Conduction and Convection Heat Transfer
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Lecture - 32
Introduction to Convection

So, today we will get started with the foundations of convective heat transfer, the mathematical foundations of convective heat transfer. Now, the mathematical foundations of convective heat transfer strongly depends on heat mechanics. And this is so because convection may be fundamentally perceived as advection assisted conduction. So, you have a situation, where conduction is a must, but that is assisted by something, which is advection or transport of heat by fluid flow.

So, to get a physical picture of why fluid flow is important for heat transfer, let us consider a situation like this.

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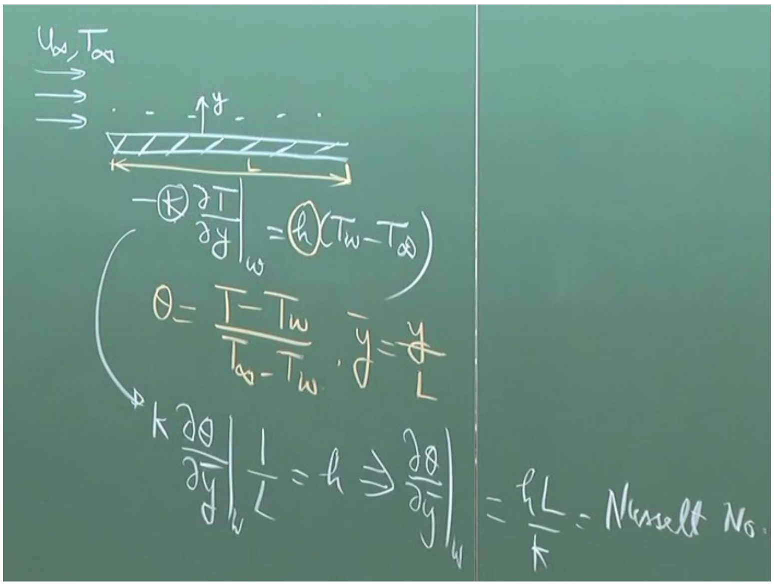

Let us say that you have a solid boundary and fluid is coming from fast stream with a velocity U infinity and temperature T infinity. Now, to write what is the heat transfer at the wall, we express it by this following boundary condition. We have discussed earlier that this boundary condition is valid for both steady state and unsteady state and only important assumption is that radiation is neglected, otherwise there will be a radiation term along with it.

Now, in convective heat transfer we are mainly interested to find out what is h. If we know what is h, then we know, what is the rate at which the heat is been transferred at the wall. The question is h will depend on what? What are parameters on which h will depend? So, you can of course, non-dimensionalise this equation and write, if you define a non-dimensional Theta, non-dimensional temperature Theta and non-dimensional y, y / L, where you can write this equation.

So, what you get is the non-dimensional temperature gradient at the wall, because this is non-dimensional, this is also a non-dimensional parameter, which is known as Nusselt number. It is very important to understand that there is a fundamental difference between this Nusselt number and the bio number that we have talked about in conduction heat transfer. We will come into that. Now, we will see what the parameters on which we depend and we will bring up context and we understand that how fluid mechanics relates to heat transfer.

So, my first question is what is this K? Is this K solid or fluid? This is my first question. You have a solid wall and you have fluid at the top of it. So, this k, is it solid or fluid? So, sometimes as I often discuss that these days it is easier to get an answer if we give in multiple choice mode. No 1. K are solid, No 2. K are liquid, No 3. K are either solid or fluid, No 4. None of the above. So, three choices K are solid, K are fluid, K are solid or fluid.

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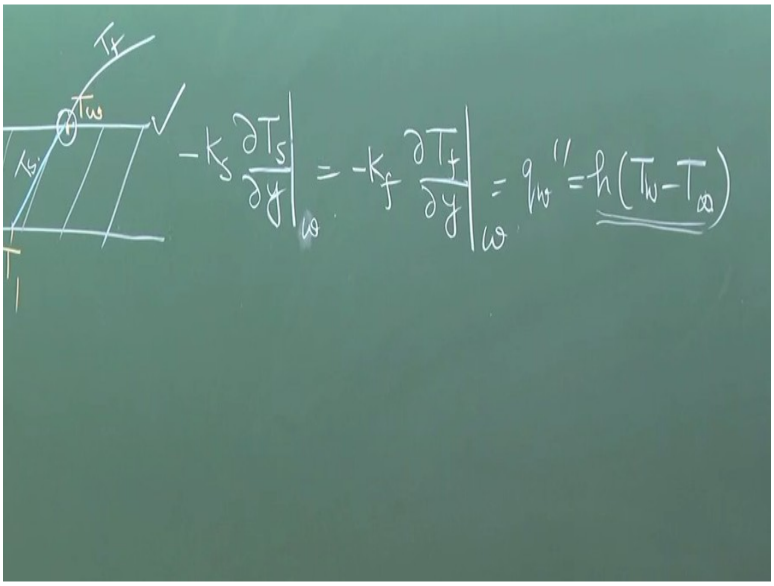

I am here enlarging a solid boundary, solid wall, this is a solid wall. Let us say this temperature is T wall (T w), which may be a function of x, this is x axis, T wall (T w) may itself be a function of x, may not be a function of x, so if this is T wall (T w), let us take an example that T wall (T w) is greater than T infinity. We can take two interesting examples, one is T wall greater than T infinity and another is T wall less than T infinity**.**

T wall equal to T infinity is not an example, because then there won't be any temperature difference between the wall and the fluid. Let us say this temperature is T 1. If it is steady state what will be the temperature profile between T 1 and T wall, liner temperature profile, right? Let us draw it with some color. What will be the temperature profile in the fluid? In general, not linear, if it is a system, where there is only conduction that means the fluid is stagnant, then it may be possible at the temperature profile in the fluid is also linear, but in general it is not so.

So, let us say that the temperature profile in the fluid is something like this, so, importantly what is common at the interface, what is continuous at the interface between the solid and the fluid, No 1. Temperature is continuous. Because at a given point, you cannot have two different temperature, one in the fluid, one in the solid. So, at a given point you must have unit temperature. So, there must be continuity of temperature.

And the second, the heat flux must be continuous. The rate at which it is transferred from the solid to the fluid here, is the same rate at which heat leaves to the fluid. The reason is that this is an interface, interface doesn't have any thermal storage capacity. So, whatever heat comes at the interface at the same rate it will leave. So, what is the rate at which it is coming at the interface? Minus K. S for solid. We will use S for solid and F for fluid.

This is the rate at which this temperature profile, this is T s and this temperature profile is T f. So, this is equal to the heat flux at the wall. Wall means interface not the bottom at this location. This is equal to h * T wall - T infinity. This is nothing but the so-called Newton flow of cooling. So, this is always valid except when radiation is present, a radiation heat flux has to be added.

Now, can you give an answer that whether it is K of solid or K of fluid? See it all depends on whether you are using the temperature profile of the solid or the temperature profile of the fluid. If here, you are using the temperature profile of the solid, then this will be K of solid. If here, you are using the temperature profile of the fluid, it will be K of fluid.

But, in convective heat transfer, our interest of domain is the fluid domain; therefore, we are interested to find out the temperature profile in the fluid and the temperature gradient in the fluid profile. So, if we are interested to do that means this is K of fluid. But fundamentally there is no restriction that it will be K of fluid. It depends on it is T of what?

If it is T within the solid, then this is K of solid and if this is T within the fluid, it will be K of fluid and you can see here that because in general K s and K f are different, therefore the slopes of these two lines are different. These two lines have same unique value at their points of intersection but you do not have the same slope because K s and K f are in general different.

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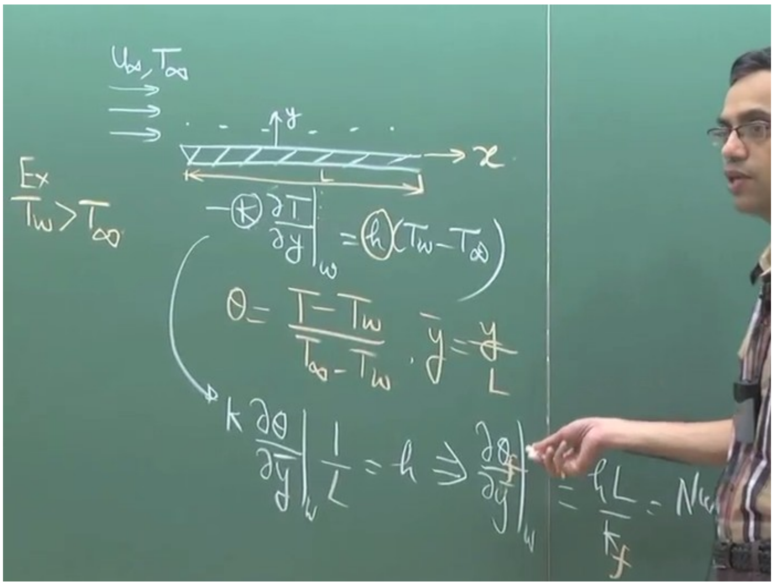

So, important thing is that if this Theta is Theta of the fluid. Now we write here the Theta of the fluid, then this is K of fluid. So, non-dimensional temperature gradient of the temperature profile within the fluid is equal to hL / K of the fluid. In the bio number definition, the K was K of solid and in the Nusselt number definition, K is K of fluid. So, typically we can say that Nusselt number is therefore a representation of non-dimensional temperature gradient in the fluid at the interface between the fluid and the solid.

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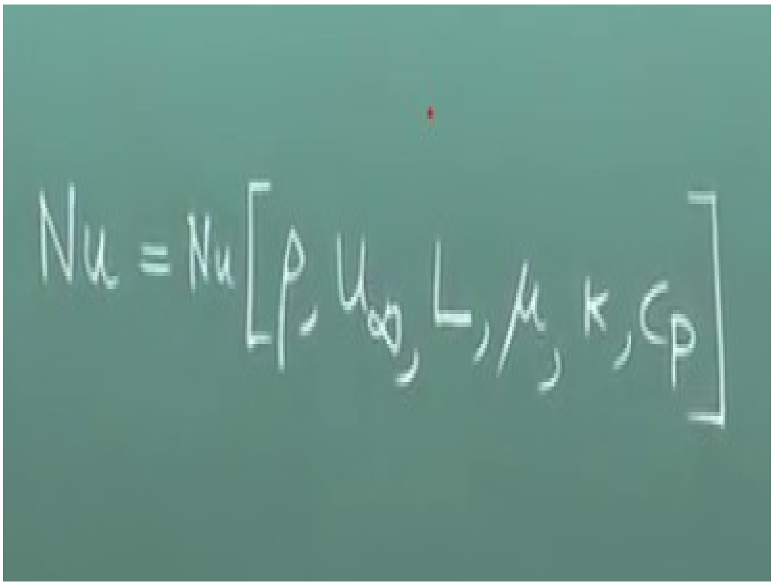

So, it will depend on what? Intuitively the Nusselt number will be a function of, these are the parameters like; out of these what are the fluid mechanic parameters? The fluid mechanic parameters are Rho, Mu infinity, L and Mu. And we will discuss about these parameters, why these parameters are important? And the non-fluid mechanic parameters are K, Cp and partly Rho also. Rho partly contributes to the fluid dynamic parameters and partly contributes to the thermal parameters.

So, the question is that why these parameters are important? These parameters are important because of the following reasons, let take an example. Let me ask you a question if U infinity is increased, do you expect a higher heat flux at the wall or lower heat flux at the wall? The question is if you increase U infinity, do you expect a higher heat flux at the wall or lower heat flux at the wall?

You expect a higher heat flux at the wall; the reason is like this, see what is happening with higher U infinity, there is a higher rate at which heat is getting transferred by fluid flow. To compensate for that to replace, more heat is to be supplied from the wall. So, let us say that there is some person who is a voracious eater, who always eats. And after eating the person becomes more and more hunger like a diabetic patient.

So then, to make that person happy you have to supply more fruits otherwise that person will not feel happy. So, this is something like this, the fluid flow is carrying heat at a very fast way. So, to make up for that more heat has to be supplied from the wall. So, that is one of the very important parameters. Then the other parameter L is of course important because the entire characteristic length scale of the problem is governed by the length scale.

Mu and Rho, the combination of Mu and Rho what does it play a role? So, it is essentially not Mu or Rho independently but Mu / Rho, which is known as kinematic viscosity. Now, the question is why Mu / Rho is important and why not Mu individually and why not Rho individually? So, what does Mu do and if you think of this typical problem, let us draw a separate sketch here.

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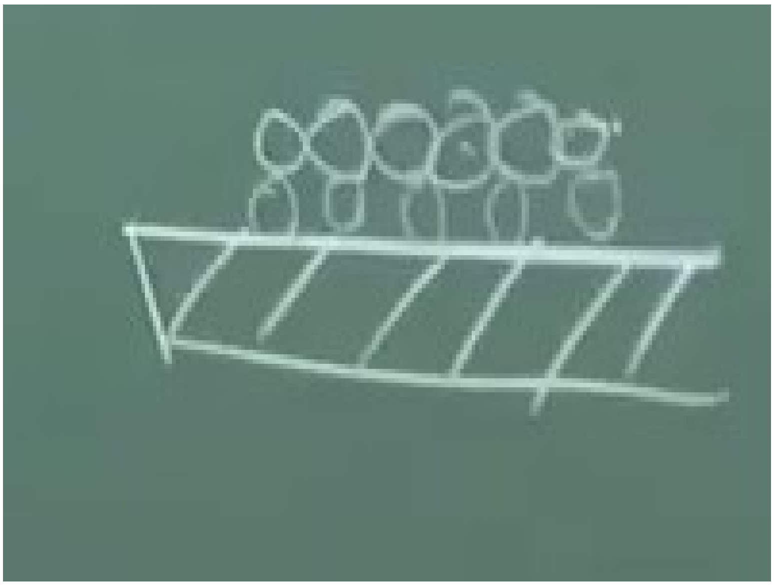

Let us say this is a wall, the first molecule, which are adhering to the wall will directly feel the effect of the wall so if the molecules are significantly compact, then they are almost like trapped by the attractive interactions of the wall and they cannot escape from the wall. This in fluid mechanics, in classical fluid mechanics is known as no sleep boundary condition. There is no relative tangential component of velocity between the fluid and the solid boundary.

Of course, there are many situations in micro fluidic and Nano fluidic, where this no sleep boundary condition is violated, but we will not discuss about that in the basic course. So, let us say that there is a situation, when the no sleep boundary condition is valid and the molecules are sitting to the wall. Now, these molecules are sticking to the wall because they directly understand that there is a wall. Now, consider the second layer of molecules on the top of these one.

These second layer of molecules, what happens to them? Does the second layer of molecules move with the velocity Mu infinity? No, it moves with a velocity much lower than Mu infinity but not zero. So, it moves with the velocity, which is much lower than Mu infinity but faster than this, that means why it is not moving at the velocity Mu Infinity. Because, it understands that there is a wall.

How does it understand that there is a wall, it is not in direct contact with the wall? So, how does it understand that there is a wall? There is a messenger in the fluid that propagates a message that, yes there is a momentum disturbance and that messenger in the fluid is nothing but viscosity of the fluid. So, viscosity of the fluid is physically a messenger for momentum disturbance. It gives a message that there is a momentum disturbance in the fluid.

Now, the fluid gets the message that there is a momentum disturbance, but the fluid has its own inertia. So, even though there is a momentum disturbance the fluid intends or tends to maintain its whole momentum. By virtue of which property of the fluid, by virtue of inertia fluid. By virtue of inertia of the fluid, the fluid tends to sustain its momentum and inertia of the fluid is proportional to the mass and mass is proportional to the density.

So, Mu is the tendency of the fluid to undergo a change in momentum or disturbance in momentum and Rho is the tendency to sustain the momentum. So, this is a relative ability of change of momentum with respect to the tendency to sustain its momentum. And that is why it is important as a basic feature. And you know it is very interesting that, there are different types of fluids which grossly vary in Mu and Rho, but in terms of the property Mu / Rho, there are many apparently different types of fluids which has close value of the kinematics viscosity.

If you go to steel plant, you will find that there is flow of molten steel in steel plants. Now to understand the fluid Mechanics of molten steel, many times in the R and D divisions of Steel plant what they do is they make water models that is instead of studying the behavior of Steel, they are studying the behavior of water in the transparent box. The question is that why it is being done like that?

Of course, the practical constraint is that molten metal flow at a very high temperature it is very hazardous to dilute it, it is very hazardous to handle. So, it is very difficult to do fluid mechanics experiments by visualizing that. Not only that molten metal, most of the cases they are not transparent medium. So, you cannot visualize the flow. So, there are practical reasons, but the question is that why you cannot visualize molten metal, how you can replace water with molten steel.

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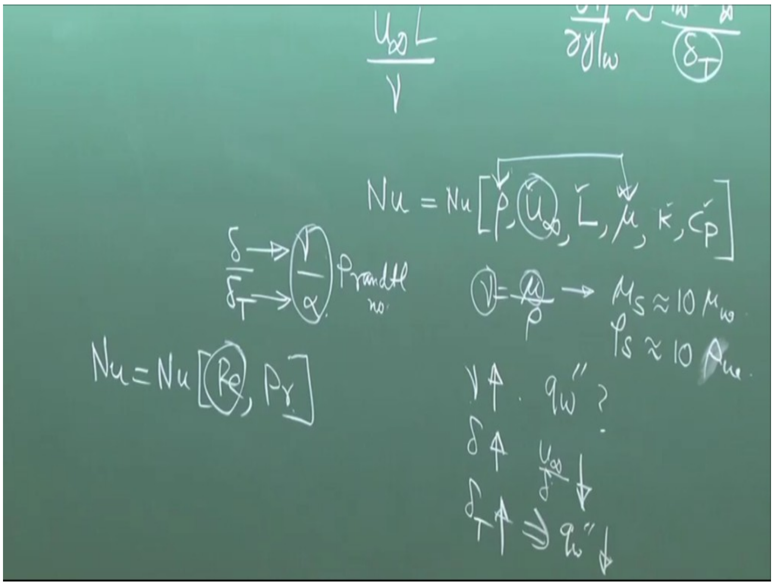

One of the basic reason is that typically Mu of steel is roughly 10 times Mu of water roughly, one order and Rho of steel is also roughly one order more than that of water. So, Mu / Rho of steel is roughly Mu / Rho of water. So, this is very remarkable behavior that all the density and viscosities of water and steel are grossly varying. But kinematic viscosity of water and kinematic viscosity of Steel are roughly very similar.

Now that make water models work but only when heat transfer effect is not very important. When heat transfer effect is important you also have to consider the thermal diffusivity, and thermal diffusivity of steel and water are grossly different. So, when heat transfer effects are very important just by these similarities you cannot say that whatever will happen for molten Steel be the same with water.

Because the thermal diffusivity wise, they are different. But, purely from fluid mechanics configuration this is the similarity. Now, we have to understand that what will be the role of this Nu that is, if you increase this Nu what will be the wall heat flux? Will it increase or decrease? See what we are trying to do, we are trying to understand physically first? influence of fluid mechanic parameters on heat transfer.

Because that will give us the motivation of why should we study fluid flow equations in the context of heat transfer. In the context of Fluid Mechanics, we study fluid flow equations, fine, but why in the context of heat transfer we should study fluid flow equations. So, U infinity role we have understood, so now we have coupled these two and we are trying to see that what is the role of this Nu? Now, if Nu is high, what will happen?

See Nu will dictate that how far from the wall you should go to assist, to attain U infinity. The distance from the wall that you go to attain close to U infinity that is known as the boundary layer thickness. So, boundary layer thickness can be anything from very small to very large. So, the question is that when Nu is increased then, what do you expect the boundary layer thickness to be large or small?

If Nu is large that means, if Nu is increased then what is the distance up to which the wall effect is faced? The distance is more. So, more and more fluid understands the effect of the wall if the fluid has higher kinematics viscosity. Because the momentum disturbance is very strong. So, if that means, if Nu is large then the boundary layer thickness is large.

If the boundary layer thickness is large, what is the velocity gradient within the boundary layer? If the boundary layer thickness is large, then the velocity gradient within the boundary layer for a given U infinity is small. Now it can be shown that, if we consider so this is related to velocity gradient. We will discuss about the velocity gradient later, but similar to the boundary layer within which the velocity gradient occurs, there is a boundary layer within which the temperature gradient also occur. That is known as thermal boundary layer.

So, in a thermal boundary layer the temperature will change from T wall to T infinity. Let's the thickness of the thermal boundary layer be Delta T. So, Delta / Delta T, what is Delta related to? Delta is related to the kinematics viscosity of the fluid. Similarly, from analogy can you say what this Delta T related to? Thermal diffusivity of the fluid.

Kinematic viscosity and thermal diffusivity are very similar parameters. Thermal diffusivity is what? Thermal diffusivity is K / Rho Cp. K is just like Mu / Rho, Mu is momentum disturbance and Rho is momentum sustenance. Similarly, K / Rho Cp, K is thermal disturbance due to conduction and Rho Cp is the ability for thermal storage or that is thermal inertia. So very similar.

So, Nu / Alpha is related to Delta / Delta T and this Nu/ Alpha is the parameter known as Prandtl number. So, for a given Alpha if you have higher Nu, you have higher Delta, right? So, if you have higher Delta, because of Prandtl number is the property of the fluid you should also have higher Delta T. Because Delta/ Delta T is peak for a given fluid. So, if you have a higher value of Delta T then what will happen? If u have higher value of Nu, then you will have higher value of Delta T.

If you have higher value of Delta T, then what is the temperature gradient within the thermal boundary layer, is it more or less? Remember that we are essentially interested about the temperature gradient. So, what is roughly the temperature gradient within the thermal boundary layer T wall - T infinity / Delta T roughly, if it was linear, this is just order of magnitude. So, this is roughly Del T/ Del Y at the wall, order of magnitude.

So, if you have higher Delta T, you will have lower heat flux. That means the Nusselt number will be low. So, you can see we started with Nu we came with heat flux something very non-intuitive, right? We started with a pure fluid mechanics parameter and we came up with an increase of Nu will have a decrease of heat flux. That means decrease of Nu will have an increase of heat flux? So, decrease of Nu and increase of U infinity and also increase of L are of same effect. So, the net effect is of increase of U infinity L / Nu.

So, if you increase U infinity, the effect is like decrease of Nu and increase of L. And these together as a combination is Reynolds number. That all of you know, right? So, the Nusselt number for post convection will be a function of Reynolds number and these parameters Prandtl number. I just try to give you a pure physical arguments without getting into any mathematics.

Just simple physical argument but this kind of physical argument is very important. Because, we should first develop a motivation of why are we interested to go through the fluid mechanics equations to understand convection? So, here is the parameter which now is related to fluid mechanics. So, this gives us a motivation of why should we study fluid dynamics for convective heat transfer.