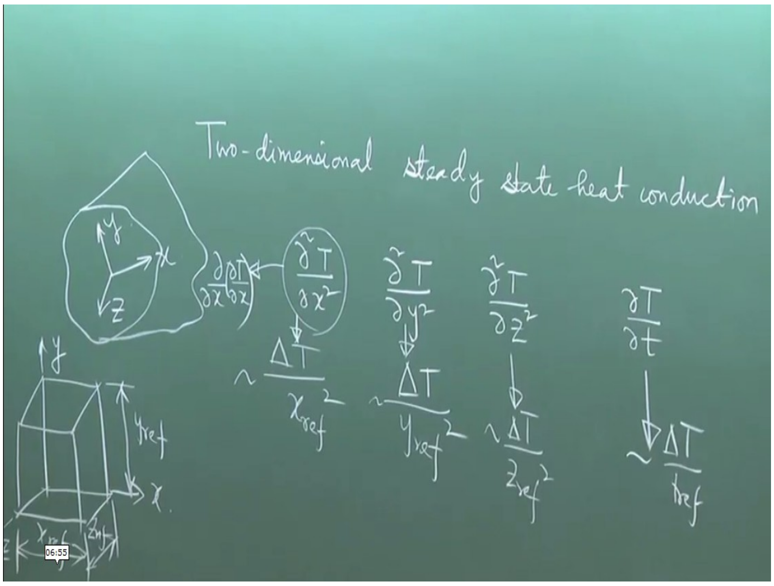
Conduction and Convection Heat Transfer

Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 24
Two-dimensional Steady State Conduction

So far, we have discussed about various cases of 1 dimensional steady state heat conduction. So, with us Prof. Som has discussed about those and today we will start with 2-dimensional steady state heat conduction. So, 1-dimensional problems have been discussed now we will discuss about 2-dimensional problems.

(Refer Slide Time: 00:54)



Now let us give a practical perspective to these type of problem. So, whenever you are solving a problem like in say Practical Engineering Industry or may be life science or whatever, nobody tells you that the problem is 1 dimensional, 2 dimensional or 3 dimensional. So, how do you assess whether a problem can be treated as a 1-dimensional problem, 2-dimensional problem, 3-dimensional problem.

Whether the problem is a steady state problem or an unsteady state problem. This must come from our own judgment and once that judgment is applied then we can apply the appropriate mathematical technique to solve the problem. So, let us think of that there is an arbitrary domain like this, some 3-dimensional domain. So, in this domain, you could have let us think of 3 octagonal directions x, y and z like this.

So, if you look at the heat conduction equation which we derive, you will figure out that in the heat conduction equation you will have terms like, these are special gradients and you also have unsteady term where the temperature variation with time govern by this type of a term. Of course, row c all these things are there I am not writing the full equation, I am just writing the important derivatives.

Now, let us say that you have a problem like this where there is a temperature difference imposed across the length of x reference. X reference is the length along x. What is x reference? So, let us may be draw cuboid like this. This is x reference, length along x. So, what is this basically? So, it is some characteristic delta T divided by some square of some characteristic length, that is the order of magnitude of these terms.

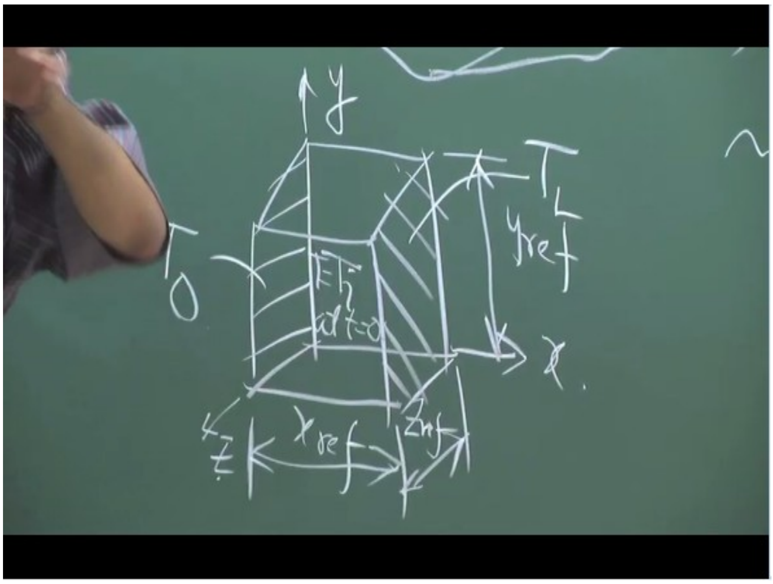
So, when we say order of magnitude by that we mean that what is like let us say maximum order. So, let us say that if x reference be the length then this is along x. Similarly, if the same delta T is imposed along y and let us say y reference be the length scale along y. Let us say this is x, this is y, this is z. So, this is of the order of remember this is not equal to this. It is just a rough estimate of its order of magnitude. So, this is delta T by x reference square.

This is delta T by y reference square. This is delta T by z reference square and this is T reference is the characteristic time scale of the problem that is the time over which you are studying the problem. It may be minutes, it may be hours depending on the time over which you are studying the problem. Now, the first keyword is steady state. I will discuss about this keyword very carefully.

So, when will a problem, a heat conduction problem be treatable like a steady state problem. When can we treat it as a steady state problem? So, of course from just a simple mathematical perspective we can treat it as a steady state problem when this term is not important right. So, when is this term not important? This term is not important when with respect to this T reference is large. That means what? That means the ratio of these two is small.

Now, how large, how small comparable to what and what we will come into that when we write the actual equation. But I am first trying to give you a very rough feel. As an engineer, it is very important that you get first an approximate rough feel for the problem and then you get into the mathematics rather than start with the bull work of mathematics from the beginning. So, if this is large and we are interested about characteristics of the large timescale. Then, it may not be important that we consider the unsteady problem.

(Refer Slide Time: 08:13)



Remember in the world, all problems are unsteady in energy because let us say that you have these kind of a situation and initially there is a particular temperature within the domain. Now you suddenly subject this body to particular boundary conditions. So, let us say you subject these 2 temperature T 0 and you subject these 2 temperature T L. T is equal to T I at time equal to 0 for all x, y, z.

So, in this problem the temperature from its initial state will start changing. It will start changing because of the impose boundary condition. So, what does the boundary condition physically do? A boundary condition helps the disturbance to propagate from the boundary to the interior of the domain and how is it possible? It is possible because of some material properties.

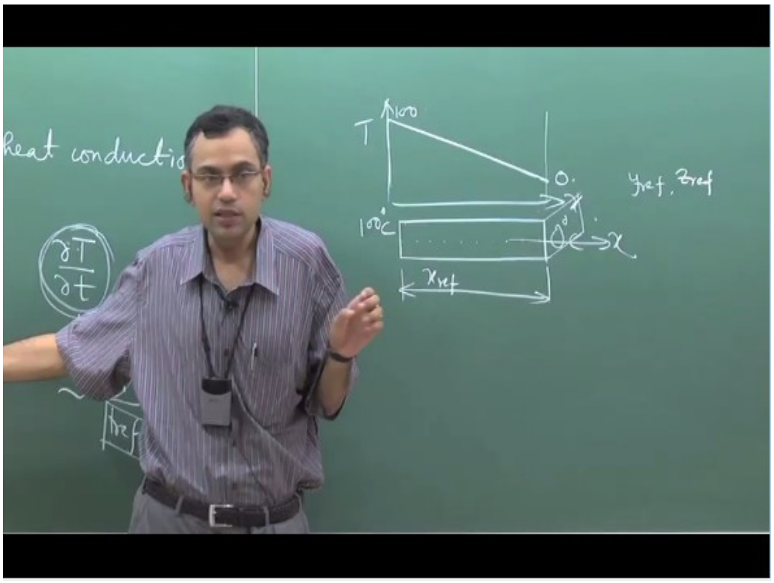
Here, in heat conduction the relevant material properties are K, row and CP or when you are talking about a solid it does not matter whether its CP or CV simply C. So, K, row and C. Now, if you are thinking about the change with respect to time may be there is some initial change because of the imposition of the boundary condition, but after that the changes stop taking with respect to time. Then, we say that steady state has attained.

So, steady state has attained when there is no further change with respect to time. When there is no further change with respect to time then what happens then the unsteady term the gradient with respect to time is no more important and then you can treat it as a steady state problem. Possibly beyond a critical time when it achieves steady state then you can use steady state formulation, but below a critical time when there is a continuous change of temperature within the body as a function of time then you have to consider unsteady state.

So, no problem is perfectly steady or unsteady state. Depends on your interest over the act, what timescale you are interested. If you are interested over the timescale over which the problem has already attained steady state then you go for steady state analysis. This is the first point about steady state. The second point about steady state is that there is a tremendous misunderstanding between 2 terminology, steady state and thermodynamic equilibrium.

So, in many scenarios people say people use these terms interchangeably. People say that the system has attained steady state and the system has attained equilibrium simultaneously.

(Refer Slide Time: 11:36)



Now, let us take an example, let us say I give an example of a 1-dimensional problem just to illustrate the concept, but same concept can be applied to multidimensional problems. Let us say there is a rod, this end is immersing steam at 100 degree centigrade, this end is immersed in ice bath at 0 degree centigrade, ok. So, the question is, is this an equilibrium? can this be in steady state.

So, in terms of equilibrium, when do we say that the body is in equilibrium or thermodynamic equilibrium? It is in thermal equilibrium, mechanical equilibrium and phase and chemical equilibrium. When all these things are satisfied we say that a system is in thermodynamic equilibrium. So, one of the requirements is thermal equilibrium. What is thermal equilibrium?

Thermal equilibrium means the entire system is at a homogeneous temperature. Because if there is some gradient in temperature then that will trigger a local heat transfer to take place. So, if the system had to be in thermal equilibrium then throughout this system the temperature had to be something, some value, but here I mean from simple analysis you can say that if you plot temperature as a function of may be x, this is 100 and this is 0, ok.

So that means the temperature within the domain continuously varies with x and it is not a constant. That means this is an example which shows that steady state need not necessarily mean thermodynamic equilibrium. There is a difference between steady state and thermodynamic equilibrium. So, when we are talking about steady state, we are not caring whether it is in thermodynamic equilibrium or not. That is not of interest to the study of heat transfers.

That is of interest to the study of equilibrium thermodynamic, but not of heat transfer. So, now the question is dimensionality of the problem. So, we are clear that out of these 3 terms if any one of the terms dominates over the other terms. Let us take an example x reference much much less than y reference, z reference, ok. So, if that be the case then out of these 3 terms which term is most important?

The first term right and then it is as if like a 1-dimensional problem. Now, let me bring out a fallacious situation here. Look at this problem, ok. In this problem, this is x, this is x reference right. Let us say this is a bar which has some dimension along y and z, y reference, z reference all those are there, those are not drawn and 1 dimensional problem is solved. So, in these problem by the shape of a rod or a bar it is quite clear that x reference is much much larger actually the other way.

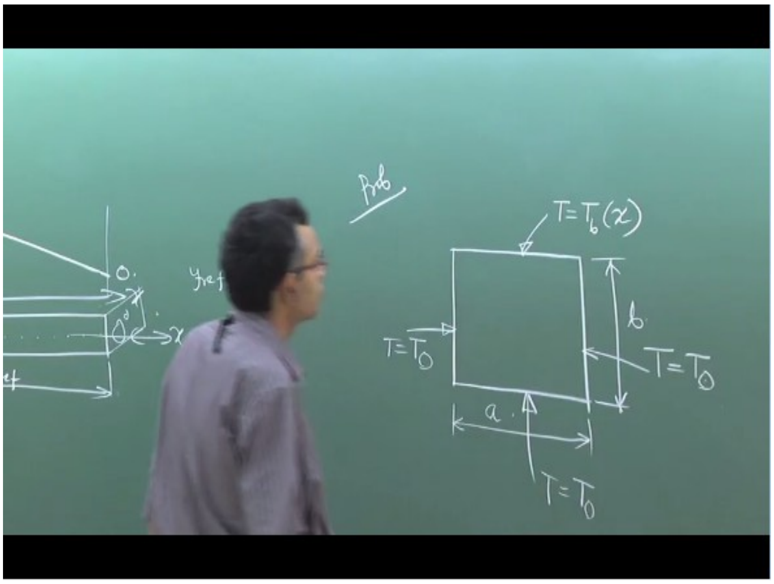
It is much much larger as compared to y reference or z reference, but still we are assuming this as a 1-dimensional problem along x, y. You understand my question? Here along x, the length is much larger. So, ideally if you go by this logic then it should be like the y reference and the z reference being small, it should be at least a 2-dimensional problem in the y, z plane, but we are considering a 1-dimensional problem along the x direction.

So, what is the inconsistency? If at all there is any inconsistency. See, here we have assumed that the same delta T is applied in all directions right. In this problem, the delta T is applied only along x direction. In other directions, no delta T is applied. So, it is not just the length. You also have to see what is the delta T that is imposed right. Here, the delta T is imposed along x, so the temperature gradient will be created along x, ok.

Now, it is quite clear that when we consider steady 1 dimensional, 2 dimensional, 3 dimensional, when we consider unsteady all these things must be very clear before we solve a problem. See when we solve a problem what we essentially try to do is we try to make a mathematical model. What is a mathematical model? A mathematical model is an abstraction of the physical reality.

It is not exactly what is the physical reality right. Otherwise every problem has to be solved 3 dimensional or unsteady, but mathematically when we simplify a problem we take into account some practical considerations and simplify the problem to an extent that it is mathematically simplified to the extent possible.

(Refer Slide Time: 18:05)



So, we will straight away get into a problem of a 2-dimensional scenario which will be our agenda for today's discussion. So, let us take a problem like this that you have a domain, rectangular domain with length a long x, length b along y, ok. Now let us assume that we are interested about a solution when steady state has already been attained. Later on, we will see that if you are interested for a solution when steady state has not yet been attained.

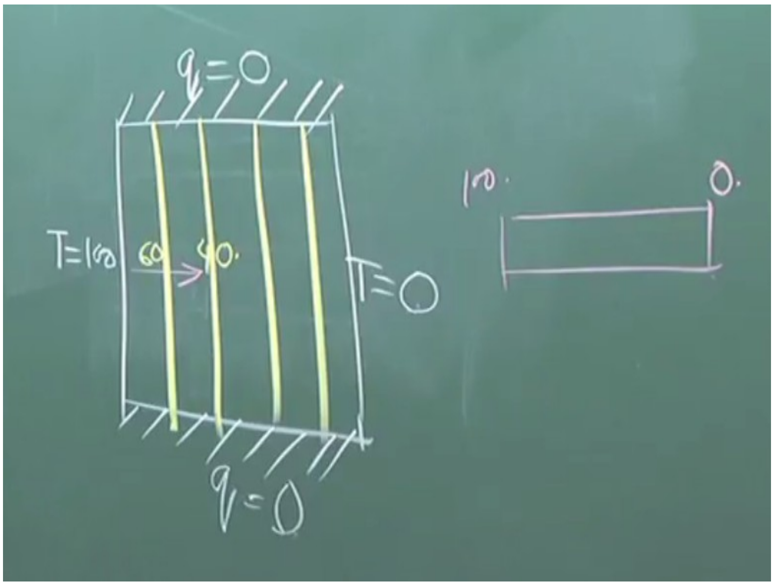
How to go for the solution, but to begin with let us assume that steady state has already been attained and we are interested for the solution. So, now can you tell whether it is a steady state problem and the largest dimensionality of this problem is 2 right. It is not a 3 dimensional problem because everything is planar and all the temperature gradients are imposed in the x, y plane.

Now, a similar problem can you think of when the problem looks like 2-D, but actually it's almost a 1-D problem. This is a 2-D problem. Why this is a 2-D problem? It is quite clear that you have a temperature gradient here. You have a temperature gradient here, you might argue that where is the temperature gradient here? This temperature and this temperature is the same, but that shows that not everywhere within the domain the temperature is same.

But there is a periodicity in the solution. Because there is a temperature gradient along y and the governing equation has some total of the effect of temperature gradient along y and temperature gradient along x. So, to make the sum total equal to 0, some temperature gradient along x should come right. So, it is a 2-dimensional steady state problem. We will solve this problem today.

But before that I want to capture your attention by looking into a scenario, when the scenario the schematic the drawing looks very similar to this, but the problem is actually a 1-dimensional problem, ok. Let us try to find that out.

(Refer Slide Time: 21:41)



So, let us say that these 2 boundaries are insulated. In heat transfer, insulated boundaries are often drawn by this hatched lines, ok. You have seen in thermodynamics also that insulated boundaries are often schematically represented like this. So, these are insulated that means there is no heat flux. Let us say this is T equal to 100, this is T equal to 0, ok. So, can you sketch the isotherm lines here. What are isotherms?

Isotherms are lines of constant temperature. So, across the isotherm you have the temperature gradient, ok. So, let us try to draw it with a different color. So, you can see that when you look into a problem, always look into 2 things, one is geometry because of course geometry dictates the dimensionality. If you have already a 2-dimensional geometry there is no question that the problem will be 3 dimensional.

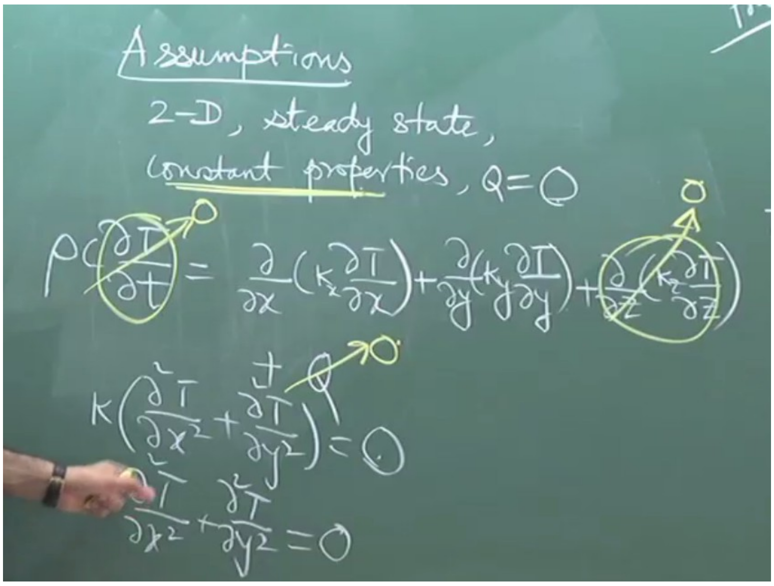
If you have a 2-dimensional geometry the maximum dimensionality of the problem is 2 dimensional, but you also have to look into the boundary conditions. So, here the boundary conditions are such that see no heat is able to get transferred along these directions. So, the primary direction of heat transfer is like this. Because the primary direction of heat transfer is like this, it is almost like the same problem where you have these 1-dimensional rod, here 100 and here 0, ok.

And the isotherms you will be getting will be like this. So, this is 100, let us say we draw it as some intervals so let us say this is 60, this is 40, I mean in this way it goes to 0 here. So that this line is the line where along which everywhere temperature is 60. So, you can see that this lines are vertical lines that means the gradient is along the horizontal. So, although the geometry is 2 dimensional, the boundary condition says that the problem is primarily a 1-dimensional problem.

Of course, again it is not perfectly a 1-dimensional problem. If you are a great perfectionist, it is a 2-dimensional problem. But for all practical purposes, the temperature gradient along y is not of great interest here. The great interest is temperature gradient along x. So now we will come back to the solution of this problem. So, let us make some assumptions. So, what are the assumptions? 2 dimensional steady state constant properties.

So, if we write the governing equation. Let us write the full governing equation.

(Refer Slide Time: 25:25)



This is a solid so any term related to compressibility is not coming into the picture. Now, first of all it's a 2-dimensional problem, because it's a 2-dimensional problem - this term is not important. Then it's a steady state problem, so this term is 0 ok. Now once you say steady state problem, when you talk about constant properties, it is not necessary that all properties have to be constant just thermal conductivity constant is good enough.

Because once you say that it's a steady state problem row and C are no more important. So, whether they are constant or they are variable that will not make any sense. It is important that what K is and of course constant properties means that K is constant and then we will assume there is no heat generation. So, when K is a constant, K will come out of the derivatives, so you will have. Because K is not equal to 0, so you must have.

This prototype is not just important in heat transfer, but in any branch of applied physics and engineering and this is known as Laplace's equation. Of course, this is Laplace's equation in 2 dimensions. If you have another term like this for z that is Laplace's equation along 3 dimensions.