Conduction and Convection Heat Transfer
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Lecture - 33
Review of Fluid Mechanics - I

So, when we say fluid dynamics, we have to keep in mind that fluid dynamics, to understand fluid dynamics, we need to understand basic equations of fluid mechanics and what are the basic equations of fluid mechanics. The basic equations of fluid mechanics at the conservation equation like conservation of mass, conservation of momentum and in heat transfer, you also have conservation of energy.

So, when we say the conservation of energy in the context of heat transfer, we essentially talk about the conservation law dictated by the first law of thermodynamics. So, energy, where you have both heat and what involved. So, we will start with conservation of mass, but before that we will try to recapitulate a general theorem, which talks about conservation in fluid mechanics and then we will apply that theorem to address the issues of conservation of mass, conservation of momentum and then eventually conservation of energy. And that theorem is Reynolds transport theorem.

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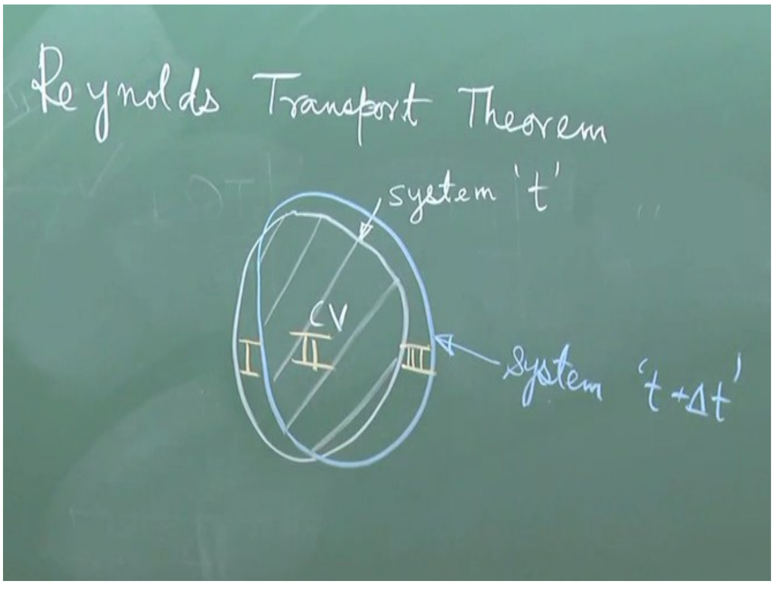

Again, everything requires a motivation. We should understand that, why are we interested about Reynolds transport theorem. I mean, as a teacher, I really say that any teaching should not start with, let us take a control volume, do this and do that all these things are fine. But, it is important to understand first that what you are trying to achieve with Reynolds transport theorem and why you are trying to do that.

See all the basic law of mechanics, classical mechanics are based on something on the concept which is control mass system, that is a system of particles of fixed mass and identity. If you think of Newton's Laws of Motion, the basic Newton's Laws of Motion they also talk about some identified particles on which you write, apply the basic law. So, you have a control mass system.

In a mechanics approach, it is also known as Lagrangian approach that is you track the motion of identified particles. The particles maybe solid particles, fluid particles whatever. On the other hand, in fluid mechanics you do not commonly rely on Lagrangian approach. Why? Because fluid has numerous number of continuously deforming particles and it is impossible to track the motion of individual particles just like that.

You may track the motion of few identified particles but the total description of motion of a fluid by identifying all the particles is difficult. So, instead what you do? Instead you focus your attention on a specific location. As if you were sitting with a camera and focusing the camera at a particular location and seeing what change is taking place across that. What fluid is coming in and what fluid is going out.

So, that approach is known as Eulerian approach or control volume approach. So, in fluid mechanics you are dealing with control volume approach, but the basic laws are all described on the basis of a control mass system. So, you require a transformation from a control mass system approach to a control volume approach, so that equations of motion can be written in terms of a control volume approach. And that transformation from control mass approach to control volume approach is given by a theorem known as Reynolds transport theorem.

That is why we need to study the Reynolds transport theorem. So, let us say that we have a system, which is basically a control mass system at time t. Let us say this blue colored outline is a system at time t + Delta t. This common region you can identify as a control volume that is a fixed region on which you focus your attention over the time interval Delta t in the limit as Delta t tends to 0. So, you can say that, this is your control volume. Now this entire region can be divided into several parts. Let us say this is zone number I, this is zone number II and this is zone number III.

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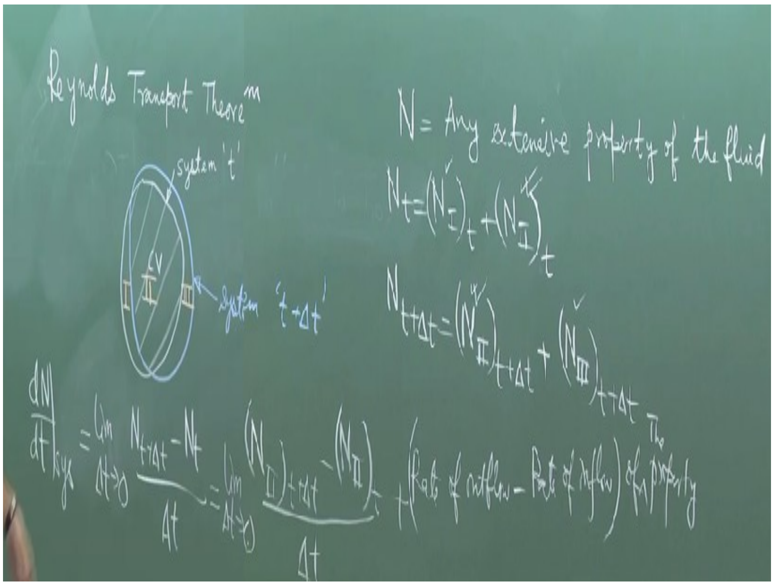

Let us say that capital N is any extensive property of the fluid. So, extensive property means a property, which depends on the extent of the fluid or mass of the fluid. So, N at time t is equal to N I at time t + N II at time t. Because at time t, the system is I plus II. Similarly, N at time t + Delta t is this. This is very straight forward. So, what we are interested is to calculate, what is dN, dt of the system?

So, by definition, this is limit as delta t tends to zero, N t plus delta t minus N t, divided by delta t. So, N t plus delta t, what is that, so it has two parts. One is two, another is three. And N t, it has also two parts, one is two, another is one. So, the two region is common for both, one has t plus delta t and another has t. So, we will isolate that. We will write limit as delta t tends to zero, N II t plus delta t minus N II t, divided by delta t plus N III at t plus delta t divided by delta t, what is that, that is the rate at which the property is flowing out of the control volume.

Remember that, as you are advecting in this way, this is acting like inflow and this is acting like outflow. So, this is plus rate of outflow minus rate of inflow of the property. Now the next job is to express this mathematically, rate of outflow minus rate of inflow of property.

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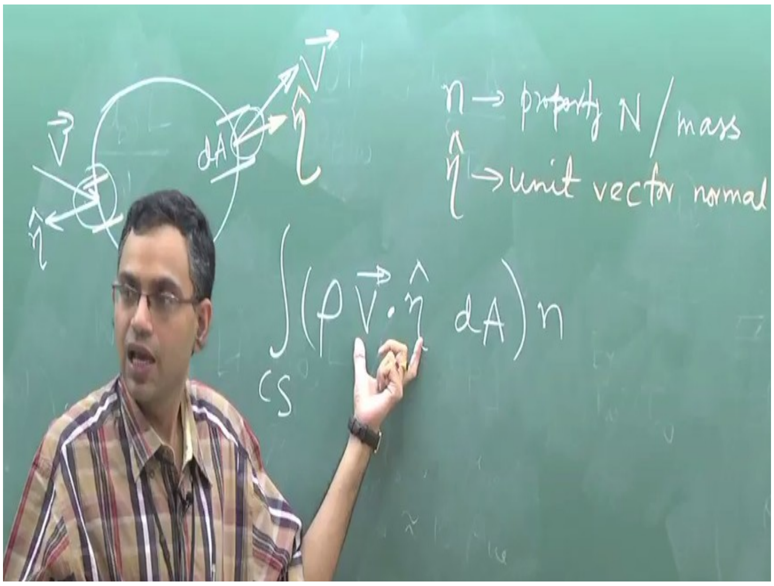

Let us make a sketch of the control volume, let us say this is an outflow boundary. Let us say dA is that elemental area on the outflow boundary. Let us say that V is the velocity of flow at this outflow boundary. And small n is the property N / unit mass. We also describe a unit vector, normal to the perceive. Let us say that Eta is a unit vector, which is, so what is Eta, unit vector normal (outward) normal.

Now can you tell what is the master unit to this dA. What is the volume flow rate to this dA. volume flow ratef is the normal component of velocity, time the area. So, what is the normal component of velocity? This is the normal component of velocity. Time the Eta is the volume flow rate. This time the density Rho is the mass flow rate. And the small n is the property / unit mass. So, this time n is the total rate of the property flow across the outflow boundary.

Let us think about an inflow boundary. Let us say this is an inflow boundary. That will be the difference between the inflow and outflow boundary. The only difference is that when you calculate v dot eta, the dot product will be positive for outflow and negative for inflow. That you can see from this figure. The dot product of these two vectors is positive here, but the dot product of these two vectors is negative here.

So, if you write outflow minus inflow, you do not have to separately take two terms. It is now integrity to what the entire control surface, a positive dot product will mean the outflow part of the surface and negative dot product will mean the inflow part of the surface. This CS is control surface, the surface that bounds the control volume.

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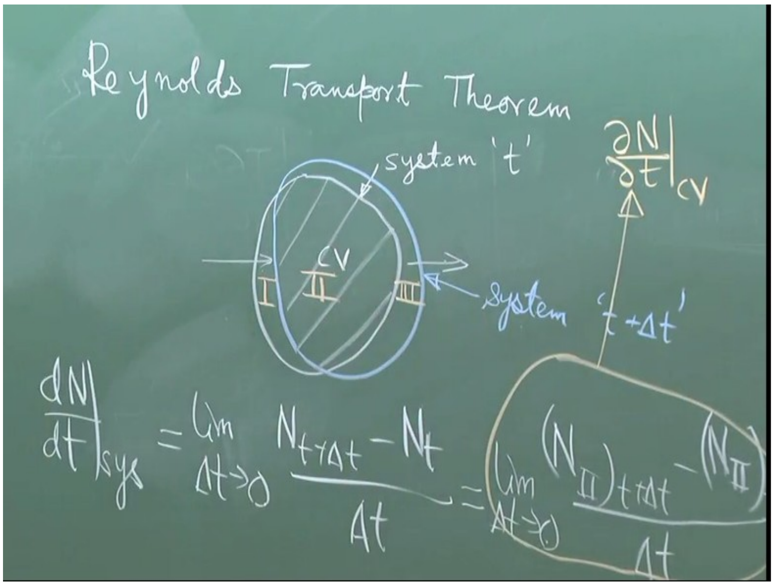

And the other term, which is this term, we can write this term as partial derivative works N with respect to time within the control volume. Why partial derivatives? Because you are fixing your location to and at the same location, you are finding the theory of property, with respect with to time. So, change of property, with respect to time, at a fixed location, that is why partial derivative and not ordinary derivative.

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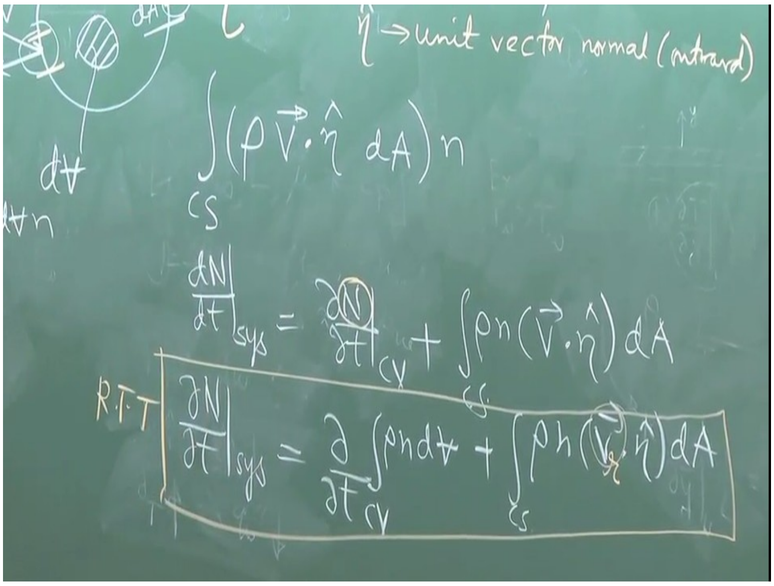

So, we can combine that and use in this expression to write dN, dt of the system. What is capital N of the control volume? Let us say this is a small element dC, a volume element that we get. What is the mass of this element? Rho * dV. What is the property for unit mass? Small n. So, the total property is this * small n.

So, this you can write as, there is one important point to note here, is that, what is this V. Let us say that it is control volume at the tank. Let us say that there is a tank, which is moving towards the right, with a velocity of one meter per second. And let us say water is also moving towards the right, at a velocity of one meter per second. There is a hole in the tank. Question is, what is the rate at which water is coming out of the tank?

There is a tank, in which there is a hole. The tank is moving towards the right with the velocity of one meter per second and water is also moving towards the right at a velocity of one meter per second. Then what is the rate at which the water is coming out of the tank. Zero. Because, the tank is moving at one meter per second, in the same direction water is also moving with one meter per second.

So, there is no relative velocity of water with respect to the tank. So, the flow across the control surface depends on not the absolute velocity, but the velocity of the fluid relative to the control volume. So, this V, must be V r, velocity of the fluid relative to the control volume, not the absolute velocity. So now, see we have been successful, in deriving a general relationship, where the rate of change with respective to a sys scale, this is the Lagrangian parameter, is described in terms of rate of change with respect to a control volume. That is an (()) (50:48) parameter. And this is known as Reynolds transport theorem.

In the next five minutes of course, we will illustrate the use of Reynolds transport theorem for the law of conservation of mass. We will start with, as I told that in fluid mechanics, we are interested about some basic laws of conservation and the most fundamental law of conservation is the conservation of mass. So, we will start with that. The first example is conservation of mass.

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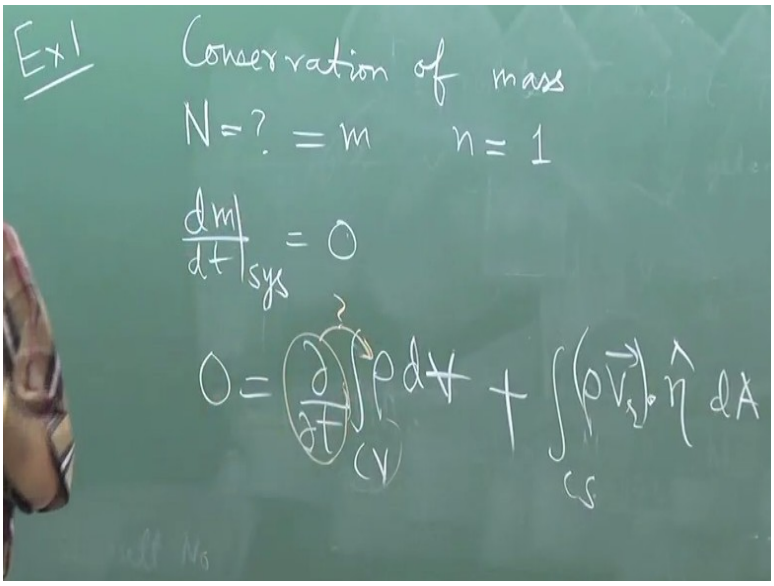

For conservation of mass, what is capital N? It is the total mass of the system. What is small n? One. Let us look at the previous state. So, what is dn, dt of the system? By definition, a control mass system, has a fixed mass. So, what is the rate of change of a mass of a system? Zero. So, this is zero. Now to simplify in the next stage, there is an immediate tendency of taking this derivative inside the integral. Question is, can we do it or not?

The answer is very straight forward. We cannot always do it. When we can do it? We can do it, only when the volume of the control volume is not a function of time. Then you can put this time derivative, outside and inside the integral, as per your wish. Otherwise, you have to correct, when you differentiate outside the integration and when you differentiate within the integration. They are not the same and they have to be corrected by an additional term, which is given by the Leibniz rule, of differentiation under integral time.

So, it is not that we can do it always, we can do this, we can put this inside, provided the volume of the control volume is not a function of time. So, when the volume of the control volume is the function of time, let us say that there is a balloon, which is continuously expanding or contracting whatever, so for solving a fluid mechanics problem in that, we cannot put this derivative within the integral.

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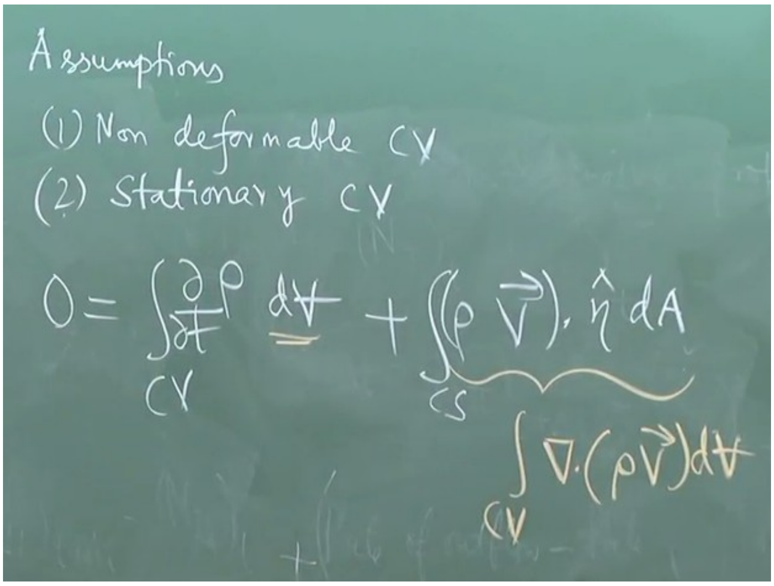

So, we will make certain assumptions. One is non-deformable control volume. That means d is not the function of time, volume of the control volume. Second is stationary control volume. Stationary control volume means that the control volume is fixed. If the control volume is fixed, then relative velocity or absolute velocity are the same. So, under those conditions, you can write.

Finally, this is the volume integral. So, we can convert this into a volume integral, by giving the divergence theorem. So, we can write. So, if you have a vector function s, s dot, say ds is equal to divergence of s dv, where the surface is over with the integral is made, is a surface that completely bound the control volume. By definition, the control surface is the surface that completely bounds the control volume.

So, this is a very important pre-requisite for the divergence theorem to be applied. So, when that is there, you can convert the area integral to volume integral. Why? What is the motivation? The motivation is that you, both of this trans are now having the volume integral.

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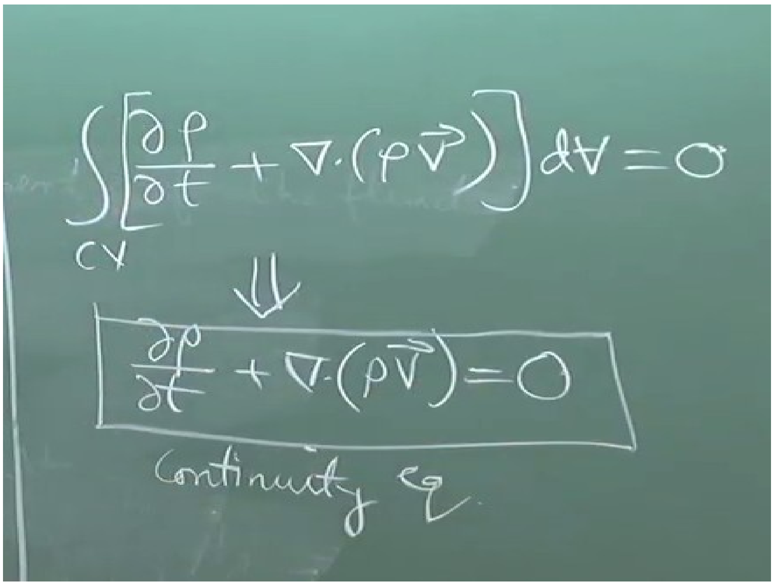

So, you can write the integral of. Now let us look into a situation. You have integral of fx, dx equal to zero. Does it necessarily mean that fx has to be zero? If it is a definite integral, not. For example, you may have a sinusoidal function, so y is equal to sin x over a period, integrated over a period, the integral will be zero.

That does not mean that sin x is equal to zero everywhere. But when this is true, for any arbitrary choice of the domain, then the function must be zero. So, this is true, for any arbitrary choice of control volume. That is what is very important. Not just a specific choice, but for any arbitrary class of the control volume and that means that, this must be equal to zero. This is nothing but the continuity equation in fluid mechanics. So, the continuity equation, physically talked about conservation of mass.

We will discuss in the next class, some implications of the continuity equation and then some special cases, what does it lead to for incompressible flow, may be for steady flow, unsteady flow with certain situations, we will discuss and then we will use the Reynolds transport theorem for a second example, which is conversation of momentum. And from that, we will derive something which is known as Navier-Stokes equation. That we will take up in the next lecture. Thank you.

In the previous lecture we were discussing about the continuity equation and we will take it up from there. So, the continuity equation was derived from the Reynolds transport theorem by applying the principle of conservation of mass.

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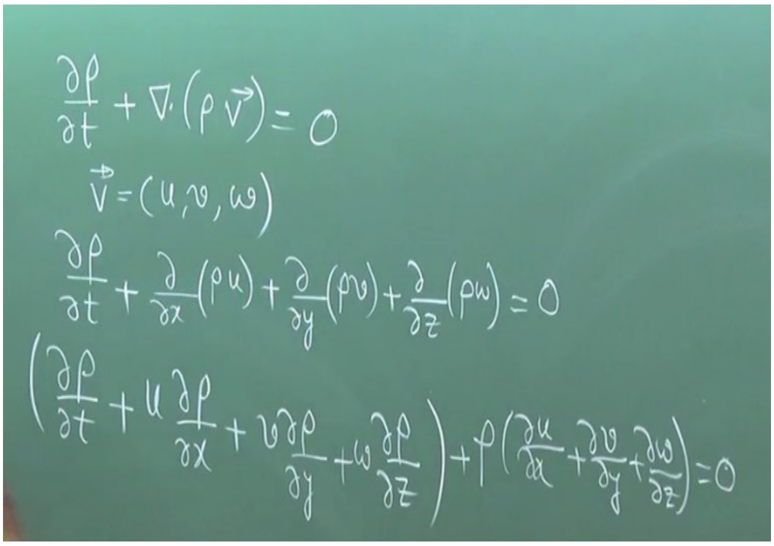

Now, we will try to develop a bit of insight into this equation. Let us say that you are considering a Cartesian system, where the velocity vector will have three components, u, v, and w, u along x, v along y, and w along z. So, it is possible to write, so you can write this as, what we have done is we have expanded each term in terms of the product rule of the derivative. So, we have clubbed three terms here and the remaining three terms with rho.

Now, we would be interested to consider the special significance of these terms, okay. To understand that, let us take an element with the length of delta x along x, delta y along y, and delta z along z. Overtime, it is possible that this element, this fluid element in this special case of consideration of a fluid will deform.

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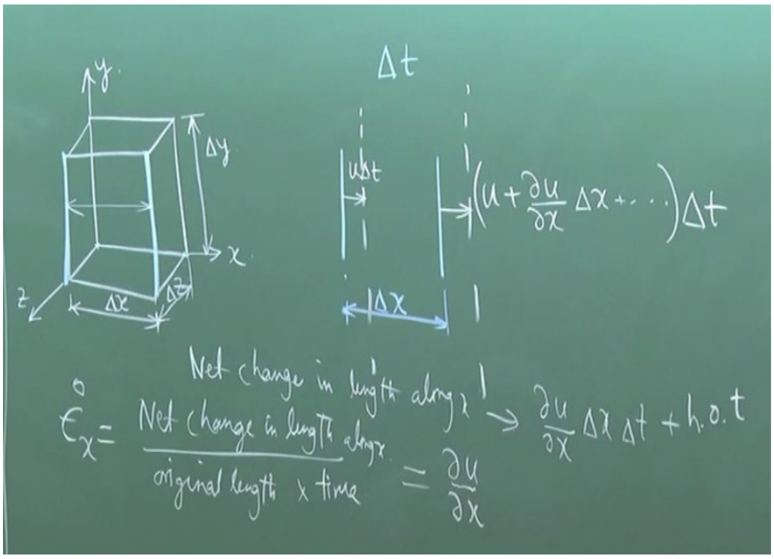

Now there are virtually possible mechanisms of deformation. But we can talk about two types of deformation broadly, one is linear deformation, another is angular deformation, these two types of deformation, we can talk about. So linear deformation will give rise to a change in volume and that is what is the matter of our interest to calculate.

So just for clarity, let us draw these two lines separately, these two vertical lines, which I have marked with blue colour, let us draw them separately. So, they were at a distance of delta x apart. Now let us say that a time delta T has elapsed, over a time delta T this line element will go along move along x by a distance of what? What is the displacement of this line element, u into delta T, where u is the velocity along x, right.

This line element will also be displaced, so it will be displaced by an amount, what is the displacement of that? The displacement of this is u into delta T, what is the displacement of this, what is the velocity of this line. If the velocity of this line is u, what is the velocity of this line, u plus del u del x into delta x plus higher order terms in the teller series, this into delta T.

So, what is the net change in the length, this was the length along x, what is the net change in length. That is the difference between these two displacements, that is the net change in length. So net change in length along x that is, plus higher order terms. So net change in length per unit original length per unit time, what is this? This is equal to, because if you now divide both sides by delta x and delta T and take the limit as delta x delta T, all tends to zero, the higher order terms will be tending to zero.

So, this is nothing but what, this is the so-called strain rate of the fluid element along x, epsilon dot x. For fluid, it is the rate of strain that is important and not the strain, because fluid deforming. So, if you allow more and more time, it will be more and more strain. So total strain depends on time, but the rate at which it is straining is more important, because that gives how fast or how slow it is changing in its dimensions.

Now this is about the linear strain, what about the volumetric strain. There is a volume element here, what is the change in volume.

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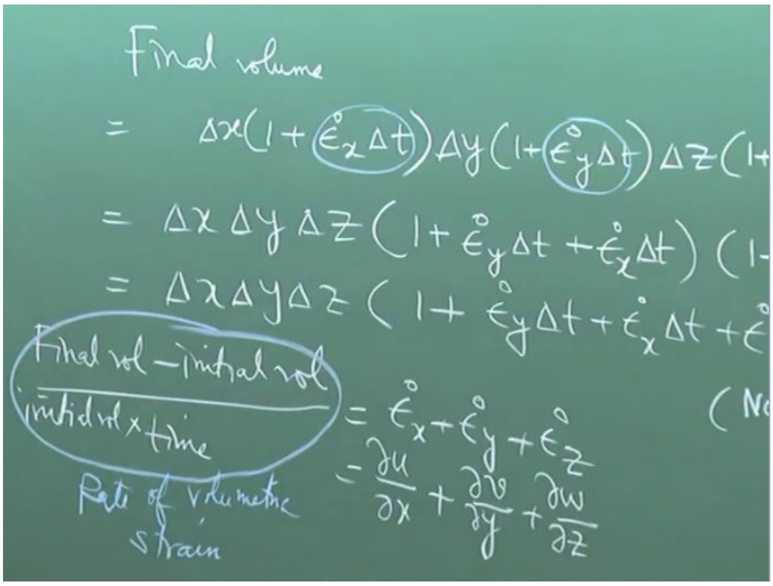

So, let us write the final volume, this is the new delta x, right. This is the original delta x plus the change in delta x, right. So, this is the new delta x. Similarly, you have a new delta y, and you have a new delta z. So, you can simplify this a little bit, so while simplification we have neglected the product of these two terms. That is of the order of delta T square as compared to the other terms, which are of the order of at the most delta T.

Again, we can do one more step, so all these are neglecting of the order of delta T square, okay. So final volume minus initial volume by time, divided by initial volume, what is this? What is this physically? Physically this is the rate of volumetric strain, final volume minus initial volume divided by initial volume divided by time. So, if you subtract the initial volume and divide by this and time you will get epsilon dot x plus epsilon dot y plus dot z, okay.

So, we have shown that epsilon dot x is del u del x, similarly epsilon dot y is del v del y and epsilon dot z is del w del z, okay. So, this what does it indicate, this is rate of volumetric strain so it is clear from here that this term, which is put in the square bracket represents the rate of volumetric strain.