Conduction and Convection Heat Transfer

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Lecture - 31
Transient Conduction: Semi - Infinite Slab - II

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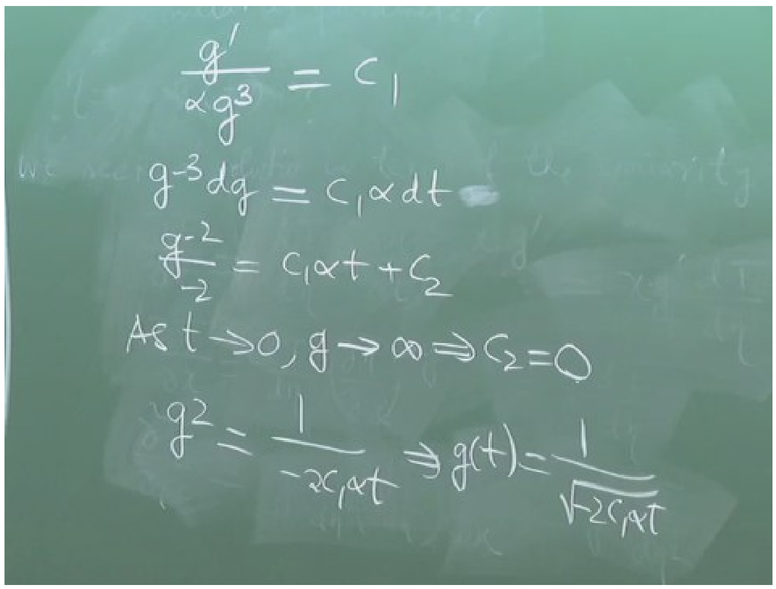

So, g to the power minus two by minus two is equal to C1 alpha t plus C2 where C2 is a constant of integration. Now, how can be find out what is the value of C2? Clearly, even if we do not know the exact value the time dependent should be such that the characteristic length scale, so x/ x reference equal to x into g(t) therefore x reference x we will scale with 1/g(t), right.

Even if we do not know what is g(t) at least we know that x reference should scale with 1/g(t). What is the x reference as time tends to infinity? Or as time tends to zero, as time tends to zero what is x reference? x reference is a distance from the wall up to which the thermal affect is filled? At the wall at which the boundary condition is given what is the distance from that wall up to which the thermal affect is filled.

So, at time equal to zero what is that it is only zero because only that wall knows that there is a heating just the boundary heating has started getting penetrated into the wall of the medium. So, the boundary heating at time equal to zero is confined still at the boundary. Therefore, x reference will be tending to zero as d tends to zero and gt is 1by x reference so gt will be 10 to infinity as x as T tends to zero.

So, we can say that as t tends to zero, g tends to infinity and this means from this equation you must have C2 equal to zero. So, from here you can write that g square is equal to 1/minus 2c1alpha t. So, this is something very remarkable because by mathematical analysis we are getting a form of the gt which is same as we did without any mathematical analysis but just by physical argument. So, the multiplying factor is not important because we say that it is of the order of.

So multiplying factor may be one, two, three, four whatever but the functional dependence gt, functional dependence is 1/ square root of alpha T and that without any deep mathematic just by physical arguments we could come up with. Now that means that we have come up with a situation when you could separate the function of eta with function of T and the function of T can be determined.

If we could not separate the function of eta with function of T we say that despite seeking a similarity parameter for the solution. There is no similarity parameter that exists for the solution. If we couldn't separate these two but because we could separate these two we can now say that yes, the similarity parameter exist which is of this from x into g(t). The next question is that when you have this xt into g t there is a parameter C1 here. So what value of C1 you should take?

See the only constraint is that C1 has to be negative because this is a real number and to make it real because it is negative and alpha is positive and T is positive C1 has to negative. So, choice of C1 must be negative. Within that what value of C1 you can take any value of C1 you can take it does not matter think about the situation see that a/ b equal to c/d this is what you have to satisfy in schools many problems of ratio and proportion.

We solve where we write a/b equal to c/d equal k, no matter whatever is the k. K is not a matter of our big concern. Big concern is some whatever may be the constant the equality of ab/cd has to be maintained and here also it is the same thing but the choice of constant only is to be constrained by negative constant. Now, we can take any value of the constant but for simplicity in algebraic manipulation if you take C1 equal to minus 2 then this two comes out of the square root.

So as an example, take C1 equal to minus two not a must. It is not that it has to be taken as minus two it is just one example it can be shown that whatever value of C1 you take within the negative paradigm you will finally get the same solution. So, it is not important that you have to take this but if you take this the only advantage is that the algebraic manipulation gets simpler because this two comes out of the square root.

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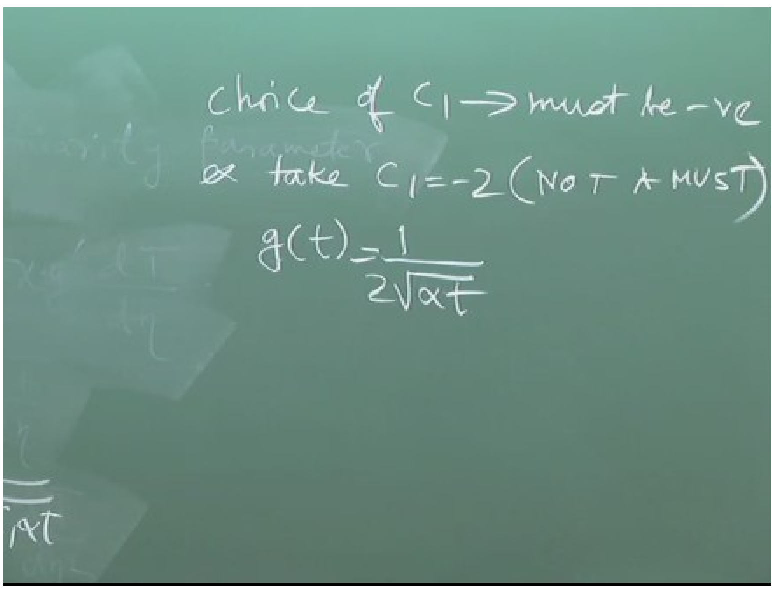

So, then g(t) if you take C1 equal to minus half then you get square root of alpha T so that is also possible but this is an example that we are taking. As a home work you should take C1 equal to minus half of another example and show that you converse to the same solution finally.

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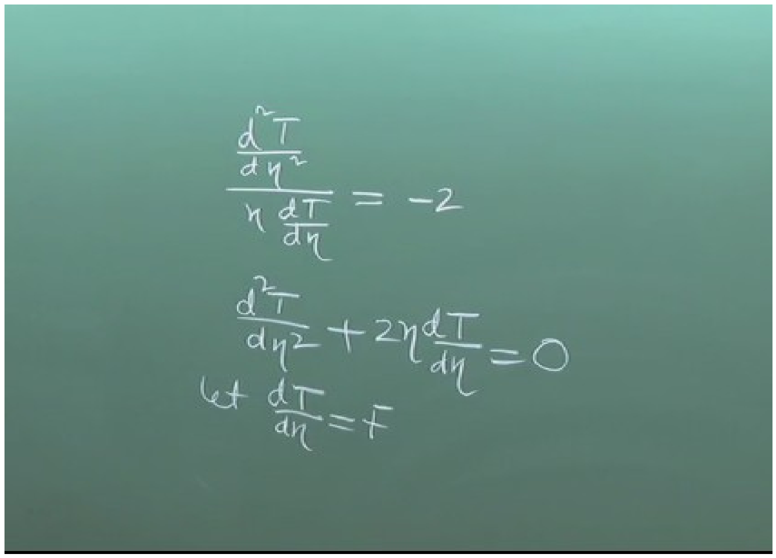

So, if you take C1 equal to minus two then you have d2T/d eta 2 by eta dT/d eta. This is equal to minus two from this equation. So basically, you have to solve this ordinary differential equation which is quite a simple task. So, let us assume dT/d eta equal to F.

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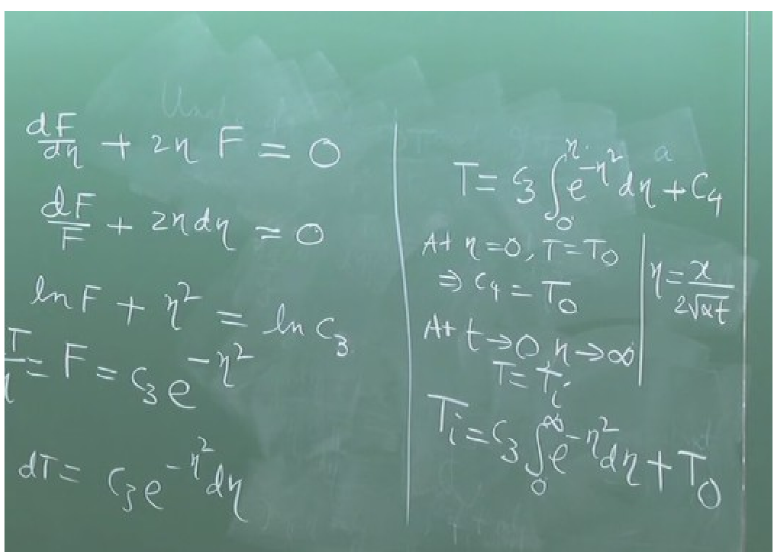

So that means you have dF/d eta plus 2 eta F is equal to zero. So, you can write dF/F plus 2 eta d eta equal to zero. So, ln F plus eta square equal to ln of C2 sorry C3, C2 we have already taken. Ln of C3 where ln C3 is a constant of integration so F equal to C3 into the power minus eta square. F is nothing but dt/ d eta. So, you have dT equal to C3 into the power minus eta square d eta. So, you have T equal to C3 integral of e to the power minus eta square d eta plus C4 that integral is zero to eta.

Now we will apply the initial condition and boundary condition to get the two constants this we have not yet applied. So, this is zero to eta. At x equal to zero what is T? T is T zero, At x equal to zero what is eta? Eta is also equal to zero. So, we can say that eta equal to zero, T equal to T zero that means C4 is equal to T zero. At eta tends to infinity. Remember what is eta? Eta is equal let us write eta here, eta is equal to x/ 2 square root of alpha t.

At eta tends to infinity sorry rather let us write at t tends to zero, you have eta tends to infinity that is what I wanted to write. As t tends to zero eta will tend to infinity. So, you can write T equal to ti at that t. So, Tl equal to C3 integral zero to infinity into the power minus eta square, d eta last T zero.

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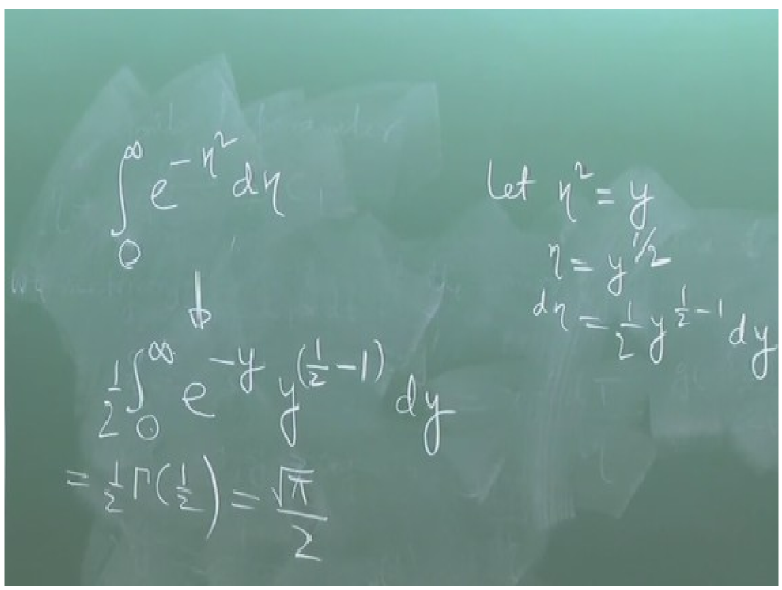

Now let us evaluate this integrals zero to infinity e to the power minus eta square d et. So, let us make a substitution, let eta square is equal to y, just new variable. So, eta equal to y to the power half, d eta so this will become integral zero to infinity e to power minus y into y to power ½ minus one. What is this integral zero to infinity, e to the power minus x into x to the power n minus one dx. It is gamma of half. Gamma function of half is root pi. So, this is root pi by two.

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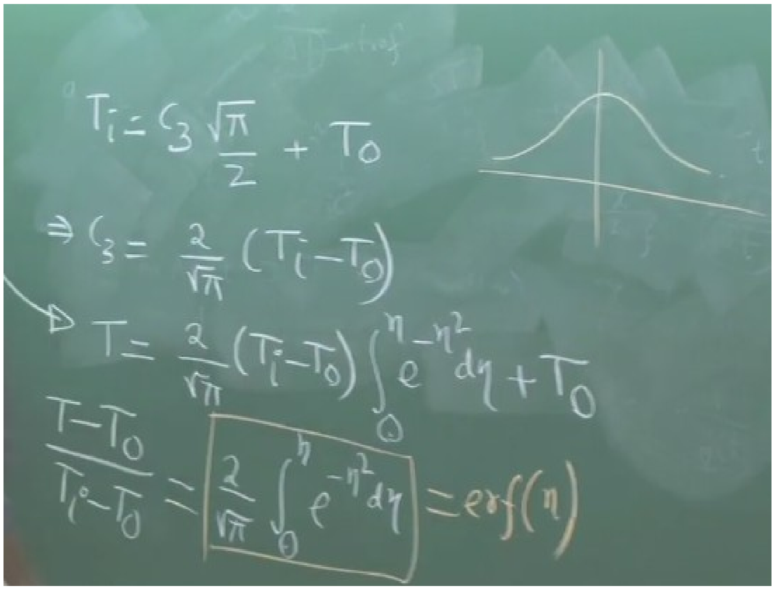

If that we the case then we can write Ti equal to C3 root pi by 2 plus To. So C3 is 2 by root pi into Ti minus To. So, then we can substitute in this equation and write T equal to 2/root pi Ti minus T zero. We have substituted C3 and C4 in this equation. So, we can in a compact form write T minus T zero by Ti minus T zero is equal to 2/ root pi integral zero two eta into the power minus eta square eta. By the definition this is what? This is the error function of eta.

So, the error function you must have studied in statistics where it is a function representing the normal distribution in a statistical sense like the probability distribution. The normal probability distribution which has a Gaussian type of distribution of probability of x as a function of x. That is that particular form is represented by this error function. So, you can also write it in a different way you can also write T is equal to T zero, let us subtract 1.

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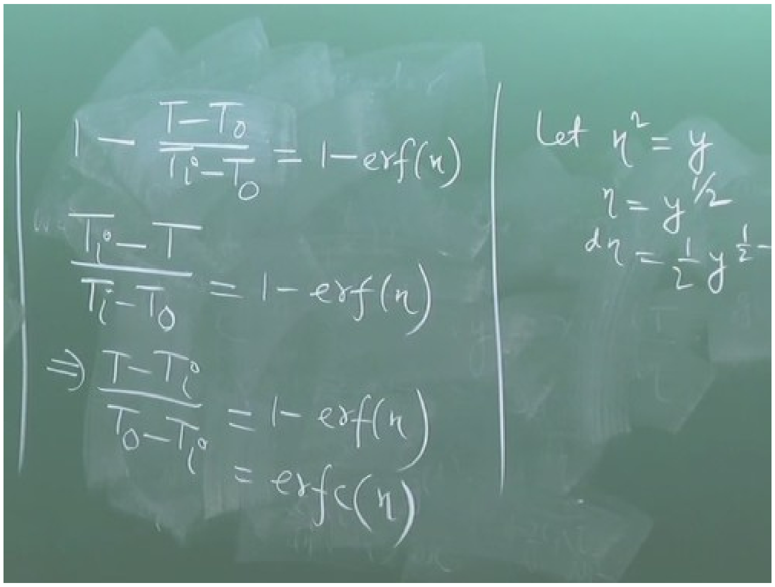

It is also called as complementary error function erfc. So, I have shown these two forms because you know different books will give different forms I do not know which specific book you are following. So, it is important to be familiar with the different forms but I mean this is just good enough to discuss about the solution. Now this is the mathematical part of the solution but again let us try to get some physical insight on the solution to this problem.

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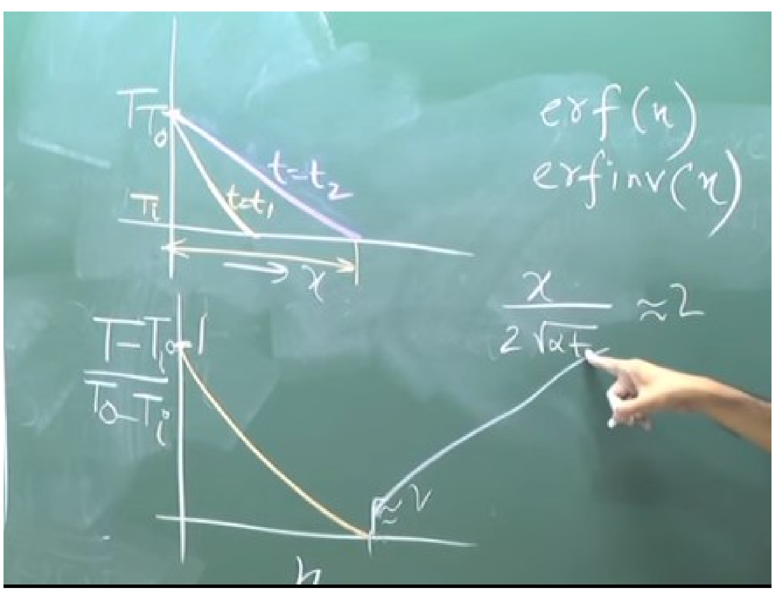

So, if you plot T as a function of x at x equal to zero. T is T zero then this these (()) (43:25). Let us consider another time. This is say time equal to T1 this is not a straight line. This is my problem whenever I want to draw a straight line I can't draw it and whenever I do not want to draw a straight line it becomes a straight line. This is t equal to t2, time. So, can you tell which time is more t1 or t2?

T2 is more because intuitively if you allow more time more and more length of the plate feels the thermal disturbance, right. So, it comes to the Ti. This is say Ti add some distance x that means beyond this critical distance x the wall does not feel the effect of the boundary temperature heating or cooling, whatever? So, when do you consider the wall to be a semi-infinite wall when the wall thickness is greater than then this thickness.

So that depends on what is the time at which you are considering. Now this is the distance, this is the thickness up to which the thermal disturbance of the boundary is failed. So, if the wall thickness is greater than this that it is as if infinity that is the notional meaning of infinity. It does not literally mean that it had to infinity it has to be greater than then the thermal penetration depth.

So that other end of the wall does not know that there is a boundary at which a temperature has been imposed. Now this is a dimensional plot but you can also make none dimensional plots. So, let us make this plot T minus Ti. This is a plot of one minus error function eta. So, if you want to make a plot in Met lab. Met lab has inbuilt function erf so you can use the inbuilt function erf in Met lab to plot the error function and to get the value of the error function.

Not only that let us say you know the error function but you have to find out what is eta? So that means you have to get the inverse of the error function. So, for that Metlab has inbuilt function erfinv. So, this problem is if you know eta what is error function? This problem is if you know error function eta what is eta? Because sometime you have to you know that this is the temperature. The question at what time and position that is the temperature.

So, then you have to find out error function inverse because you have to get the eta then you have to do this. See in old days you will find that still in books that is there. There is a table of error functions. So, for even eta what is the value of error function that is tabulated basically the numerical evaluation of the Gaussian distribution function. So, if you have some value of eta from the table you can find out the error function or if you know the error function from the table you know for eta that error function is there.

But I would say that in modern times that kind of approach is no more necessary because you already have Metlab you can sit with Metlab or any other numerical computational software. If you the value of beta just write the corresponding function type the corresponding function it will give you back error function. If you know the error function just write inverse of that, that will give you the value of the function you do not have to have a real physical look of table.

That is what you can do but that's just for implementation purpose now you can see can you tell what the value of this. What is the value of this point? At eta equal to zero what is T? T zero, so what is the value of this. One. Typically, if you calculate this I would request you to plot this using Metlab. You will see that this value is I can't remember exactly but it will be close to two. So, anything beyond this is like semi-infinite.

So, question is say x by two root alfa t that is roughly equal to two at this point. So which x is considered to be good enough for the plate to be considered to be semi-infinite it depends on what is the time. Larger and larger time you plate width also has to be larger and larger to make sure that this assumption is justified. But very short time even a small thickness can be infinite. So, let us say that you have a laser pulse by which you are treating a material.

This is very common in manufacturing process, very common in surgery that you have a laser pulse you are treating with the laser pulse treating the material. Now typically if you are doing the treatment for a very short period of time. Let us say (()) (49:58) for a medical treatment or milliseconds for a manufacturing process. Over that period of time the penetration depth may be a few microns to millimeter so in that respect even a plate with one centimeter is like infinity.

So, again I'm repeating the same concept over and again to make you understand that infinity is not a literal type, literal name that we are giving it here. It is a notional thing it is that if the thermal penetrating depth is shorter than the width then the width is like infinity. Now let us quickly work a problem and then we will call it a day today.

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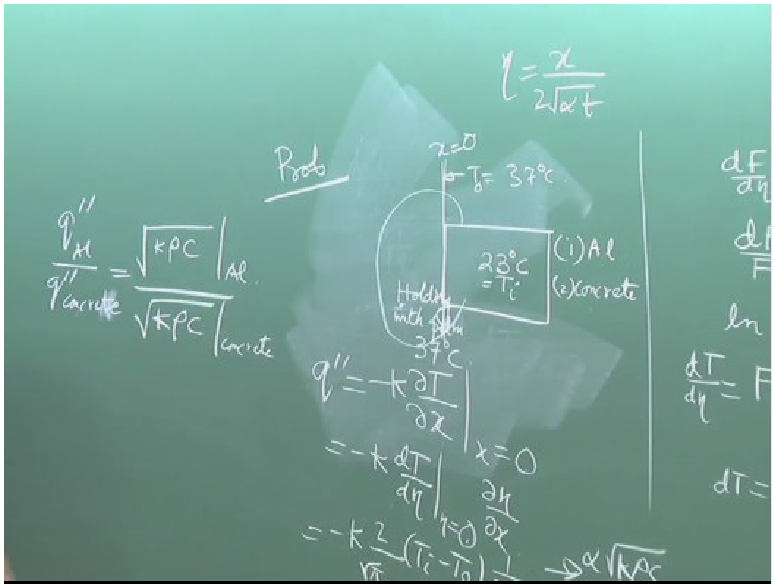

Let us say that this is a cold block at 23 degree centigrade, now you are holding with your palm that is at 37 degree centigrade. This may be of two material one material is aluminum the other material is concrete. Out of these two materials which will fill colder to your hand? That is the question, right. So out of these two materials this block can be of these two materials either aluminum or concrete. You hold it with your hand which is at 37 degree centigrade.

This is at 27 centigrade so colder then you hand but which will feel colder? Now I do want to get an answer from you what I want to get always is how to logically arrive at the answer not what is the answer because I mean once you logically arrive at the answer you can arrive at the answer but many times we arrive at the answer with a common sense without knowing that logic and that is actually very dangerous in science.

So, it is important that we know that or we figure out how to arrive at the solution. In fact, this is one of the assignment problems this chapter of unsteady heat conduction that we have given to you. So how will you know that which feels hotter or colder basically you have to find out in which case the influx is more so the heat flux, this is the boundary. This is basically Ti and at the boundary this is T zero equal to 37 degree centigrade, this is x is equal to zero.

So, minus K Delta T, delta x as x equal to zero. So, what is eta? Remember eta is equal to x/ two root alfa t. So, what is dt/ d eta, at eta equal to zero. Look at this expression dt d eta is equal to 2/ root pi Ti minus T zero into e to the power minus eta square. At eta equal to zero that e to the power minus eta square part will be one. So, it will be 2/ root, Ti minus T zero. So minus k, 2 by root pi into Ti minus T zero into delta eta, delta x is 1/ 2 root alpha t.

That means what is alpha, alpha is k / rho into C, right. So, this will be proportional to square root of k rho c, right. This K divided by the square root of k becomes square root of k and 1/ square root of rho c becomes square root of rho c. Other, parameters remaining constant so for aluminum and concrete you have to substitute the corresponding values of k rho c. So, the ratio of heat flux with aluminum by heat flux by concrete is root over k rho c aluminum by root over k rho c concrete. So, let me write it down here.

So, heat flux aluminum by heat flux concrete so you can substitute these values get the values from this table and table which is there table of properties which is there in your book and figure out that what is the ratio of heat flux. Whichever heat flux is higher that will make you feel colder. Let us stop here today and in the next class we will discuss about the tutorial problems and the solutions.