Conduction and Convection Heat Transfer
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 22
Boundary Condition at the FIN Tip – II

Good Afternoon, I welcome you all to this session of Conduction and Convection Heat Transfer. Last class we were discussing heat transfer through extended surfaces know as Fin. The situation is that extended surfaces are attached to a base surface to enhance the rate of heat transfer from the base surface and these extended surfaces provide additional surface area for convective heat transfer from its lateral surfaces along with conductions through these surfaces and through this.

That is by providing additional area for convective heat transfer from lateral surfaces the fin or extended surfaces enhance the rate of heat transfer from the base surface. And what we discuss we formulated the differential equation for temperature distribution by applying the conservation of energy to an element in consideration of one dimensional heat conduction through the field.

That is the extended surface along with the convection from the lateral surface and we are (()) (1:38) for the temperature distribution for different cases or situation of the field which impose the different boundary condition for the solution of the differential equation for temperature. Today, we just have an overall summery before that I tell you different types of Fins that are used.

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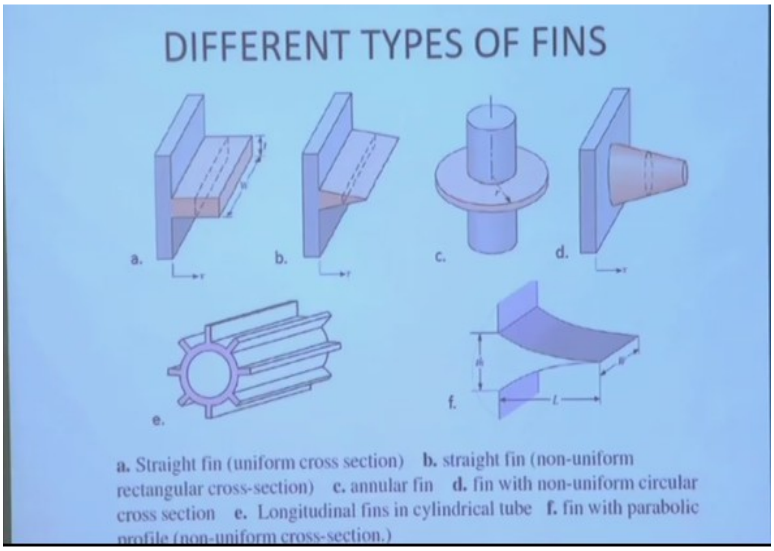

Overall view, this is the Straight rectangular fin is the most simple case that the cross-sectional area of the fin is constant, same. This is the base surface as I have just told this is the extended surface know as fin which provide additional surface, this surface top, bottom this surface these are the surfaces, this lateral surface all this surfaces additional surfaces for convective heat transfer along with the conduction to these fin along this direction for which the heat transfer from this base surface where it is (()) (02:52) in hand.

This was discussed and from the conservation of energy principle. We developed the differential equation for temperature and we solve this under different boundary condition, different situation. These are different types of fins, straight fin but non-uniform cross-section. See this is an annular fin sometimes it is known as circumferential fin, in a cylindrical chamber.

These are used sometime in heat exchanges this type of fin are used the circumferential these are annular fin, number of fin may be used here also there may be number of fins only one fin is shown. This one fin with non-uniform circular cross section like a conical shape, circular cross section this is sometimes known as pin fin. Pin with non-uniform circular cross-section this is same thing in a cylindrical tube where the heat transfer from the surface.

For all these case, the heat transfer from the base surface is enhanced by attaching these extended surfaces or pins. Here the pins are longitudinal, parallel to the axis of the cylinder that’s why this is known as longitudinal fin pin cylindrical tube. This is a fin similar type like this is a parabolic profile non-uniform different types of pins are used. This you can get in any classical text book nothing great.

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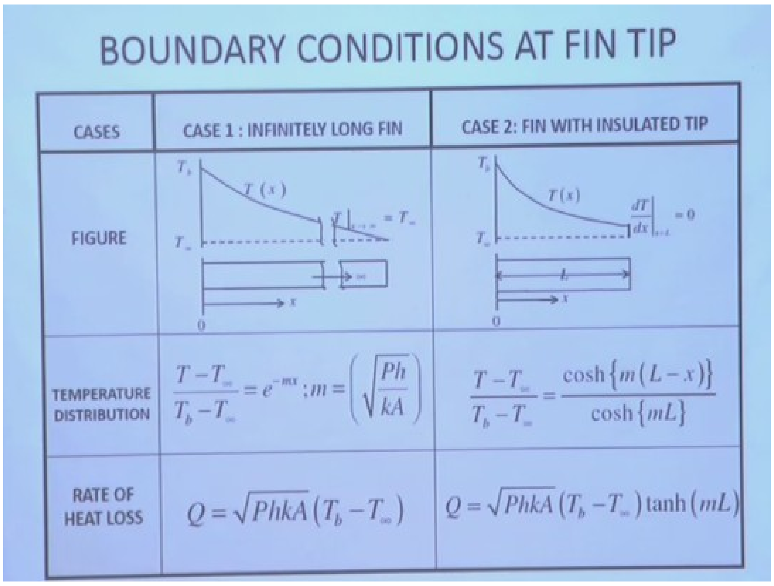

Now these are the things which we discussed last class that we first discussed infinitely long fin where the base is at a temperature Tb, base temperature is same for all cases. Now if the fin is infinitely long we impose a boundary condition from our physical fin that ultimately the temperature at these exposed ends becomes the surrounding fluid temperature T infinity. In that case our temperature distribution is exponentially decreasing.

It will be bar minus mx, T minus T infinity by Tb minus T infinity. Tb is the base temperature. T is the temperature at any section x. So, x ten to infinity, it is the infinity so both for T infinity another thing is that this type of solution at the same time gives a zero (()) (5:23) dt dx is zero x, x is got to infinity and x ten to infinity and the heat transfer rate accordingly is given by this hP or PhkA root over Tb minus T infinity.

Case two discuss fin with insulated tip where we impose the boundary condition dt and dx, zero. That means fin tip is insulated no heat is being transferred from these exposed surface, this surface that means all the heat which is being extracted from the base is being convected from the lateral surfaces. So, that here the heat transfer is zero that is the physical function. Boundary condition is imposed in terms of the temperature gradient whenever there is an insulated surface, adiabatic surface heat transfer boundary condition is in conduction dt by dx is zero because heat transfer is zero like fluid mechanics shear stress is 0, du dy zero, one dimensional case.

Similar thing so that the temperature profile does not attend the T infinity temperature. This temperature is higher than T infinity because of this insulation but the slope is zero. The shape of the curve is such, it becomes parallel to the (()) (6:38) so that dt, dx is zero.

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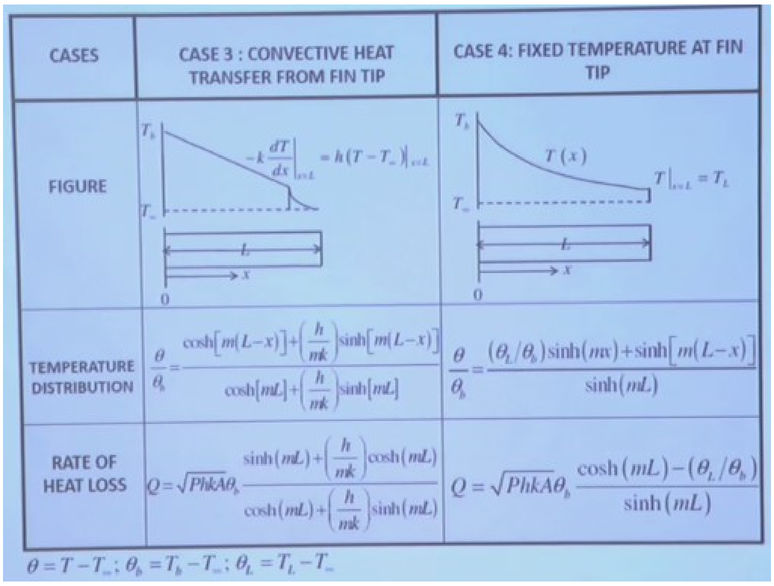

CASES	CASE 3 : CONVECTIVE HEAT TRANSFER FROM FIN TIP	CASE 4: FIXED TEMPERATURE AT FIN TIP
FIGURE	T_b, $-k\frac{dT}{dx}\Big\vert_{x=L} = h(T-T_\infty)\Big\vert_{x=L}$, T_∞, L, x, 0	T_b, $T(x)$, $T\vert_{x=L} = T_L$, T_∞, L, x, 0
TEMPERATURE DISTRIBUTION	$\frac{\theta}{\theta_b} = \frac{\cosh[m(L-x)] + \left(\frac{h}{mk}\right)\sinh[m(L-x)]}{\cosh[mL] + \left(\frac{h}{mk}\right)\sinh[mL]}$	$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh(mx) + \sinh[m(L-x)]}{\sinh(mL)}$
RATE OF HEAT LOSS	$Q = \sqrt{PhkA}\,\theta_b \frac{\sinh(mL) + \left(\frac{h}{mk}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{h}{mk}\right)\sinh(mL)}$	$Q = \sqrt{PhkA}\,\theta_b \frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$

$\theta = T - T_\infty;\ \theta_b = T_b - T_\infty;\ \theta_L = T_L - T_\infty$

Number three case that also we discussed in the class that here we impose the most practical situation that we do not insulate these it is open through the atmosphere that means the fin shared heat by convection through this lateral surface, this bottom surface. The side surfaces, both the sides and all through out these exposed surfaces at this end, extreme end. So, this exposed surfaces heat transfer we prescribe the condition of governing through the infinity.

So, therefore while solving the equation within the fin as x is equal to a here well this diagram is little wrong I am sorry to tell you probably it is wrong from the place where it is taken this point this line should coincide with the length of the field because this part is outside. So, it is wrong from the place where it is taken you see event the published thing are sometimes this is because of our overlooking the error while reading the proof.

So, therefore here what happens the temperature changes and it has a flow not zero flow which determine the heat transfer minus K, gt by dx and should be matched with the convective heat transfer at these interface that is this solid exposed surface with the surrounding gas so that this slope is matching with this and this is the temperature distribution by convection after this that means from this surface there is a decrease in temperature in a thin film due to convection which will be dealt in details in convection classes when we will take convection heat transfer.

So, this comes to b this is to understand. And in this case with this type of boundary condition it becomes a routine calculation with little complication as a big manipulation are little tedious so that we get a temperature distribution theta by theta b nomenclature you know theta is the express temperature over the surrounding fluid temperature. Theta d is the base temperature over the surrounding fluid temperature that means theta, the temperature over the excess fluid temperature.

So, accordingly the heat flux is given by these expressions. This gives hyperbolic functions. Another case was pick the pin temperature at the end that means pin temperature is fixed at the end by some arrangement of pulling fluid we keep a fixed fin temperature we are not much bothered how it is done but we think mathematically as if the temperature is given as x is equal to Lca.

So, solve the equation we know the boundary condition x equal to Lt is equal to Tl in that case the temperature distribution is like that and it does not reach the infinity and the temperature distribution if you find out it will be like this. So, this is the fixed temperature at, sorry fixed temperature at the end of the field. Temperature distribution and the heat flux. What is meant by Fin Efficiency? Let us have this base plate.

This is the base and this is the rectangular fin, now Fin efficient, what I meant by fin efficient? Fin efficiency eta f. The fin efficient is defined like this now if the base temperature is Tb the heat is transferred and the amount of heat is transferred Q from this portion of the base where the fin is attached and this portion of the base where pin is attached a huge amount of heat transfer take place because of this heat transfer from this lateral surfaces.

You can show this surface like this also that means the top surfaces this surface, bottom surface this surface again this surface back side all these surfaces the front side this surfaces and this exposed surfaces everywhere the convection Q convection, from this surface also the lateral surface Q convection, through this we already we deal in the last class. Q convection and from this surface also finally convection heat transfer is there.

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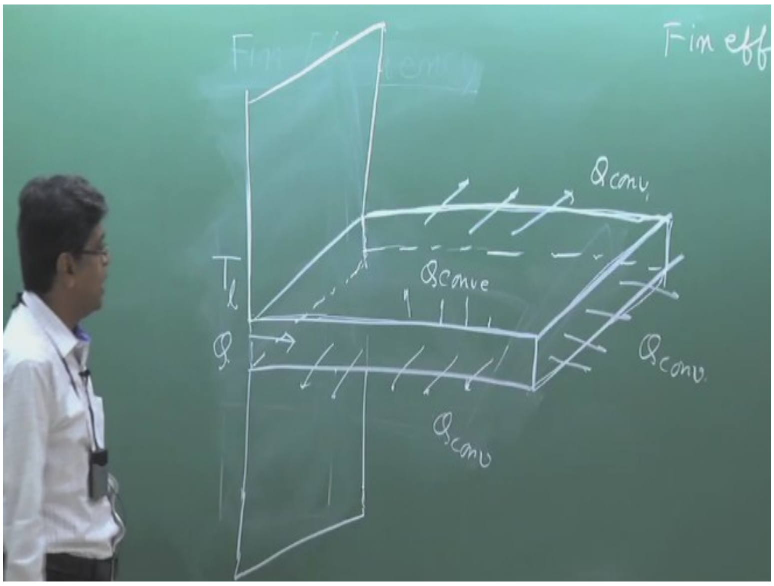

If it is not insulated that mean this side, this side top surface, bottom surface that means all lateral surfaces convection heat transfer is there due to this large surface area the convective heat transfer is huge. So, that finally this heat that is coming from this base where it is attached is in hand that is the basic principle of fin. But one thing what we assume that it is one dimensional problem usually for the last fin.

So, that we consider this direction is the x that means this is the x direction from this plane so at any x the temperature is T which is same in this direction fin thickness of the fin or height whatever you can tell that means same temperature but it changes (()) (15:56) and this T is always less than Tb that we have already seen that temperature is maximum at this point and then slowly decreases to a temperature and free stream temperature is T infinite now with this knowledge in background how does the fin enhances the heat transfer.

The efficiency is defined as the actual heat transferred from the base that means this portion of the base where the fin is added let us write this as Qa that is the heat transfer through the base I am not writing it I am telling heat transfer through the base with the attachment of the fin in actual case. So, why I am telling actual case, this case is actual why? That is because the denominator defines a quantity Qb which is the heat transfer, rate of heat transfer from the base where the fin is attached.

If the entire fin would have been at the base temperature that means the entire fin attains the uniform temperature that of the base when it can attain if the thermal conductivity of the fin is infinitely high. Infinitely high thermal conductivity of the fin otherwise in actual case the fin temperature decreases the along the direction x but at any x this is same for all the lateral surfaces.

This we took in deriving the temperature distribution the differential equation for temperature distribution from energy conservation field. So, this is the definition that heat transfer I may write this. Rate of heat transfer from the base with fin. This is the actual rate of heat transfer and this one is rate of heat transfer from the base if the entire pin is at base temperature.

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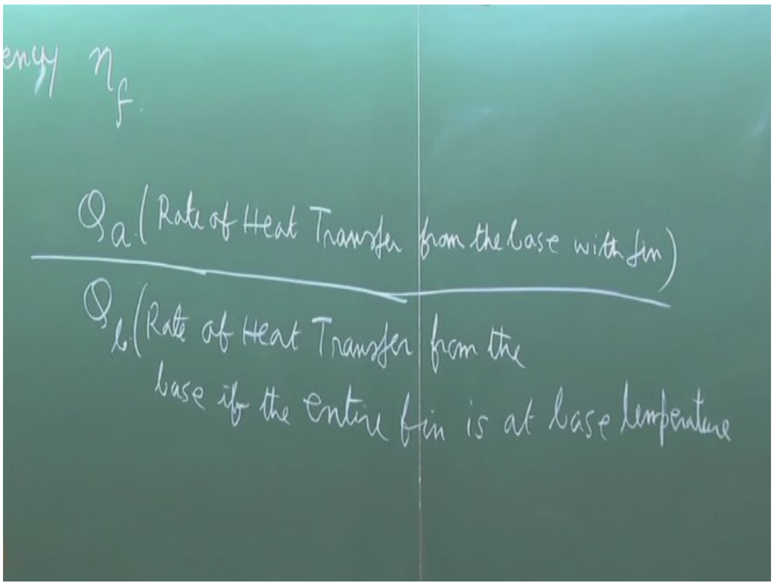

Now Qb therefore can be written like this if h is the heat transfer coefficient with our nomenclature, p as the perimeter of the field that means some of this dimension this plus, this into two this side and the bottom surface that is the perimeter into the length of the fin, times the Tb minus T infinite, because the entire fin is that base temperature. So, there for the ideal heat transfer that means considering the fin to be infinitely thermal conductive so that having infinite thermal conductivity hPL theta b.

Therefore, the expression of fin efficiency depends upon the expression of QA if you take long fin where the fin length L tends to infinity very large then we know that Qa is equal to this Qa root over hPKA theta b. So, there for in this case theta f is equal to root over hPAA theta b divided by hPL theta b and this becomes if you cancel it 1 by mL where m is given by our nomenclature that root over we denoted m as root over hP by kA.

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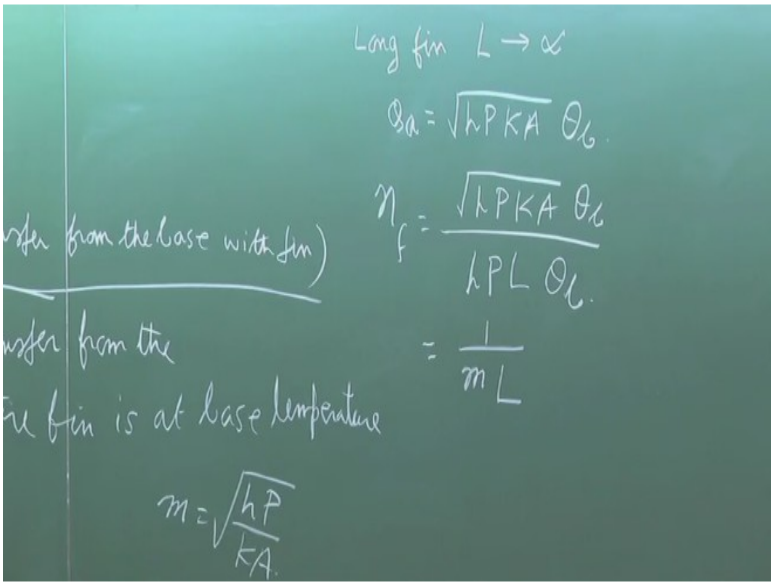

So, it is very simple that you substitute the expression in the numerator for the actual heat transfer in case fin with insulated tip. What will be theta f, theta f will be this is hPL theta b and heat transfer which we drive. I have shown it also it will be root over hPKA same thing but tan tan hyperbolic mL into theta b. m is the nomenclature root over hP by K and this becomes tan hyperbolic into mL divided by mL.

So, depending upon the cases you can find out the efficiency expression so there for the question of fin efficiency comes because of this conducting at the first class I told that we are putting a conduction resistance so we are adding a more convection area to enhance the heat transfer. But if the conduction resistance is very large because of very low thermal conductivity. So, that there is a drastic dropping temperature along this direction.

So, heat transfer from the lateral surface may not be augmented that way because the heat transfer depends not only on the surface area but also on the temperature difference. So, therefore there is a race between the two that we have to be careful that the fin shouldn’t have a large temperature drop we should have a high thermal conductivity and in the most ideal case is that thermal conductivity is that the entire fin at base temperature.

So, if this you compare you define a parameter like this which is known fin efficiency.

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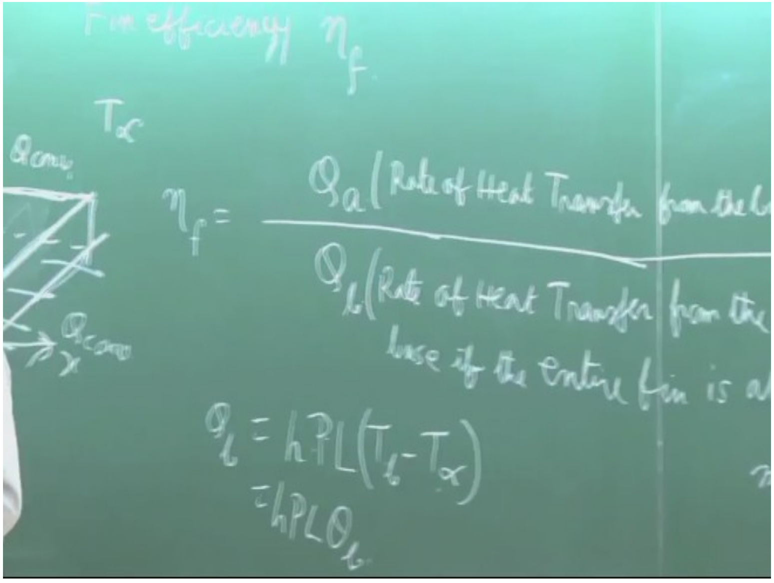

Now here we see a number of fin, array of fin a system with array of fin. This is the base plate number of fins are there. Here there are three fins it is shown. This wall of a temperature of Tb and the surrounding fluid at T. So, here how do you define the fin efficiency? Same thing but little calculation that fin efficiency here it is defined as the Q from the system fin array system Q Fin array system divided by Q from the same Fin array system at Tb.

That means, the entire system is at Tb. What is the difference? Difference is that here Fin array system when you consider Q then we have to consider the heat transfer from the Fin also the heat transfer from the un fin portion. Now if we had a nomenclature like this let us consider Af is the total surface area of fin and if make this nomenclature At is the total area, total heat transfer area, total surface area.

And this surface area means heat transfer area which takes part in heat transfer. Total surface area including both the surface all surface area fin plus uncovered portion that means these are the uncovered portion AP.

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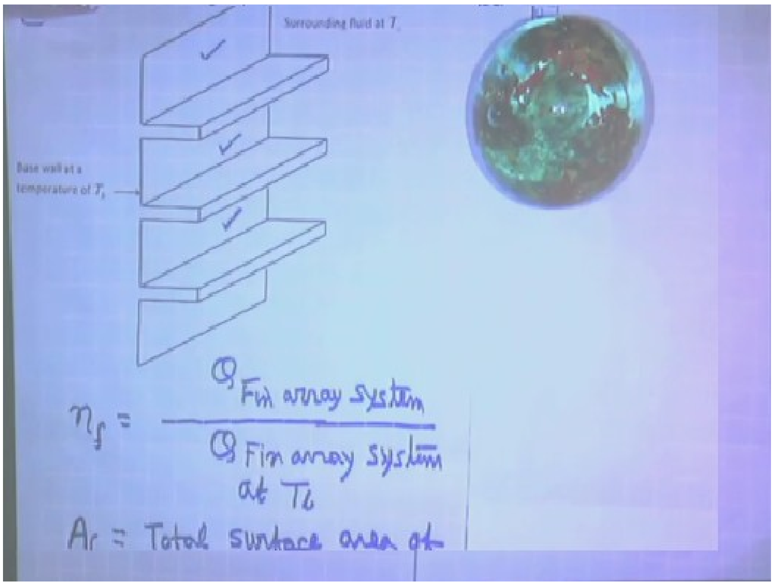

AP is the total surface area that means uncovered portion plus fin then we can write here Q the numerator QF, system can be written as QFin system is the heat transfer from the fin that means A is the total surface area of the fin then h into Af into theta b, what is theta b, here I write theta b is the Tb minus the infinity. Now here what I want to tell you that how we can write the efficiency of fin system in terms of the individual fin efficient.

If we assume the individual fin efficiency is beta f which is constant, same for all the fin then we multiply the this with eta f gives the heat transfer from all the field, h A theta b considering fin at base temperature. So, this is the ideal heat transfer and this is multiplied by the fin efficacy of the individual fin as it was defined earlier is the heat transfer from all the fin. Last the un fin portion that is A total minus Af that is the area into h into theta b and the denominator will be the total area At into h into theta b.

That means how to express this if we know the total surface area of the fin and the total surface area including the un fin portion plus fin so minus the fin portion this is the area of the un-fin portion At minus A. This can be written in a simple expression taking At, h theta b separate term 1 minus theta b canceled out, h canceled out At h, theta bh canceled out one minus Af by At into one minus theta f, efficiency theta f sorry, it will be the efficiency of the array system, efficiency fin system not Qa.

I am sorry, it is efficiency of –I wanted to first start with this with the numerator. So, I have done at the same time. I first started to write this expression in the numerator however does not matter the numerator is the Q that is heat transfer of the fin system that is the heat transfer from the fin, that is the heat transfer from the un-fin portion so theta for the fin system is divided is by At, h, theta b divided by the heat transfer of the entire base with the fin when they are all at the temperature Tb.

So, when eta f then equal to 1, eta fin system also equals to one. That means we can express the efficiency of a fin array system in terms of efficiency of individual fin.

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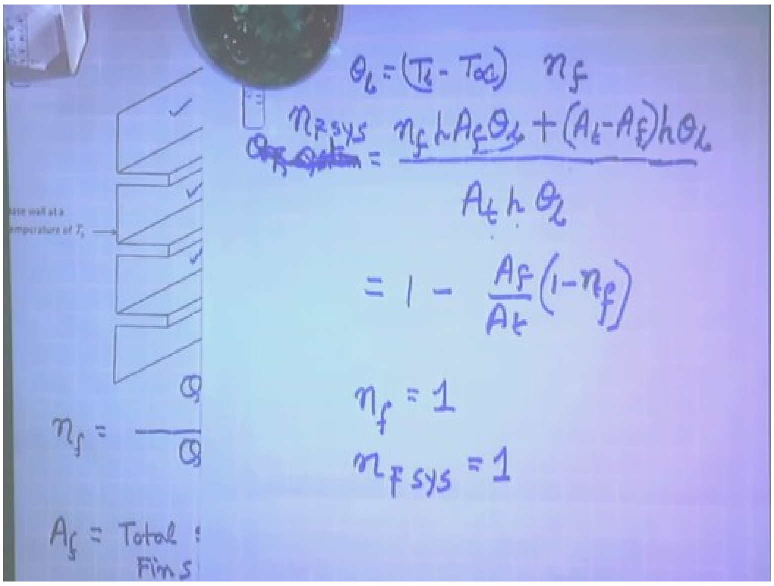

Now, another important concept in practice I will tell you which is very important for engineer we define fin efficacy okay this is the definition we understand the ideal case a highly conductive fin within superconductor whose temperature is equal to the base temperature and we define the fin efficiency the actual heat transfer with fins and what is the heat transfer if the entire system with fin that is the same base temperature.

Now question arises whether how much we gain by adding fin sometimes it happens that okay there is a monotonic gain but what is the return whether it is economic to use that after certain stage. Under certain case whether fins are really required or not if we have a marginal gain in enhancing the heat transfer then cost will be provide you cannot judge from the economic point of view.

Because you have purchase fin you have to go for manufacturing attaching the fin surfaces the entire fin system has to be fabricated. So, there for a question comes effectiveness of the fin to judge whether it is effective to use fin. Effectiveness of fin that means to (()) (31:48) Let us define a parameter the similar way as the efficiency but e effectiveness that means it is known as effectiveness but frankly speaking physically efficiency and effectiveness is same.

Effectiveness of performance is a efficiency but here it is defined in a little different way. Efficiency is the effectiveness of his performance when it is performing but now I will judge what is the difference between the two that is performed or it does not perform that means if we define effectiveness as Q with fin or Q without fin. Efficiency by definition is always less than one.

Because fin can never attain Tb temperature like (()) (32:48) cycle like one is the theoretical limit but here also it is always better than one we have seen always heat transfer will increase but how much what is the gain? Let us see that. Now in a very simple expression if I write that this is equal to efficiency, fin for example h fin area, surface area of the fin into theta b.

That means simply I multiply with the efficiency of the field h, heat transfer coefficient A,b,c surface area that is provided by the fin and what is this without fin. Without fin base area theta b that means I get an expression Af by Ab into eta f is so simple but it says many things.

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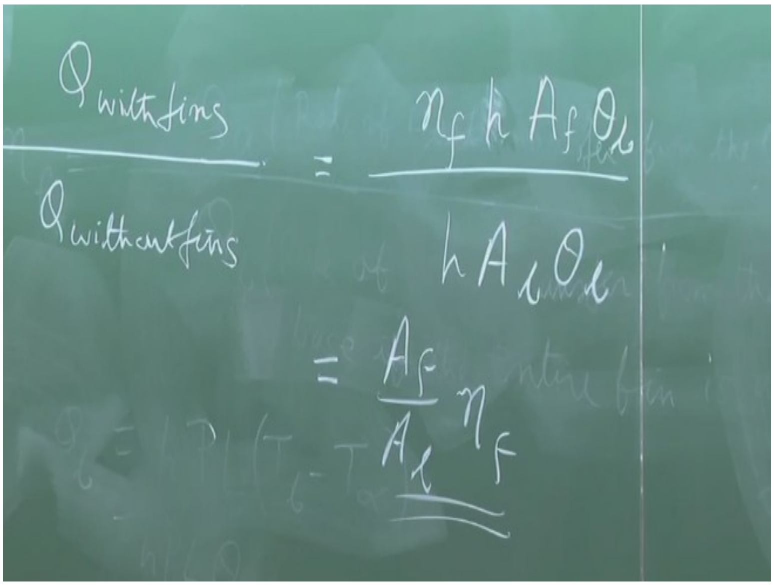

That means I gain by using fin because I have a huge value of Af by Ab but I should also consider if eta f is not also not very low. If you multiply with a high value with a very low value eta is very low my gain may not be very high that means I have provided additional surface. But efficiency is not a function of surface area, you go through the expression. The answer to the question lies in the fact that with increasing value of h with eta h decreases.

You have to be careful about eta h if we have a low eta f, there is no point gain is less, very first class that is why I started fin by telling that this provides a conduction resistance though we create more surface area to enhance the heat transfer by convection but provides a conduction resistance and this conduction resistance becomes more and the efficiency becomes less if you have a high heat transfer coefficient.

So, there for one has to judge it but without going for any detailed analysis which is beyond the scope of this class I tell you as an information that where h is high usually fins are not provided because of low eta and we cannot provide a very large surface area because not only the cost but the space, space is a requirement where I am he is telling the (()) (35:37) so therefore we have to fragment enough to have a meaningful gain which is a product of Af by Ab into eta f.

For fluid, especially liquid which has a high thermal conductivity in both convections if they very high heat transfer coefficient in boiling condensation phase change process have high heat transfer coefficient there, theta is usually low we do not usually give fin. And from a very ready common-sense case when the heat transfer coefficient is very high the exposed where surfaced will be enabled, will be capable enough to transfer heat.

Why you are adding, why you are giving him, you are brilliant student, why are you giving him a tutor. He himself can make it up like that. So, therefore sometimes these question is asked if you have a heat exchanger one side liquid is going and turbulator are there to create more turbulence in the flow field and another side gas is moving with a low velocity which side you will provide fin if you tell gas side you get full mark.

If you say (()) (36:55) liquid side flowing with high turbulence, get out. Zero. Because this is the concept that effectiveness of the fin that means fin shouldn't be used where the heat transfer coefficient is high this is because this side.