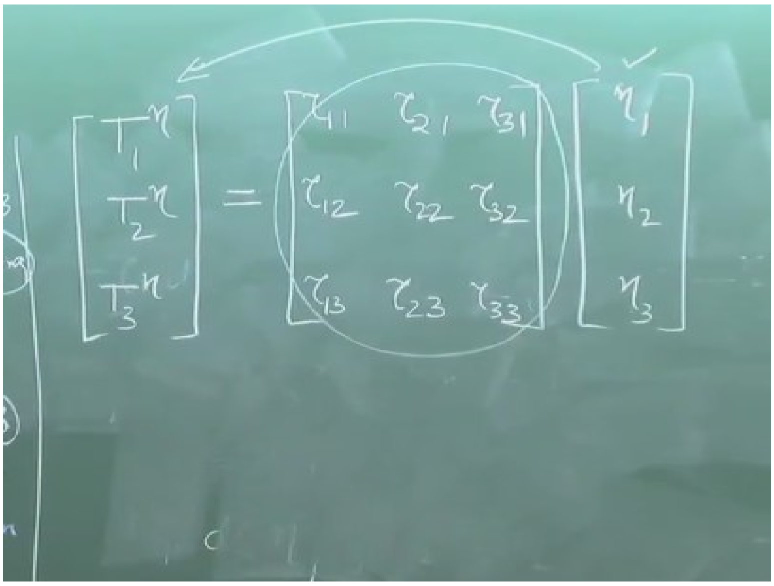
So, you can represent force on any arbitrary oriented surface, surface force on any arbitrary oriented surface in terms of the stress tensor components, which are specific to surfaces having normal either along x1 or along x2 or along x3. This in continuous mechanics is known as Cauchy's theorem, one of the very fundamental theorems and in the next lecture.So, we will write the corresponding expression which is already written here.

(Refer Slide Time: 00:46)



T with subscript i, subscript i is the direction of action of the force and superscript theta which is a direction normal of the surface arbitrary oriented surface in terms of sublimation of Tau j i eta j. So, we will be writing that now in a matrix form.

(Refer Slide Time: 01:20)



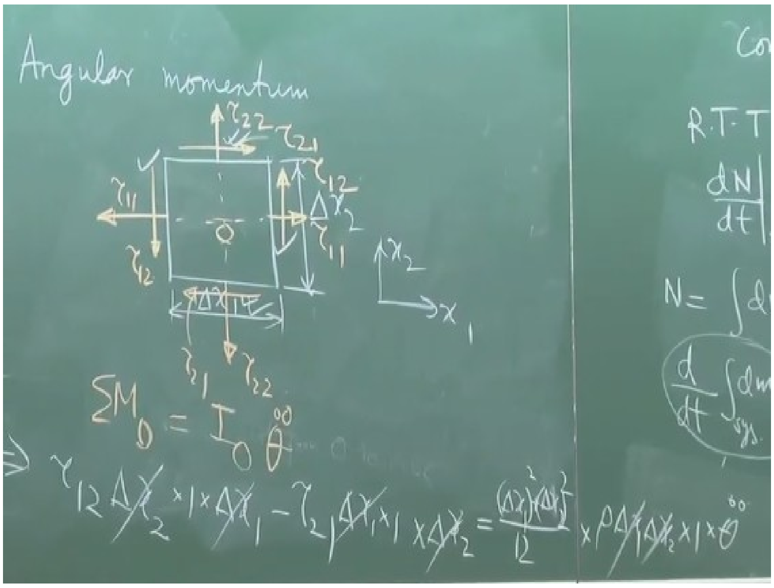
So, we can write the components. The first component you see T1 is equal to tau 1,1,1 eta 1 plus tau 2, 1 eta 2 plus tau 3, 1 eta 3. So, the first row of this multiply with this column vector. Similarly, T 2, the second row of this times the column vector and T 3 is the third row of this times the column vector. So, you can also represent the stress tensor components in the form of a matrix. It is another way of way representing it.

So, here you can see that this is a second order tensor we had already discussed and we had discussed the second order tensor map a vector on to a vector and that you can see from here. This is a vector this is map on to another vector by means of this second order tensor. If this is the case, then we say that the tensor is a second order tensor. It is capable of mapping a vector on to another vector.

Now the question is how many components are here total three plus, three plus, three, total nine components. But out of this total nine components, all are not independent and how they are dependent or how they are independent we have to figure out from a consideration which is conservation of angular momentum. So still now, we have used conservation of linear momentum we have partially done it.

We have not yet written a final form of the equation but we have never used the angular momentum conservation so that we will do in a moment.

(Refer Slide Time: 04:30)



So just like linear momentum relates to translation angular momentum relates to rotation. So, we will consider a simple example when we consider rotation in the x, y plan that is rotation with respect to the z axis. So, let us say that delta x1 in the dimension along x1 and delta x2 is the dimension along x2 of a fluid element. We have taken a two-dimensional fluid element purposefully to simplify the situation.

Because any rotation will always be having a plain of rotation and that plain of rotation let us say this x1, x2 plain. So, let us represent the forces. Let us say the center of this element is O. On this four faces, we have shown the relevant normal and the shear components of the forces. The tangential components are also known as the shear components which you know from basic mechanics.

What we are studying here is actually not anything different from what you have learned in basic mechanics but here we are learning the grammar of basic mechanics. Which is very important see I mean when you start learning how to sing you do not look into the notation you just simply spontaneously learn how to sing but if you want to become a very professional and learned singer or a learned musician then you have to at the end understand the notations.

So, without notations you cannot go further forward. So, what we are trying to do is trying to bring everything in mechanics that you have learned in the perspective of momentum conservation but in a bit of structured manner. So now we will write the equation of resulted momentum of all forces with respect to the axis, perpendicular to the plan passing through O is equal to the moment of inertia with respect to the same axis times the angular acceleration.

This is like equivalent to Newton second law for rotation. So, clearly you can see when you are talking of moment of forces the normal components of stresses will not matter because they are all passing through O. Only the share components will give rise to moments. What kind of moments? So, if you take these two share components they will form a couple. Therefore, their moment is couple moment so what is the couple moment of these two?

Tau 1 to, one is the force, tau is not the force. Tau 1 to times delta x to times lets us say width of the elements perpendicular to the plain of the figure is one. So, this is the force times the length of the couple arm is delta x 1. This is a clockwise couple or anti-clockwise? Anti-clockwise couple moment is there so positive. Then what about tau 2, 1? This one and this one they form a clockwise couple moment.

So minus tau 2, 1 into delta x 1 into 1 this is the total force times delta x 2. This is the couple moment. So, we have represented all the couple moments equal to moment of inertia. So, moment of inertia will be proportional to mass into some length square. So, by sum 12 into the mass, what is the mass? Mass is density times the volume. So, in place of delta y it is delta x 2, in place of delta x it is delta x 1.

So, delta x 1 plus delta x 2 square by 12 into rho into delta x 1 into delta x 2 into 1. This is like m into A square plus B square by 12 something like that. Whether it is by 12 or by whatever it does not matter actually for these derivation. Because we will see that these terms will be zero at the end so this into the angular acceleration. Now you take the limit as delta x 1, delta x 2 tends to zero because these are tending to zero and not equal to zero.

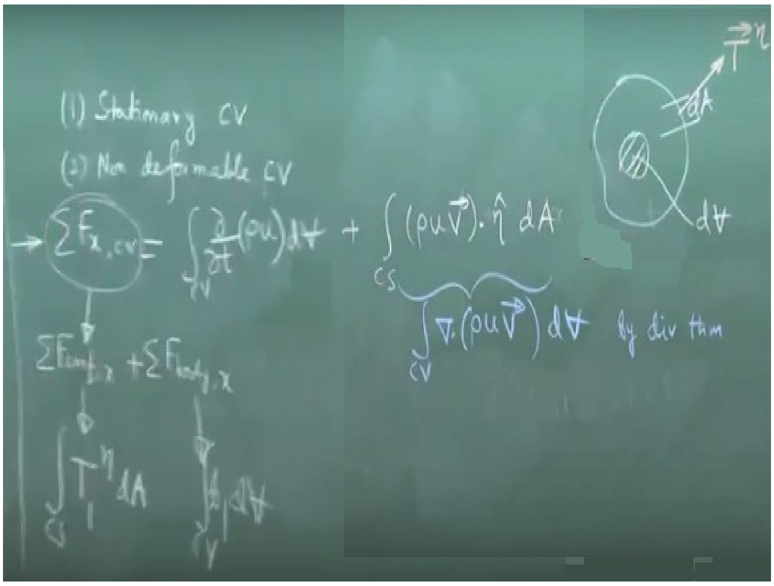
You can cancel it from both sides. This is the first thing. The second thing is that because delta x 1 and delta x 2 are tending to zero this term will into zero. So, from here we are left with one very important thing that is tau 1 2 is equal to tau 2 1. Or tau i j is equal to tau j i in general. Now here once we have derived this you can see that out of this nine components now you can say that tau 3 2 and tau 2 3 are equal, tau 1 3 and tau 3 1 are equal and tau 1 2 and tau 2 1 are equal.

So how many independent components are there in the stress tensor, six. So, you have six independent components in the stress tensor. So now the Cauchy theorem is also you can write as tau i j eta j. Now, one important thing is that while deriving this equation we have assumed that there is no body couple that means there is no couple which is acting on the volume of the fluid.

That is true in general for almost every fluid but there are exceptional fluids which can sustain body couples. One such type of fluid which is used in the research analysis is known as micro polar fluid. Micro polar fluids are certain types of fluids in which you have particles in the fluid which can induce certain rotation in the flow I mean not just in the flow but the fluid can sustain body couple.

Having a rotation in the flow does not mean that it can sustain body couple but the particular medium ensures that there the fluid can sustain somebody couple. So, if the fluid can sustain body couple then tau i j is not equal to tau j i. But here we are not considering that type of special fluid but in general fluid cannot sustain body couple. So, you will have tau i j equal to tau j i with this understanding now let us come back to this equation of motion. Or conservation of linear momentum that we discussed in the previous lectures.

(Refer Slide Time: 15:08)



So, if you can recall that with the assumption of stationary control volume and non-deferrable control volume we arrive at this form of the Reynolds transport theorem. Now what we will do is we will write the left-hand side the resultant courses. The resultant force is summation of surface force along x plus summation of body force along x. How do you represent the surface course along x? So, you consider

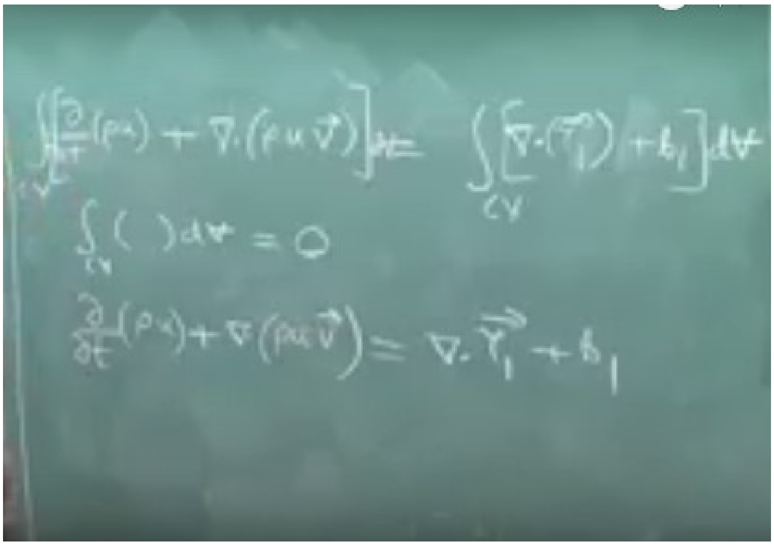
An arbitrary surface dA then the force per unit area on this arbitrary surface is what? So, you can write the surface force-- why this subscript 1 because we are interested for component along x. And what is the body force? Let us say you take a small volume and let us say b 1 is the body force per unit volume. So, b 1 is the body force along x 1 per unit volume. Now you can write T1 because Ti is tau i j eta j so T1 is tau 1 j eta j.

So, this we can write as tau 11 i plus tau 12 j plus tau 13k dot with eta 1 i plus eta 2 j plus eta 3. We had made a vector artificially made of course of vector like this which we represent in this way. Let us give this name tau 1. We can give any name just let us say we that we give a vector this vector I named tau 1. So, this is tau 1 dot eta. Why we have given it in this way is that in this term you are able to write this as f dot eta da so you can convert it into divergent f db.

So that you can convert it into volume integral. So, if you see right hand side both the terms are converted into volume integral left hand side body course is automatically volume integral. Only the surface force has to converted into volume integral and for that divergence theorem has to be used that is why we have manipulated this in this way so that it is expressed as a dot product. So that you can use the divergence theorem on this dot product.

So, you can write what we will do is we will write the right-hand side of this equation in the left hand and left-hand side in the right hand. So that is what is the standard way of writing, it in fluid mechanic.

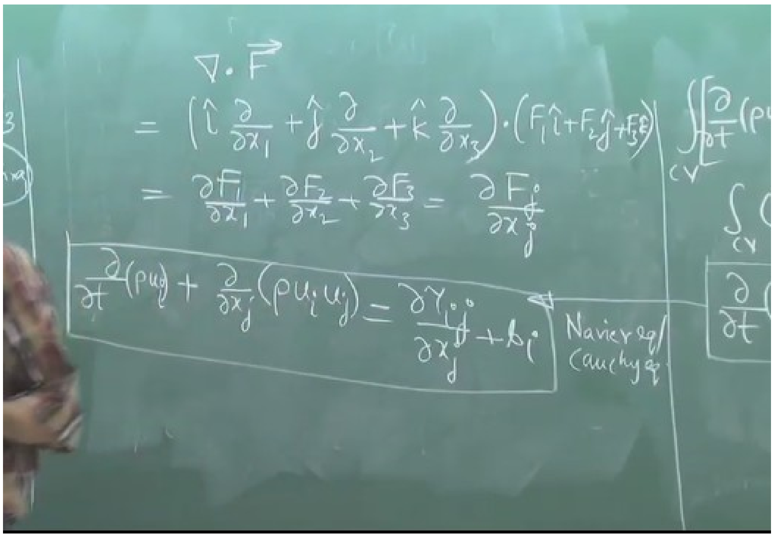
(Refer Slide Time: 20:05)



So, we will write integral of delta/rho t. So, this term is tau 1 dot eta dA it is divergence of the tau 1 dV that is what we have written here. So, this is as good as writing something dv, control volume is equal to zero. Again, the choice of the control volume is arbitrary that means this something must be zero. So, you have –so see we have derived the differential equation for fluid motion starting from the integral form.

It is written with vector as one of the essential components but we will use the index notation to write it in an alternative index notation so to do that –so see divergence of F if you write so what is divergence?

(Refer Slide Time: 22:46)



So, in index notation delta Fj/delta x j. So, we will use this term, use this equation right the index notation version of it. So, let us say that we are interested about before writing this equation let us say instead of x component of velocity we are interested about the highest components of velocity then u will be y and in place of tau 1, it will be tau i, instead of b 1 it will be b i. Instead of the x components, if you are interested about highest component just replace one with i that is all. If you add the index one, that was representing x component.

If you are interested about the highest component replace the index one with i that will give you the highest component. So now, if we are using this definition of divergence. This is alternative index notation of writing the same equation. This equation, I mean either the vector form or the index form this is known as Navier equation or Cauchy equation. What are the assumptions under which this equation is valid?

We will look into this equation very carefully but first what are the assumptions under which it is valid? See while deriving this equation first of all this is equation of what? Conservation, of what? Like continuity equation is conservation of maths. This is conservation of what? Again, I'm giving you four options linear momentum, angular momentum, both linear momentum and angular momentum none of the above?

Both linear momentum and angular momentum because we started with the linear momentum but we substituted tau i j equal to tau j i within the derivation that comes from angular momentum. So, it looks like a linear momentum conservation but angular momentum conservation inbuilt within the linear momentum equation. So, this we call as normally momentum equation in terms of differential calculus based representation of equation of fluid motion.

So, momentum equation but what are the assumptions that we had made? So, we had made two assumptions, two major assumptions one is stationary control volume another is none deformable control volume but both of them can be relaxed. Why both of them can be relaxed? Because if you considering a none stationary control volume and if you considering a none deformable control volume both of these can be manifested in terms of an extra body force that can be shown.

So, eventually stationary and non-deferrable control volume also can be relaxed because that will give rise to additional body force. For example, if you have an accelerating control volume that may give rise to centrifugal force so you can put that in the terms of body course. So, pretty general but one very important assumption there is no body couple. Because if there is any body couple then tau i j is not equal to tau j i.

So, the only and the most important assumption that goes behind this equation is that there is no body couple. Other than that, it is pretty general and why it is so important is because it does not refer to what is the fluid. It does not refer to Newtonian fluid, none Newtonian fluid whatever. So, you can use it for all class of fluid so long as you do not have to consider body couple.