Conduction and Convection Heat Transfer

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Lecture – 27
Transient Conduction – Lumped Parameter Approach

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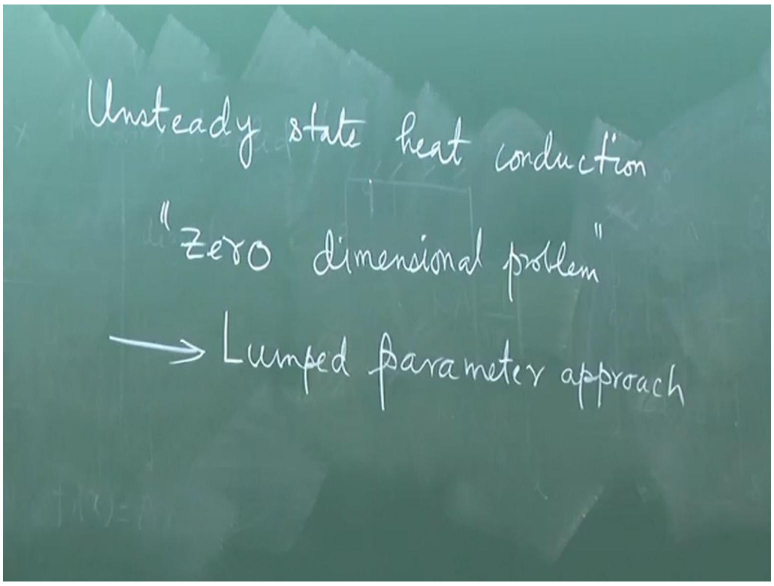

Now, we will move on to unsteady state heat conduction. When we think of unsteady state heat conduction, the situation is that we are considering that temperature is also a function of time. So, as I told you earlier that all problems are fundamentally unsteady so we are doing unsteady state analysis when our time domain of interest is such that over that time domain temperature is changing with time.

Now, temperature is changing with time, but temperature is also changing with respect to stress. It is not just change with respect to time, but also change with respect to space. So, with respect to space, it can be 1 dimensional, it can be 2 dimensional, it can be 3 dimensional, but we will start with something, which is easier than all these that is zero dimensional.

That means with respect to space there is no variation and with respect to time only there is variation. So, we will consider something these terminology is not used in books, but in a lose sense it is like a zero-dimensional problem and the corresponding mathematical modeling is known as lumped parameter approach. In lumped parameter approach, what we are trying to do we are trying to neglect the temperature variation as a function of position and only consider temperature variation as a function of time.

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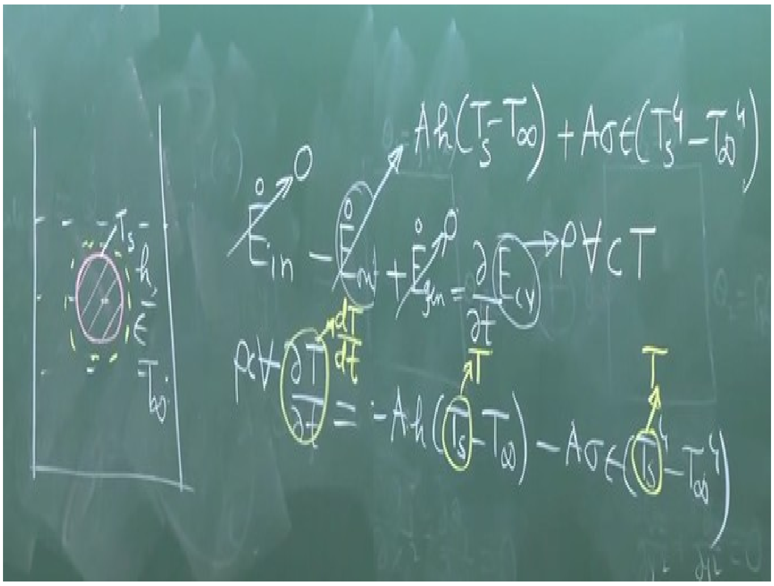

Let me give you an example, let us start with an engineering example, in manufacturing or in processing of materials, there is a process which is called as quenching. Quenching is a heat treatment process, so in quenching what happens let us say you have a metal block which you had heated, so heat treatments are done in materials industry to give sudden material property to impart certain properties to the material.

Now, once the heat treatment is done that it is heated, it may be heated in air, it may be heated in furnace or whatever then the material is cooled and sometimes it is cooled very rapidly by immersing it in some fluid. So, the material is hot and then it is suddenly put in a cold fluid and the material will cool down almost instantaneously. This process is called as quenching.

Now, this process from material science point of view is very interesting, because quenching will have very little time for heat transfer given and very little time for change to take place and overt this little time the grains in the material will not be able to grow by a long way so there will be fine grain microstructure and that can give rise to a good amount of hardness to the material.

So, see this is how an engineering problem is, when you go to an industry nobody will tell you this is the problem of mechanical engineering, this is the problem of chemical engineering, this is the problem of material science, this is you have to use a holistic approach. So, this is the problem that you want to solve so let us say you want to predict the microstructure.

Why do you want to predict the microstructure because the microstructure will dictate what will be the mechanical property of the material say that is a steel ball and that steel ball will be used for making some structures, so depending on the microstructure the property of the steel ball will be different and it will behave mechanically different. So how do you know that what will be the microstructure.

The microstructure is the function of the cooling rate that means the rate at which the temperature of the steel ball decreases with the time in this case. In some other case, it might also increase but for a quenching process of course it decreases. The rate at which it decreases accordingly the microstructure develops, so the microstructure has a very important relationship with the heat transfer in general and the temperature gradient with respect to time in particular.

So, our objective will be to figure out how temperature of the steel ball is changing with time during the quenching process. So, let us draw a schematic of the problem. Say this is the bath and this is the ball, ok. So, for zero-dimensional analysis, we consider this ball to be a lumped mass. What is the analogy in mechanics, it is a particle. In mechanics, when we say particle what we assume even rigid bodies we can assume as particles if there is no rotation.

So, what does it mean? so let us say that we write Newton second law of motion for a particle. Now sometimes a card is also modeled as a particle. As practical engineers, we can argue that is card a point mass, card is not a point mass, but for all practical purposes the entire motion of that may be conceptualize as a transalatory motion may be say center of mass which is the point mass.

So, similarly for under certain conditions, we will answer this question that under what conditions, under certain conditions we can study the heat transfer here by neglecting the temperature variation within the ball. That is, we can assume that the entire ball is at a spatial uniform temperature. I will discuss that under what condition that is valid and under what conditions that is not valid, but let us first assume that is valid.

So, if that is valid then let us say that you have a system which is the block, so energy in we can write an energy balance, energy in minus energy out plus energy generator, ok. This is the energy balance that in the very first lecture which I delivered I talked about this the energy balance of the system. Now this is losing heat fluid surroundings by say convection and radiation.

So, let us say h is the convecting heat transfer coefficient and Epsilon is the emissivity of the surface, so what is the rate at which energy is entering the control volume. There is no energy it is only losing energy because of cooling, so energy in is zero. What is energy out, let us say that temperature of the bulk fluid is T infinity, so A h T minus T infinity, right. This is heat loss by convection.

This is heat loss by radiation. Of course, the minus sign is there before both of these. This T s is what T s is the temperature at the surface of the ball. What is energy generation? There is no energy generation in this case, but I will work out a problem in the class where there is energy generation. Now, what is energy of control volume. This is the solid one, so it is internal energy that is what row into the volume this is the mass into C into T.

So, we can write. Now if we assume that this is like a lumped mass then the temperature everywhere within the mass is same. So, this T s is as good as temperature anywhere within the body. This is the lumped parameter analysis and then because temperature is everywhere uniform spatially temperature variation with time is the only derivative of temperature, so you can write this as ordinary derivative instead of partial derivative.

So, this is an ordinary differential equation governing the temperature as a function of time. Now, if you consider all these terms together, the integration may be a little bit cumbersome, but you can consider 2 limiting cases. In one case, convection is dominating, in another case radiation is dominating.

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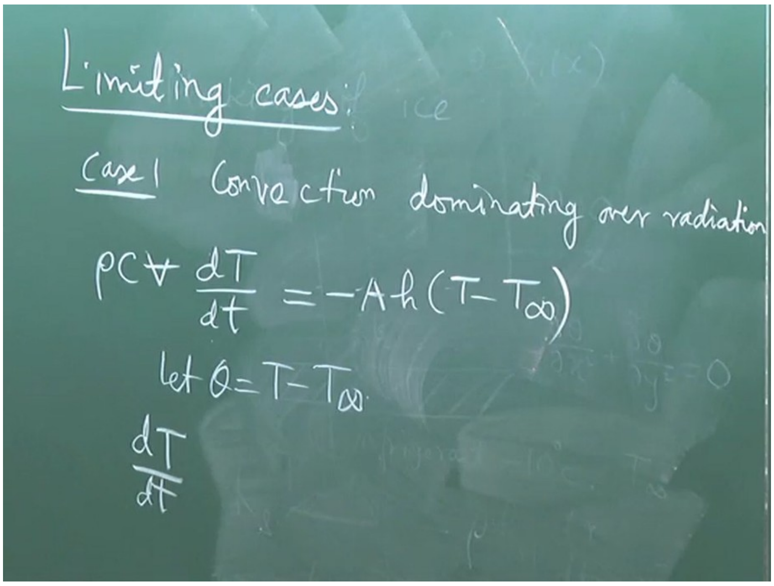

So, limiting cases: Convection dominating over radiation. Now, you can solve this equation. Let us say that theta is equal to T minus T infinity. So, you can write d theta by theta is equal to minus A h by row CV d t. So, if you integrate this ln theta is equal to minus A h by row CV t plus ln C1 where ln C1 is a constant of integration. So, theta is equal to C1 into e to the power minus A h by row CV t.

How to get C1, you can use the initial condition at time equal to 0. T is equal to T i which is the initial temperature, so theta is equal to T i minus T infinity that is theta i. So, you will get theta is equal to theta i e to the power minus A h by row CV t, ok.

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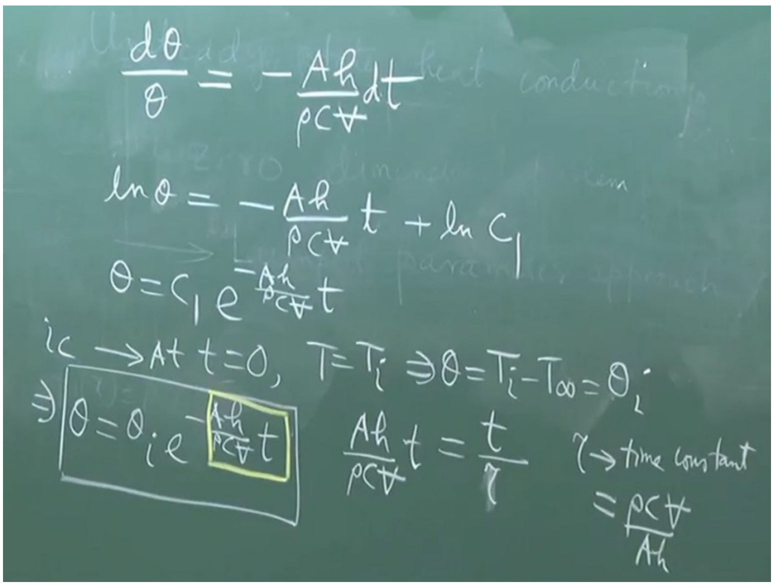

Now, let us try to develop some physical insight on this. First of all, this term in the box must be non-dimensional because when you write e to the power something that something must be non-dimensional. It is quite clear when you write e to the power x, 1 plus x by factorial 1 plus x square by factorial 2 like that. So, if x as a dimension then dimensionally it will never match. So, e to the power something that something has to be nondimensional.

So, this is something which is a non-dimensional number, so A h by row CV t that can be written as t by some tau. What is tau, is called as a time constant of the problem. Now this can also be written with some electrical analogy. What is this, this is like a capacitance, this is a storage ability of the system to store thermal energy. So, this is like capacitance and this is what this is one by resistance. So, this is like RC in a RC circuit in electrical circuit theory.

So, you will get similar type of characteristics for the current voltage type of relationship in an RC circuit in circuit theory in electrical engineering, so that is an analogy with electrical thing, but what is left for us to understand is what is the physical significance of this time constant. How can you use it for a practical problem? Not only that under what conditions this solution will remain valid.

Because we have assumed that the lumped model is valid that the entire solid is at a uniform temperature, but where is the guarantee that the entire solid is at a uniform temperature. So, we will figure that out that under what conditions the entire solid is at a uniform temperature and under what conditions it may not be consider to be so. We will take a short break of about five minutes and then we will continue with this discussion.

So, let us continue with what we are discussing in the previous lecture. We were trying to solve a problem of unsteady heat transfer, unsteady heat conduction using the lumped parameter analysis and what we are trying to do is we are trying to figure out the temperature as a function of time assuming that there is no temperature variation within the body as a function of position.

Now, when is that assumption valid? So, we will try to answer the first question.

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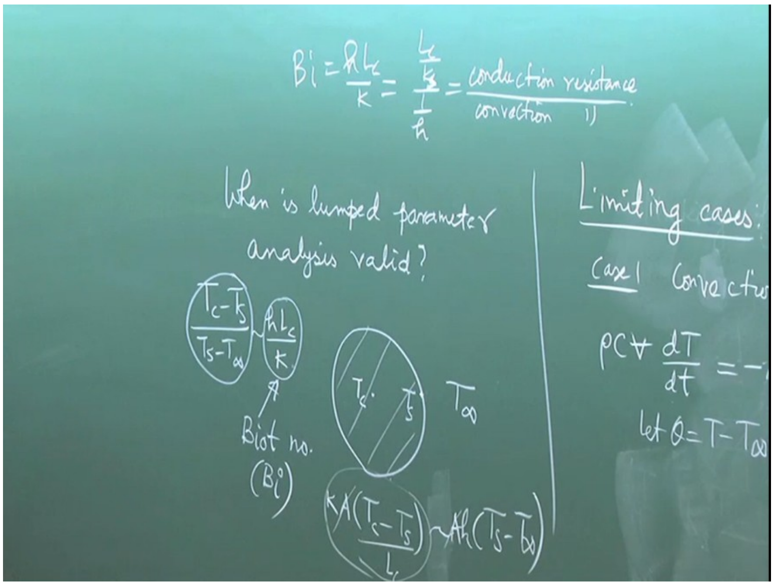

So, let us take this example. Let us say that you have a spherical ball with T c as the centile temperature, T s as the surface temperature and T infinity as the temperature of the surrounding fluid. This is a solid ball. So, you can write not exactly equal because it is not a steady state problem, so the heat transfer due to conduction is not exactly minus K a into delta T by L.

But we can write order of magnitude wise whatever is the rate at which heat is being transferred here due to conduction at the same rate heat is being dissipated by convection and this is an approximate estimation of the rate of conduction heat transfer. Had it been one dimensional steady state heat conduction that would have been exactly equal to this then instead of this order sign we could have put the equality sign.

But here it is just the order because it is not a 1-dimensional steady state problem. So, you can write there will be a here right, A into h into T minus T infinity. So, this A will get canceled from both sides and you will get T c minus T s divided by T s minus T infinity is of the order of h L c by k. What is this L c? We have not discussed what is this L c, this is called as characteristic length scale of the body.

So, typically the characteristic length scale is a length that describes the physical change or that change of temperature within the body. So, you can see that in this expression that characteristic length comes out to be volume by area. So, you can take it as a volume by area as the characteristic length. So, here just to symbolize we have retreat it as L c without mentioning what is this L c. So, this is a non-dimensional number, right?

Therefore, this is also a non-dimensional number. What is the name of this non-dimensional number? The name of this non-dimensional number is called as Biot number, ok. So, what does it physically represent? So, the Biot number which is h L c by k right. What is the numerator? It is the conduction resistance and denominator is the convection resistance. So, it is a ratio of conduction resistance to convection resistance.

Remember this k very very important. This k is k of solid, not k of fluid. We will later on see a non-dimensional number in the context of convection where it looks very similar, but the k is k of fluid and not k of solid. So, now when can we think of that the entire solid is at a uniform temperature? When can we think of like that? We can think of the entire solid to be at a uniform temperature when T c minus T s is much much less than T s minus T infinity.

That is temperature variation within the body is much much less as compared to temperature variation outside the body.

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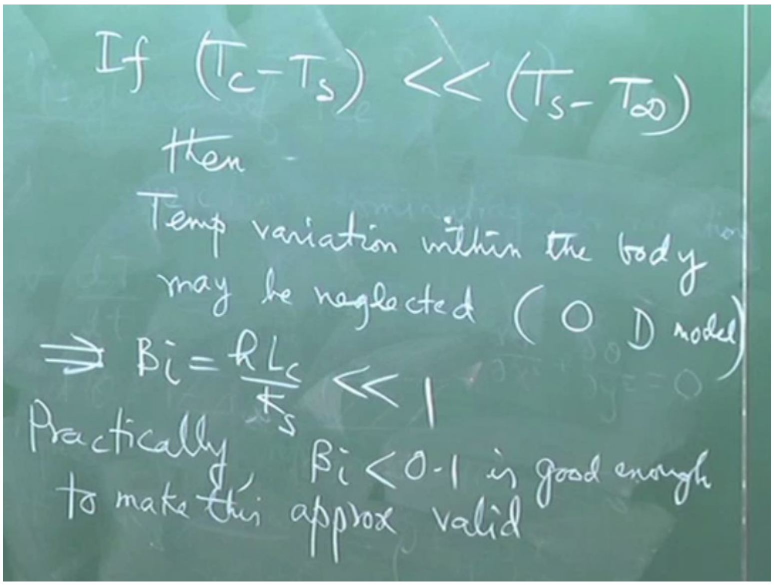

So, we can write if T c minus T s is much much less than T s minus T infinity then temperature variation within the body may be neglected. This is zero-dimensional model. So, the model is valid see you cannot adjust this (()) (7:47) because you do not know what are the temperature, so you cannot put this values and check. So, what you can check? You can check this parameter.

So, this parameter implies that the Biot number which is h L c by k of the solid much much less than 1. So, please do keep in mind that for the lumped parameter analysis to be valid the Biot number must be very very small, much much less than 1. Now, question is, what is much much less than 1? For all practical engineering purposes, see for mathematical purposes it is in the limit as it tends to zero, but for all practical purposes the Biot number may be considered to be small if it is 1 order less than 1 that is less than 0.1.

So, practically Biot number less than 0.1 is good enough to make this approximation valid. So, let us say that condition is true.

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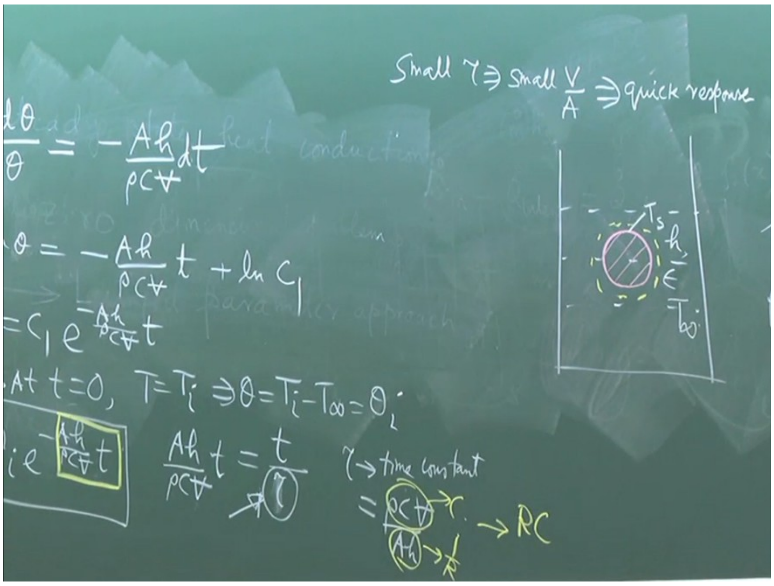

Now, what is the physical interpretation of the time constant that is the first question that would like to answer. Second question is based on that physical interpretation can we write it in terms of the Biot number, ok. So, now the first question? Now look at it, let us say that tau is small, if tau is small then what will happen? See let us say this is a lump body. You create a change and this body is adjusting to that change in temperature.

So, question is how fast it is adjusting or how slow it is adjusting. If tau is small will it adjust fast or will it adjust slow? If tau is small then for even a small value of T, T by tau is large right. T by tau is large means e to the power minus large will be tending to zero that means theta will be tending to zero. Theta is T minus T infinity. That means in a very short time T will come to T infinity.

That means the solid body will very quickly come to the ambient temperature. That means the response of the solid body is very fast. So, a short time constant means it responds to the temperature change very fast and short time constant means what? Short time constant means small volume by area. Small volume by area means short small tau. So, small tau means small V by A. This means quick response.

Why is it so? because volume is what, volume is the storage of thermal energy and what is area, area is the surface over which heat gets transferred. So, if the area is much much larger as compared to volume then there will be rapid transfer of heat as compared to thermal storage. So, V by A small means actually area much much large as compared to volume. So, then the body will quickly adjust to its change.

So, in other words in this quenching example it will cool fast if V by A is small, right. So, if V by A is small it will cool fast. On the other hand, if tau is large then what will happen? If tau is large then for a short time it will be what, e to the power zero, right. If tau is large then for a short time divided by a large time, so that means it will tend to zero. So, e to the power zero will tend to 1.

That means theta will tend to theta i. That means the temperature of the solid will remain same as the initial temperature even for a substantial time. The change will be slow, clear. So, we can say that tau is an indicator of the rate at which the time rate at which the body adopts to a thermal change. If it is small it adopts quickly. If it is large it adopts slowly. So, now how can this be applied?

This can be applied to many engineering design problems including say the design of a riser in a casting system. So, in a casting process you know that there are shrinkages at various levels and when there is a shrinkage during solidification what will happen, so there will be a volume that needs to be replenished and the riser is there as the supply of the molten metal which will supply to the mould till the entire mould solidifies that is the requirement of a riser in a casting problem.

So, when you are having such a requirement the riser should solidify at the last because it must supply the molten material to the mould till the final mould solidifies. So, riser must solidify last. So that means riser must lose the heat most slowly.

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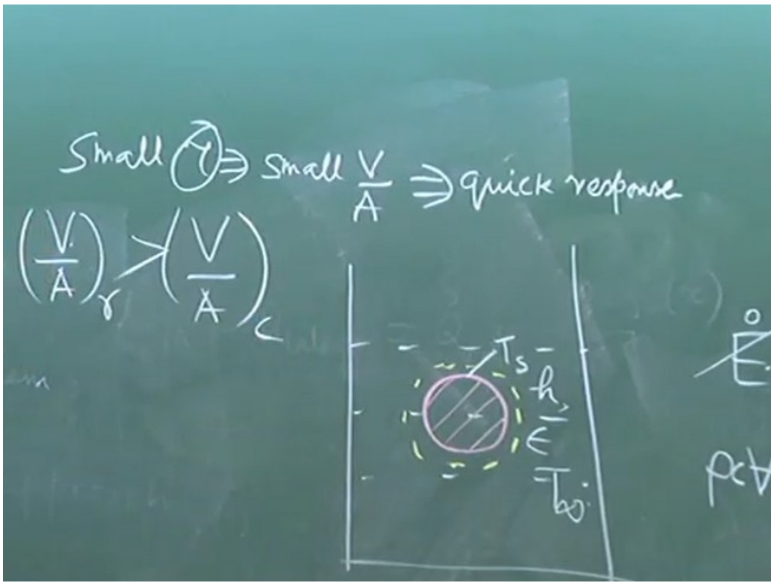

So, if the riser must lose heat most slowly then the time constant of the riser must be large, right. That means volume by area of the riser must be greater than volume by area of the casting because its time constant should be large so that it should respond slowly to the thermal change so that it will not immediately freeze. This in riser design problem is known as (()) (15:00) rule.

So, there are see classically manufacturing process or studies in manufacturing in many cases have been experienced based so before the science of this was properly known people from experienced found that this should be the case for the riser design. So, now you can see that the rule basis which are taught in manufacturing processes can be fundamentally derived if you think of a mathematical analysis following heat transfer, ok.

Now, how can you apply it for a practical problem? Let us say you are measuring the temperature of a body with the aid of a thermometer or a thermocouple. What kind of thermometer or thermocouple will you want? Something with high time constant or with low time constant. In that case you want a low time constant because you want the thermometer to quickly adjust to the temperature of the body with which it is in contact, right.

Otherwise, it will take a long time for it to read the actual temperature.

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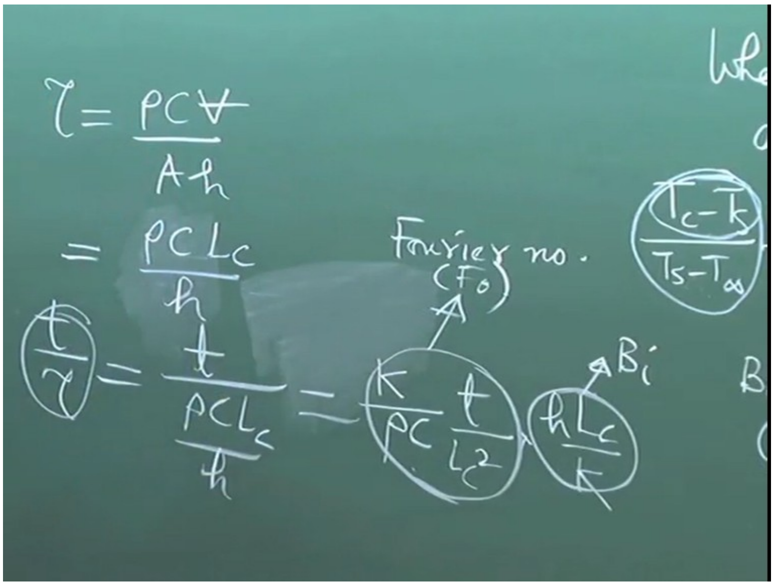

So, when you go for purchasing of a temperature measuring instrument you should look into the time constant and make sure that the time constant is small. Then how can we write this A h by rho C v into T, so you can just manipulate the algebra and you can write T by tau in this way. So, T by tau is T by rho C into L c by h, L c is viva A, ok. So, now you can multiply by K. Why you multiply by K? because you want to get the Biot number.

H L c by k, so this is a non-dimensional number. We have discussed about its physical significance earlier. This is also non-dimensional that means this is also non-dimensional. This is called as Fourier number.

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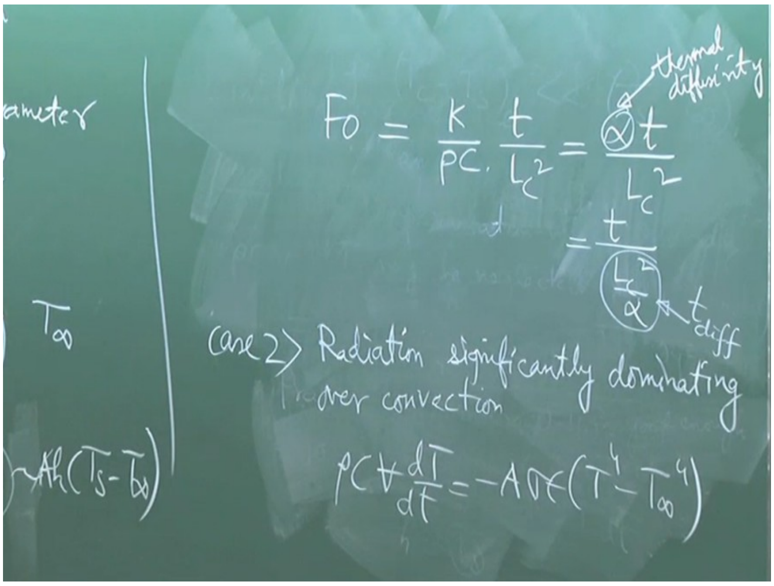

So, how do you define Fourier number? Fourier number where alpha is the thermal diffusivity. Alpha is K by rho C that is the thermal diffusivity. So, this can be written as T by

L c square by alpha. What is this? If this is unit of time, this is a non-dimensional number. This is also unit of time, this is called as diffusion time. Diffusion time is the time over which heat diffuses over a given length.

So, the time over which heat diffuses by a length L c is L c square by alpha, ok.