Conduction and Convection Heat Transfer
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Lecture – 23
Problems on Heat Transfer From Extended Surface

Now after this we will solve some problem.

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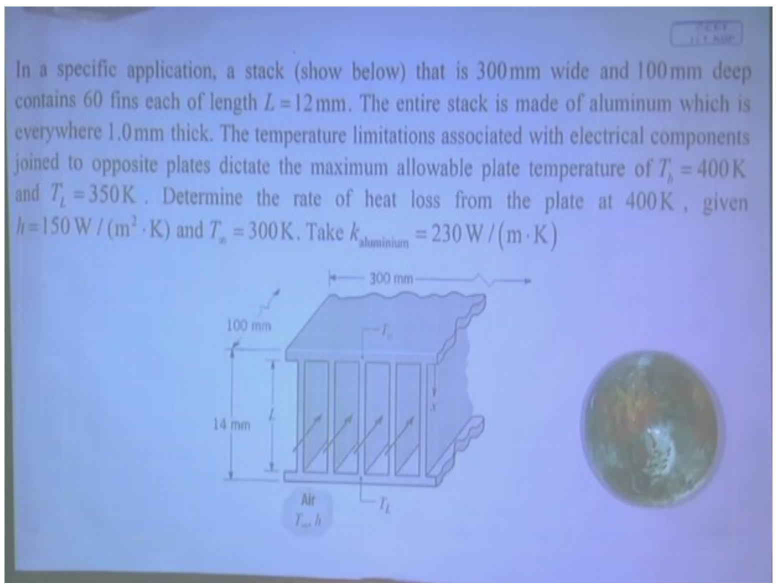

Let me have a very interesting problem that you can take this problem. In a specific application, a stack shown below this is a stack that is 300 mm wide and 100 mm deep contains 60 fins, five are shown that means there are number of fins is a very big size 60 fin. Each of length L, 12 mm. This is the length of the fin this is 12 mm and this is the thickness of the stack plate this is 1.0 mm that is why this is quoted in mm.

This is the length of fin. 60 fin each of length 12mm. The entire stake is made of aluminum which is everywhere 1.0 mm thick that means this one 1.0 mm this one. This was not shown in dimension this is width. The temperature limitation associated with electrical components jointed to opposite plates that this plate. This side of the plate dictate the maximum allowable plate temperature of 400 K.

That means this plate should not be at temperature above 400 K, electronic components will be damaged and TL is 350 K this side the plate is kept at 350 k by an arrangement of air with a T infinity and h heat transfer coefficient we are not much bothered in this side because the problem is prescribed by fixing the 350 K at this temperature. Determine the rate of heat loss from the plate at 400 K that means what is the rate of heat lost from the plane at 400 K.

What are the value given h is 150 watt, per meter square K, T infinity is 300 k. These are the value given h is 150 watt per meter square K, 300 K. Aluminum thermal conductivity that mean the material thermal conductivity 230 watt per meter K. This is precisely the problem so we have to find out the heat transfer from the plate.

Now heat transfer from the plate is due to the heat transfer from the fin which is the heat transfer from this area of the bases where the fin is attached plus the heat transfer from the un-fin portion of the base plus the heat transfer from the top surface of the plane that you have to recognize to solve this problem. Problem is very simple but one thing this type of problem in examination if it comes we will give you the formula here.

We have to use this heat transfer from the fin where the fin is fixed at two temperatures that means this you consider as base temperature of 400 K and if you consider the fin of length L which is given as what is the length 12 mm the other end is, what is the temperature, 300 K that means fixed temperature at the end. No question of mugging the formula but the formula will be provided or even a big problem you may be told that you derived this formula.

That means you start with governing differential equation theta is C1 into the power mx plus C2 into the power minus mx by control volume energy balance then theta B is C1 plus C2 and x is equal to zero then theta L is C1 into the power ML plus into the power minus ML from there you can also derive but if teacher is relatively better he will not ask that routine derivation because it is tedious. He will still use this formula for things with temperature prescribed at 2M, base temperature and other. So, problem boils down to that. So, now let me solve this problem. This is 100 mm. This is the flow of air.

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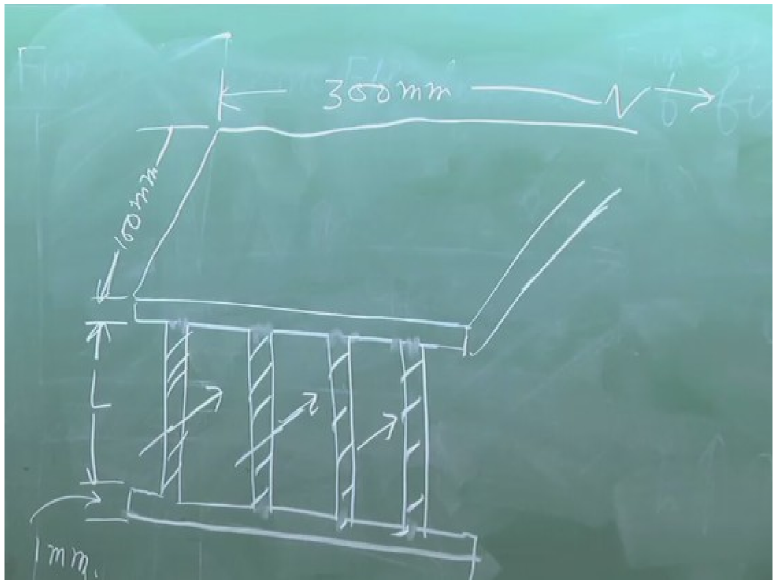

Now this is the geometry 100 mm depth, 300 mm width, this is continued this is a cut section, numbers of fin 60, fin length L and this is air flowing through this which give the convective heat transfer coefficient which takes the heat from the lateral surface of the fin this is the problem. Now let me first write this is the plate, Plate temperature let we write it as Tb base temperature that plate this is the temperature Tl is 300 K and Tb is 400 K and T infinity.

That is the surrounding fluid is at, the infinity is what? The infinity is 300 K TL is not 300 K, 350 K sorry and this is 300 K. Now let us write the equation, the expression for Q plate is what? Q plate is first two fin, numbers of fin there are 60 fins, identical fin, 60 identical fin. If fin efficiency could have been given I could have been happiest man then we could have found out that a surface area times eight, times TB minus the infinity times the efficiency.

But it is not given that means either I have to mug up that formula or I have to derive. Mugging mean what now a days no teacher gives that you have to mug up and you just write that. However, if it is given then it is good but it is a tedious calculation. Let me see do not mind. That formula is per fin root over hPKA all the nomenclature is known to us as they are conventional to thermal conductivity, cross sectional area, perimeter, heat transfer coefficient into theta b.

Then it is cos hyperbolic mL minus theta L by theta b with all nomenclature known to us divided by Sin hyperbolic mL. So, this is the expression of heat transfer when theta b, theta l are fixed for a fin and number of fin 60. This is the heat transfer from the fin surface then plus heat transfer from the un-fin surface what is the un-fin surface that area is what un fin surface is .1 into .3. Now I will put this value afterward.

Now un-fin surface we write this way A un-fin that I will find out afterward into h into Tb minus T infinity, write simply theta b then A plate, h theta b this is only a tedious calculation nothing else is there. Now you write down the value h is what? h is 150 watt power meter square K. P is what perimeter tell me what is P? P is the perimeter that means this way 100 plus that means .1 plus .001 into 2. It is almost equal to 0.2-meter perimeter.

Perimeter is what this side you understand the perimeter that means this plus this. Perimeter is all right. Lateral surface take this 100 then 1 mm, clear. This is the fin, perimeter now what we require K. What is K? 230 watt per meter K. What is cross sectional area A of the fin is .001 into .1 is equal to 10 to the power minus 4 meter square. A is PKA, everything is there. Now A un-fin, what is A un-fin equals to 0.3 into 0.1 minus 60 into 10 to the power minus four.

This is very simple meter square. Clear? that means, this is the un-fin portion this portion let area A plate is 0.3 into 0.1 meter square that means .03 clear?

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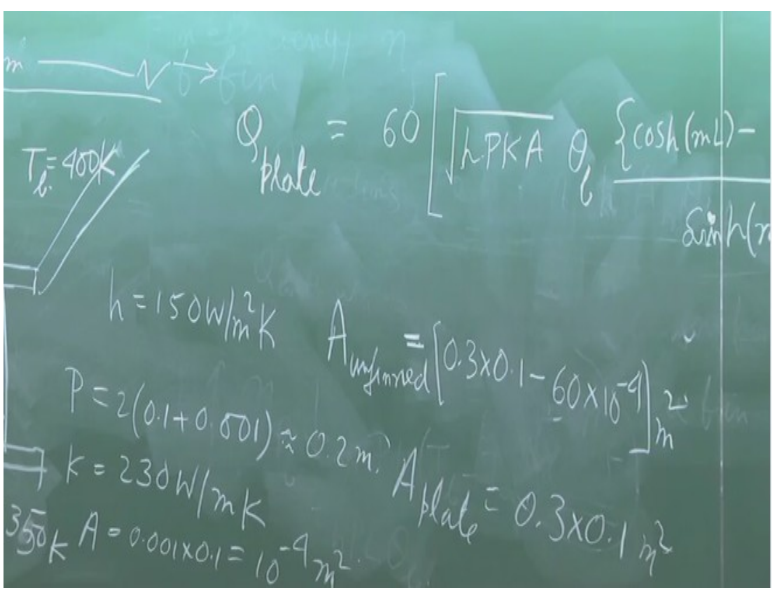

This is the plate area 300 mm into 100 mm. Un-fin portion, this area minus the cross-sectional area of each fin times 60 times. That's all. So, fin problem I am tell you are all direct applications of the equations that we have developed but you don't have to remember the equation you will only derive the equation. This problem is clear now. If you put all the values in the proper unit then you will see that the order is like this value comes out to be I tell you 6631 watt.

Un-fin portion heat transfer is 375 watt and the top surface of the plate 451 watt. This plate means top surface. A plate means this is top surface this is top surface. Now one thing which I like to mention a comment that you see most of the heat transfer take place through the fin is increased order of magnitude. This is because of what? The huge surface area provided by the 60 fin. Well here.

Bottom surface, this is the bottom surface un fin portion only heat transfer from this place this is the bottom surface. This plate this is not required the question is, what is the heat transfer from this top plate. This is not required this entire stack this part of the stack is not that is not required. Yes, correct you are interested in the entire system heat transfer but this is not asked for. It is only from the top plate.

So, top plate has two surfaces bottom surface through fin and un-fin and you see through fin and un-fin the ratio is more than an overall magnitude. Which side, no that is insulated these lateral surfaces these surfaces all these surfaces fin they transfer heat to the air. That is the convective visual from this side that that is taken care by this formula fin heat transferred from the plate top surface of the plate. This surface this end surface of the plate from the end surface of the plate?

Oh! that is neglected end affect are neglected. He is telling that end surface of the plate that means 100 mm into 1 mm that cross sectional area, very good. That is neglected. Good question. That is neglected. That is not taken. Very good. By neglecting that means from this exposed faces. This face and this face which has a cross sectional area 100 mm into 1 mm eight times the Tb minus T infinity. Good. No that is neglected. Okay. Any other question?

Now I will show you some other problem.

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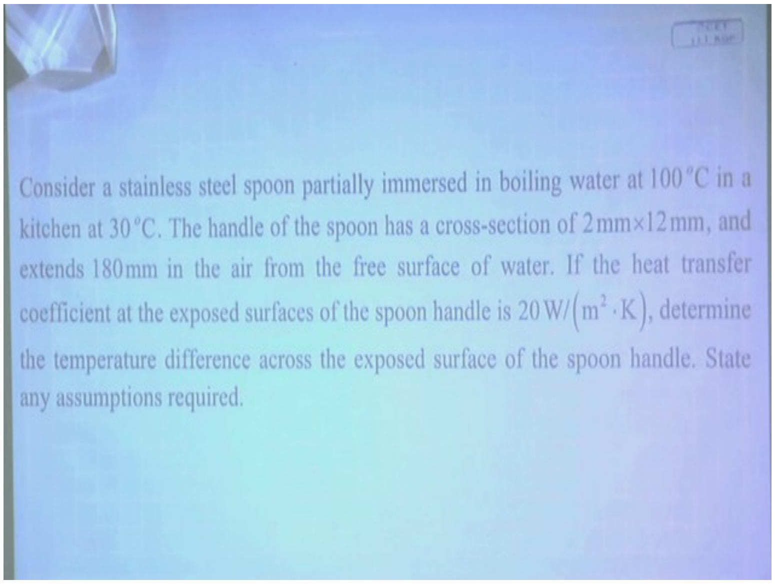

Consider a stainless-steel spoon partially immersed in boiling water at 100 degree Celsius slowly you may write it in your own language is time is less so all the words can be written or you can in a kitchen at 30 degree Celsius. Now immediately you compile in your own language 100 degree is the base temperature and 30 degree is the kitchen temperature means surrounding fluid T infinity Tb.

The handle of the spoon has a cross-section of 2 mm x 12 mm, 24 mm square is a and extends 180mm in the air that is the length of the EP from the free surface of water. The soon is extending 180 mm in the air from the free surface of water that means the length of the fin. If the heat transfer coefficient at the exposed surfaces of the spoon and handle is 20 watt per meter square, K that means in all in languages.

Similarly, compiled h is 20 watt per meter square K, determine the temperature difference across the exposed surface of the spoon handle. That means Tb minus Tl take any assumptions required. So, can you tell me this is a fin problem immediately you compile Tb T infinity, cross sectional area length so what is the boundary condition which formula I will use or which formula I will derive theta is T1 into the power of mx plus into the power of minus mx.

I asked one boundary condition know that 100 degree Celsius is the Tb what is that boundary condition? What does it say? HKFC. Very good, speak loudly KfC that means it is a practical case no insolation is there. Oh god. No insolation is there. This is not very infinitely long end temperature is not given that mean there is conductive heat loss at the end. So, that n negative field as a convective heat loss.

So, KfC you will derive the equation we have to derive the equation and use it and get it done. That's all. So, I am not solving this problem because these are only application for power reason. Now another problem.

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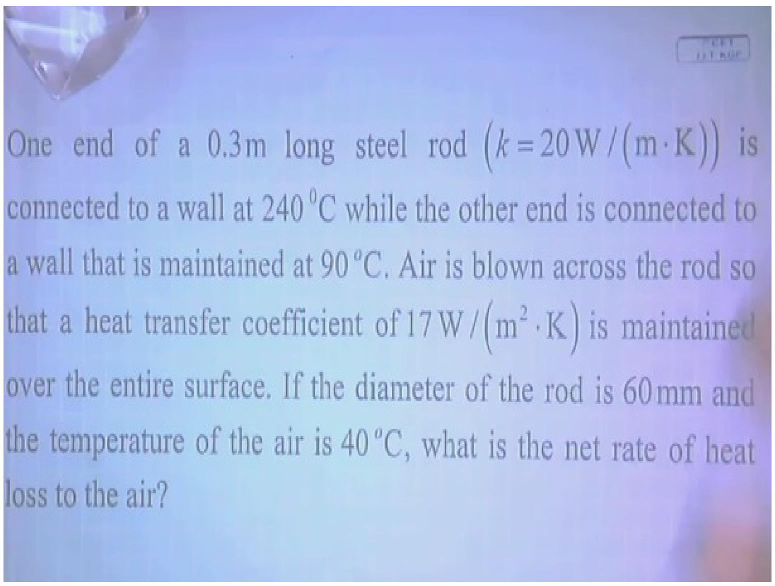

This is one very popular type of problem most of the book has this problem. One end of a 0.3 meter long steel rod 20 watt per meter K this is usually the thermal conductivity of steel is connected to a wall at 240 degree Celsius while the other end is connected is a wall that is maintained at 90 degree Celsius. Air is blown across the rod so that a heat transfer coefficient of 17 watt per meter square K is maintained over the entire surface.

If the diameter of the rod is 60 mm and the temperature of the air is 40 degree Celsius what is the net rate of heat loss to the air? That means fin with 60 mm. These are the representative fin problem. So, before concluding the session a generalized formula coming up my lecture about one dimensional steady heat conduction with generation of heat thermal energy without generation of thermal energy cylindrical, spherical or plain wall.

And also fin with lateral convection can be summed up like that. If you consider a variable area wall and take this as a direction of heat flow x which may be x which may be r whatever may be I am denoting it as x where like heat flux is there which may not be same at all section if heat generation is there we cannot say so. If the lateral convection is there we cannot say so that we have seen.

Now let us consider a case that there is lateral convection. I take an element at a distance x with a thickness delta x and I donate Q x as the heat coming in to this face with the in accordance with this nomenclature x and Q x plus delta x is the heat going out and this wall generate heat which is specified by heat generation per unit volume q G at point which is a function of x in general or may be constant whatever may be.

q G may be a function of x, may be constant, may be a function of (()) (01:02:10) function of x because T is a function of x that means T G may vary with x and T and also there is convective heat transfer as I have shown you Q convection from the lateral surface. Now if I have to write the energy balance then I can write that Q x plus delta x which is coming out from this section is equal to Q x plus q G into the area A x. A x is here which is function of x.

That is why I am writing a x into delta x and at the same time, it continually call this heat just like just like leaking pipe. It is very similar to fluid flow through a pipe where the surfaces are perforated as the fluid is flowing the water is leaking and ultimately the entire water is drained out at the end, no fluid is coming. No liquid is coming. This type is problem is interesting that from lateral surface the liquid is drained out.

Similarly, the heat is being also convected out minus. Now this convected heat transfer to take care of h, is specified T infinity specified and I use a nomenclature perimeter. So, the P delta is my area and at that location if the temperature is T at x then this will be T minus T infinity. That means Q x plus delta x is qx coming plus this is generating, generational energy minus the convection.

Now Q x plus delta x minus I take it in this side or I take this side that Q x minus Q x plus delta x plus q G S x delta x minus h P into T minus T infinity delta x is zero. Now Q x minus Q x plus delta x we have done several times extending this in delta series and neglecting the higher order term and then substituting the Fourier heat conduction equation Q x is minus k d t, d x. K x a d t d x. Q x plus delta x is Q x plus d d x of delta d x.

That means it will be minus d d x of Q x delta x that is the minus, minus plus that means d by d x of K A x d t by dx delta x plus q G A x delta x minus h P into T minus T infinity delta x is zero.

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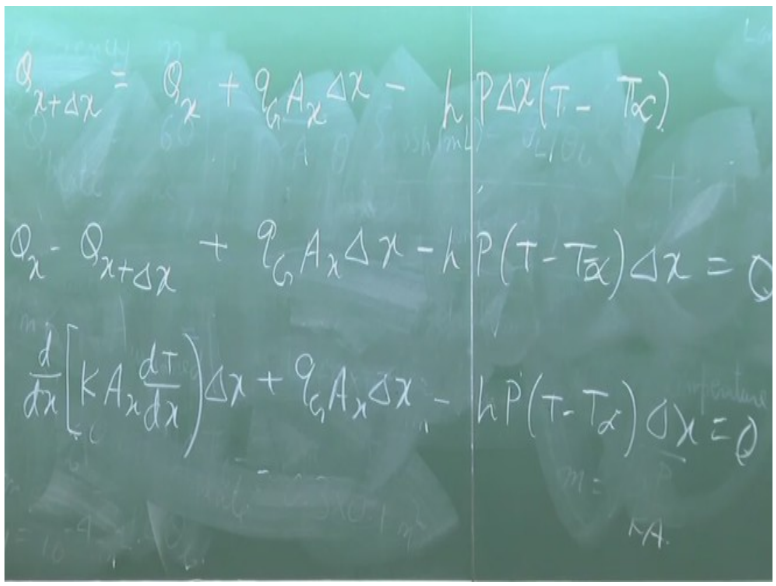

Or we can write again this thing that d by d x of K A x, dt by dx plus qG Ax minus hP, T minus T infinity is zero and this is precisely the expression for one dimensional heat conduction, with generational of thermal energy and lateral convection heat is going out laterally. Now here in a plane geometry now in cylindrical coordinate x is A r, x is r, cylindrical rod, cylindrical wall axis twice pi rL flowing in the r direction.

In a spherical coordinate x is R and AR will be 4 pi R square. Now if we consider that there is no thermal energy generation this is the fin problem usually fins no thermal energy is generated. If you don't allow the liking of heat laterally I told you stimulation liquid is flowing through a pipe if perforation is there at the surface all liquids will be (()) (01:07:17) that means, there is no lateral convection which we discussed earlier then this plus this is equal to zero and if you take no heat generation this part will be zero.

If we take thermal conductivity constant this will come out and if we take x is constant that is happening only in plane area, plain surface with the same cross-sectional area than the most simple case is d dx or dt dx, zero that means we have a linear temperature profile. So, this expression gives you or if you write this in terms of Qx d dx of Qx then you get the expression for that means another version of this is Q x plus q g A x variable area that means it is the most general expressions.

These two are the same equation in terms of total heat transfer rate in terms of the temperature you can get a generalized expression for heat transfer rate and temperature distribution if any steady one-dimensional flow with or without thermal energy generation, with or without convection from the lateral surface. So, fin is one application of the one-dimensional steady heat conduction problem that that extended surface.

Where one dimensional heat conduction is associated with lateral convection and this is take care of in the simple energy balance. All this thing oh very good. Minus Q a nice, nice very good. Any question? So, with this derivation this is the summery of all my lectures in steady state one dimensional heat conduction. Application is the extended surfaces fin. Any question, ask. The end of the entire conduction chapter we will have one session for tutorial.

Though I have solved number of problems in the class while taking this theory few more problems will be solved, teaching assistance will be there they will solve problems we will there also. Any questions before conclusion I tell you that next class we will discuss two-dimensional steady state heat conduction and that will be taken by Prof. Suman Chakraborty.