Conduction and Convection Heat Transfer
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Lecture - 28
Problems on Lumped Parameter Approach

So, we have studied the lumped parameter analysis for a limiting case when conduction is dominating. We will consider the other limiting case briefly before working out a couple of problems. The other limiting case is when radiation is significantly dominating over convection. So, let us look into the governing equation.

This equation which we derived in the previous lecture. Now, this term will not be important as compared to this term. So, you can write rho C V d T by d T is equal to minus A sigma epsilon T to the power 4 minus T infinity to the power 4, ok.

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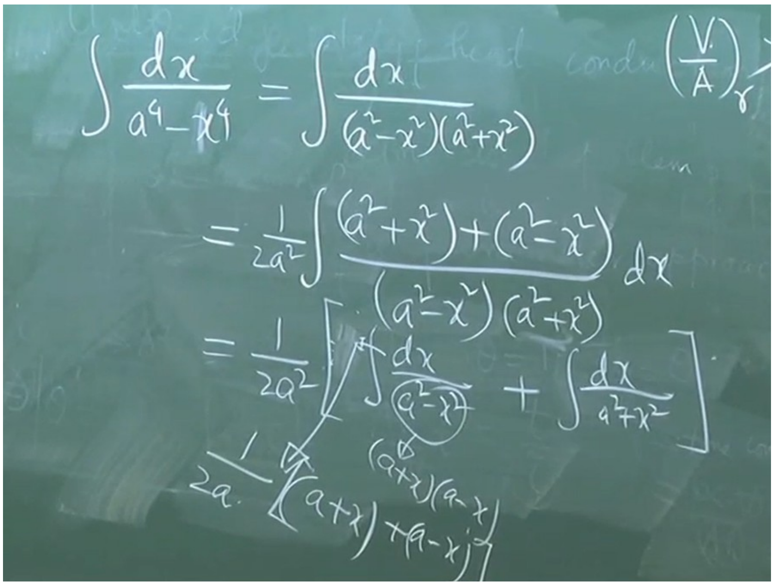

So, this is also a very straight forward problem. In the previous case the integration was d x by x. now the integration is of the form d x by say A to the power 4 minus x to the power 4 right. So, I will not go through that, but I leave it on you as an exercise but very simply you can do that. This you have done in school level. So, integral of this is the form of the integration a is T infinity this is T.

So, you can write this as integral of d x by a square minus x square into a square plus x square. So, this you can write integral of 1 by 2 a square, a square plus x square, plus a square minus x square. Again, for this term you can write a plus x into a minus x and numerator you can write 1 by 2 a, right. So, one of the integrals will be L n of a plus x, another will be L n of a minus x and this if you substitute x equal to a tan theta it will be a straight forward integration.

So, the integration is quite easy even if you do not remember any formula you can reproduce it quickly. The remaining part of the problem is just purely algebraic simplification. Concept wise we do not learn anything new. So, we will work out a couple of problems related to the use of the lumped parameter method and we will start with working out the problems.

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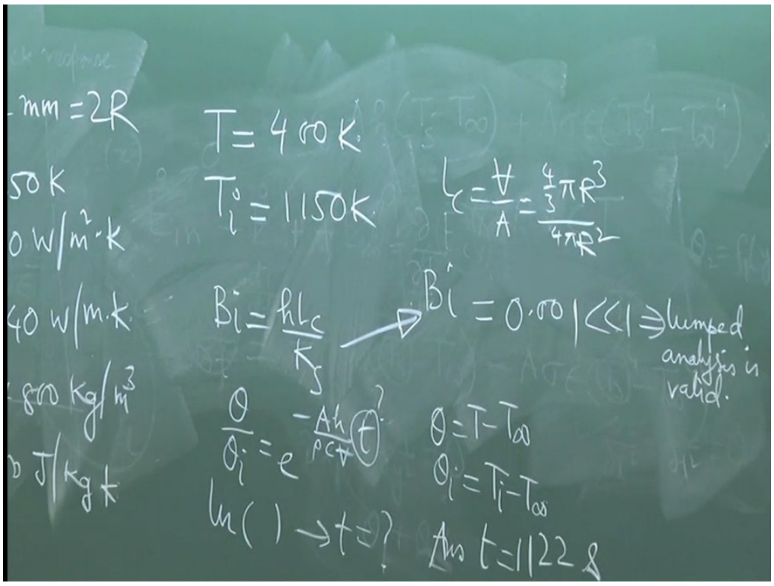

So, the first problem, please note down the problem. This is a problem from incropera book. Steel balls of 12-millimeter diameter, so D is equal to 12 millimeter, this is 2 R or annealed (annealed means annealing is a heat treatment process) by heating to 1150 Kelvin and then slowly cooling to 400 Kelvin. I am repeating steel balls of 12-millimeter diameter or annealed by heating to 1150 Kelvin and then slowly cooling to 400 Kelvin.

In an air environment for which T infinity is equal to 350 Kelvin, so T infinity is equal to 350 Kelvin. H is equal to 20 watt per meter square Kelvin. Assume k of the steel is equal to 40 watt per meter Kelvin, rho of steel is equal to 7800 kg per meter cube. C of steel is equal to 600 joules per kg Kelvin. Calculate the time required for the cooling process, how much time it will require to cool. So, what is the final temperature?

Final temperature is 400 Kelvin right. What is the initial temperature? 1150 Kelvin. So, we can straight away use the lumped analysis provided we justify that the lumped analysis is valid. So, for solving any problem using lumped analysis, first you have to calculate the Biot number and then tell. So, what is the Biot number? H L c by k. what is L c here? volume by area. So, L c is volume by area is equal to 4 by 3 pi R cube by 4 pi R square, ok.

So, if you calculate these, this will come out to be if you substitute the values this will be 0.001. Sorry not the L c the Biot number, ok. So, we can assume this to be much much less than 1 that means lumped analysis is valid. So, then we can write the governing equation and solve the governing equation, I have already done it. So, I am not going to write it again, but I will just the final solution.

So, theta by theta I is equal to e to the power minus A h by rho C v t, right. What is theta. T minus T infinity and theta, I is equal T I minus T infinity. So, in this expression you have to find out what is the time. So, you can take log of both sides and find out what is the time. So, log of both sides and what is the time, so the answer I am giving T is equal to 1122 second. This is the straight forward use of the theory that we have discussed in the class.

Now, let me ask you a question? For the lumped analysis to be valid is it necessary that the Biot number has to always be small. See why did we impose the condition of Biot number small, we wanted to make sure that the entire body is at a uniform temperature right. Let us think of a situation.

Let us say that you have one glass of hot milk and the milk is slowly stirred by a spoon because of the stirring with the help of the spoon the entire milk will come to a uniform temperature close to practical purposes. So, then whether the Biot number based on the volume and area is small or not is not important because we have ensured by stirring that the milk is at a uniform temperature.

So, the requirement of Biot number much much less than one is not a fundamental requirement. The fundamental requirement is the uniformity of temperature. So, we can say that Biot number much much less than 1 is a sufficient condition for lumped parameter analysis to hold but not a necessary condition, right. If it is true then the lumped analysis will be valid.

But even if it is not true but the substance is homogeneous to a uniform temperature then you can use the lumped analysis. Now can you tell that why I was telling that the milk is slowly stirred? If the milk is stirred very fast then convection within the system is important and here, we are assuming that the convection is taking place outside the system. Within the system it is pure conduction, ok.

So, conduction analysis will not work for a system where because of very vigorous stirring convection within the system is itself important. Then you have to solve the fluid flow equations, ok. So, finally we will work out another problem and call it a day today.

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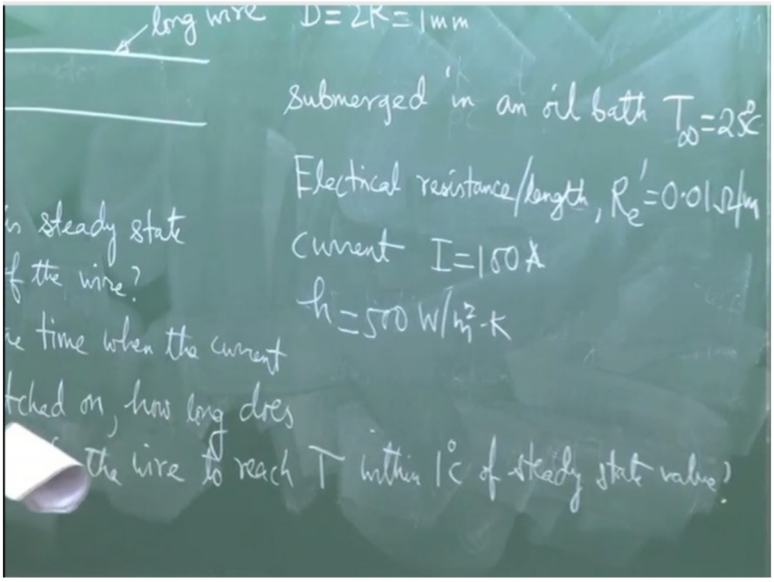

So, that is the last problem that we are solving today. So, the problem statement is like this: this is also a problem from the exercise of incropera textbook. So, you have a long wire of diameter is equal to 1 millimeter. It is submerged in an oil bath of T infinity is equal to 25 degree centigrade. Electrical resistance per unit length is 0.01 ohm per meter. The current that flows through the wire is 100 amp here.

And the convection heat transfer coefficient is 500 watt per meter square Kelvin. The first question is what is the steady state temperature of the wire? This is the first question. Second question is from the time when the current is switched on how long does it take for the wire to reach a temperature within 1 degree centigrade of steady state value. Other properties are given rho is equal to 8000 kg per meter cube, C is equal to 500 Joules per kg Kelvin and K is 20 watt per meter Kelvin, ok.

So, the physical problem is that there is a current which flows through a resistance so there is a Joules heating because of Joules effect there is a heat generation and that heat is dissipated to the surroundings. At steady state the rate of heat generation is same as rate of heat dissipation to the surroundings but during unsteady state, what will happen? The temperature within the wire will change with time before achieving steady state.

So, the first part of the problem deals with the steady state, second part is the unsteady state. So, we will write first develop the general formulation and then we will apply it to both steady state and unsteady state.

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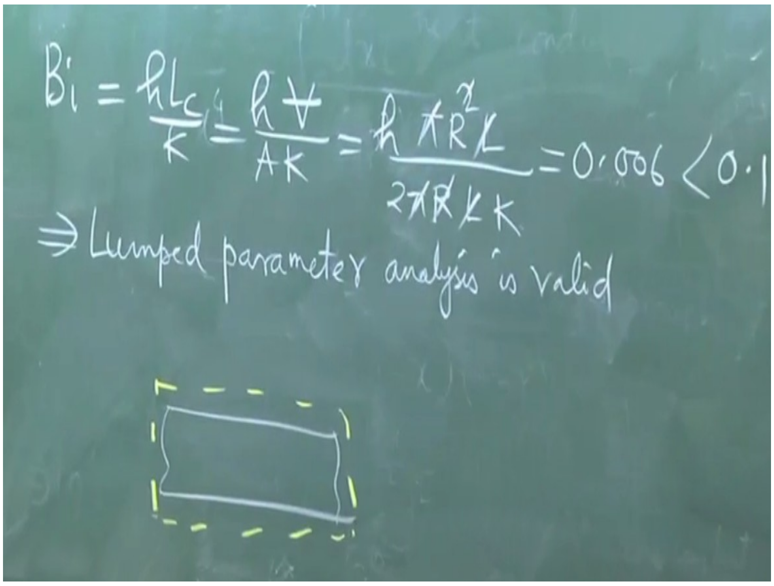

First of all, before applying the unsteady state situation can we use the lumped model here? So, let us say that length of the wire is L. So, the Biot number h R by 2 K, ok. So, if you substitute these properties this will be 0.006. This is less than 0.1. So, it a sufficient condition for the lumped parameter analysis to be valid. So, now let us try to make a control volume analysis of this electrical wire.

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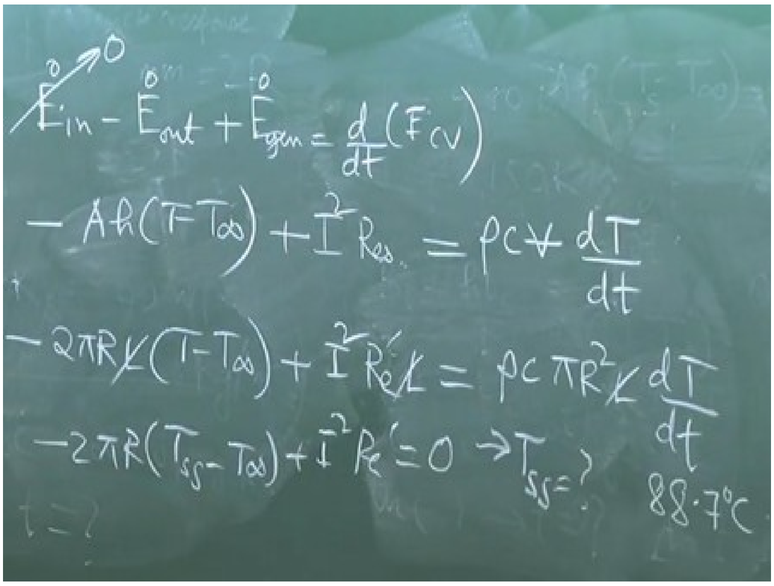

So, energy in minus energy out plus energy generated because it is a lumped body we can write d t instead of the partial derivative, ok. So, what is the rate of energy in? There is no rate of energy in. What is the rate of energy out? So, there are 2 modes: convection and radiation. We assume that convection is the dominant mode as compared to radiation. So, minus A h into T minus T infinity plus what is energy generation?

I square R because R I have used already for radius, so I am using a different symbol Res. So, we can write what is A? A is 2 pi R L into T minus T infinity plus I square. Now resistance is given see Re dash ohm per meter. This multiplied by the length will be the total resistance. So, I square Re dash into L is equal to rho C Pi R square L into d t by d t, ok. So, L gets cancelled from both sides. It is given that it is a long wire.

Otherwise, it will not be a 1-dimensional problem. In fact, 1 dimensional here becomes zero dimensional. It is not even 1 dimensional, it is a zero-dimensional problem. Now, all the values are known so what is the first part? What is the steady state temperature of the wire? So steady state temperature of the wire is obtained when this temperature stops changing with time then its steady state. So, when this term is zero, then generation balances heat loss.

So left hand side equal to zero will give the temperature, the steady state temperature. So, minus 2 pi R into T steady state minus T infinity plus I square Re dash equal to zero. This will give you what is T steady state. So, if you calculate this. So, T steady state will be 88.7 degree centigrade.

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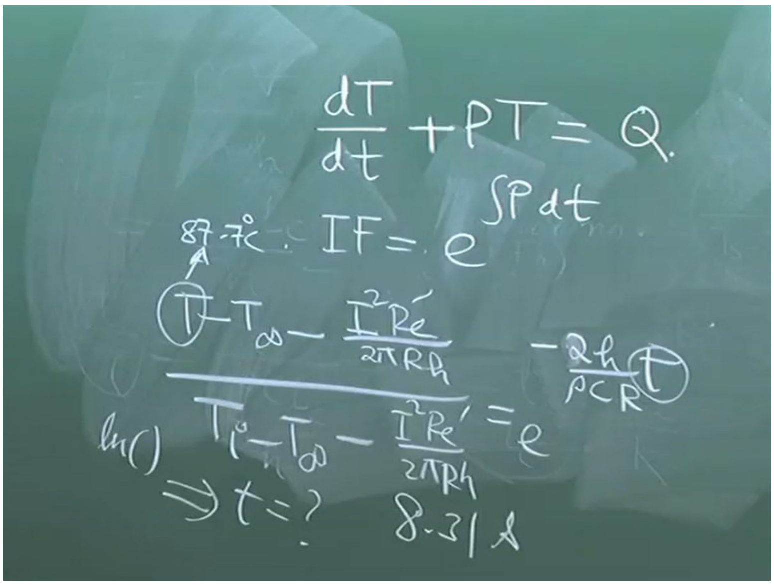

Now what will be the unsteady state variation because for the second problem basically we require to find out temperature as a function of time, right. So, if you look into the differential equation. The differential equation is of this form right, where P and Q are some coefficients. This is the form of the differential equation. So, for solving this differential equation, we have to multiply with an integrating factor e to the power integral P d t.

That is the integrating factor by which you have to integrate. So, if you do that it is a straight forward solution of the differential equation, ok. So, let me give you the solution to the problem T minus T infinity. So, this is the solution to the equation. Now, you have to find out the time at which this temperature becomes 1 degree less than the steady state temperature, right. So, the steady state temperature is 88.7.

So that will be 87.7 and then you take log of both sides and find out what is the time. The answer is 8.31 second ok. So, you can complete the calculations, but you have understood the method of how to solve this problem. So, to summarize we have mainly discussed today about first some aspects of lumped parameter analysis and then what are the conditions under which the lumped parameter analysis works.

And we have used the lumped parameter analysis to solve a couple of problems of practical interest, ok. Thank you very much. Let us stop here today.