Conduction and Convection Heat Transfer
Prof. S. K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 21
Boundary Condition at the FIN Tip - I

And the boundary conditions comes from the physical problem define to it boundary and other conditions that means the boundary conditions have to be physically defined.

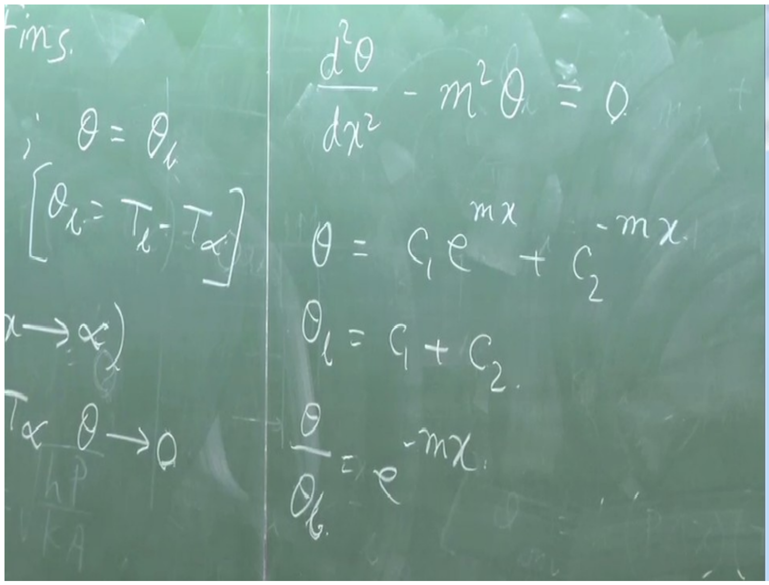
One boundary condition that is defined or prescribed by the physics is that, base temperature is Tb where from x is measured. So, therefore that x=0, T=Tb which means theta is equal to theta b. Theta b means Tb minus T infinitive that mean Theta in terms of Theta it is Theta b. What is another boundary condition? One another boundary condition, you have to search for another boundary condition with respect to x that means you have to find out what is the boundary condition at x is equal to in.

If L is the length of the fin, so you have to know whether it is insulated or something else. So, one very simple case, there are various cases, one case very long fin, sometimes in problem we tell that fin is very long, very long fin, engineers are always smart, they always take an approximation, they do not like mathematician. Very long mean consider x tens to infinitive that means the very long fin.

And if x is very long tens to infinitive then eventually the trailing surface the excrement of the fin will attend the environmental temperature, very good. So, this boundary condition is that at – as it is written as x tens to infinitive T tens to, obviously this is mathematics T infinitive and Theta tens to 0. This is the simple one. So, first boundary condition gives you Theta b C1plusC2. Now in this case if you consider a very long fin so automatically you will see C1 is 0.

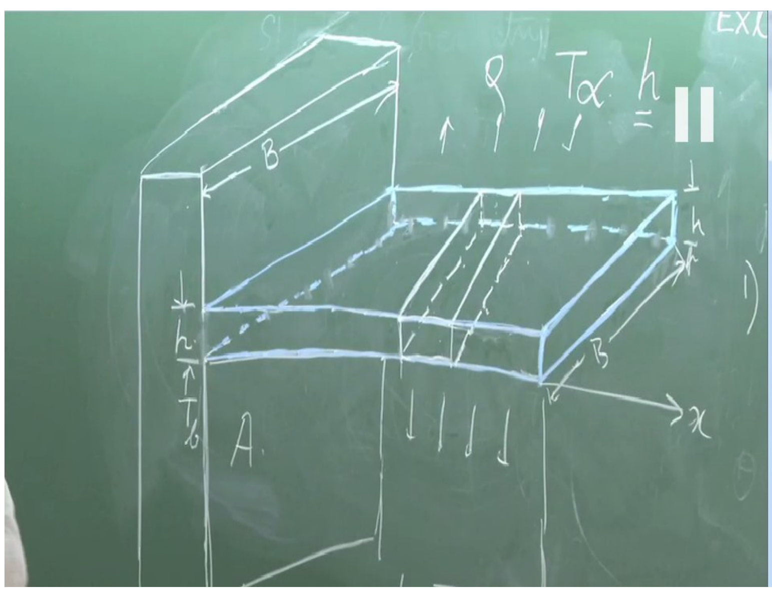
Because if you make this x tens to infinitive this term vanishes and if Theta is 0 means C1 is 0, that means the solution is that Theta by Theta b is e power minus mx that means there is an exponential of temperature to what 0. Very simple, that means if you draw this graph exponential, sorry I will show this thing afterward.

(Refer Slide Time: 29:35)



Theta by Theta b is e power minus mx at x at x tens to infinitive, Theta tens to 0. So, what is the value of Q? Now Q means what? What is Q? Try to understand.

(Refer Slide Time: 29:53)

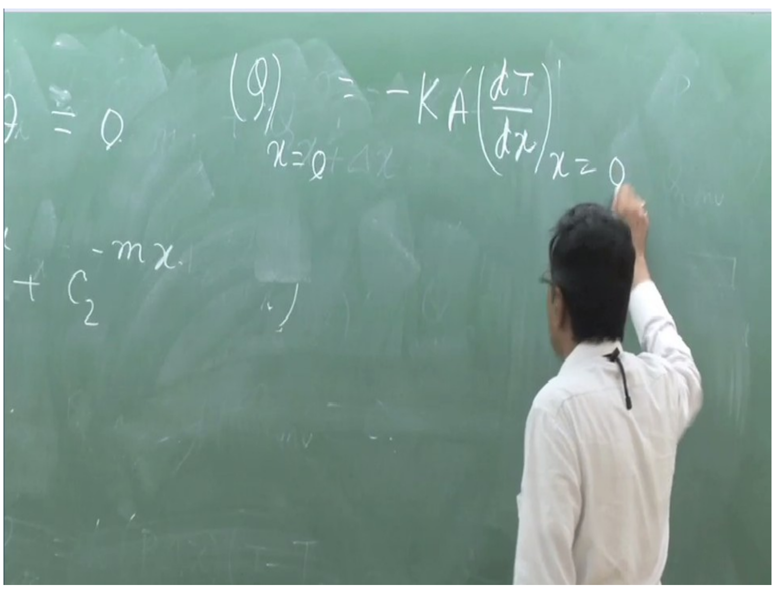


Q at every section is changing. What happens again try to understand this, these surfaces can transfer heat that means that any section that heat which is coming in is getting transferred by convection from the lateral surface and the rest part is being conducted. That means the heat conduction is getting through this is reduced that means the heat which is taking from that base extracting from the base by the fin is almost giving to the atmosphere why is lateral surface due to convection and a very less amount is being convicted from this surface.

And in a long fin, the entire heat which is takes from the base is being convicted by the lateral surface, because when it reaches this surface expose the atmosphere at the excrement, it has reached almost T infinitive, because T tens to T if you know, heat transfer, heat transfer is 0, delta T, delta T is 0. However, this is the concept, we will go by mathematics, we are interested at Qx is equal to 0. What is the heat transfer from the base?

Engineers are interested, heat transfer how much one-minute base I have attached a fin what is my heat transfer rate from the base that means how much heat it is extracting from the base.

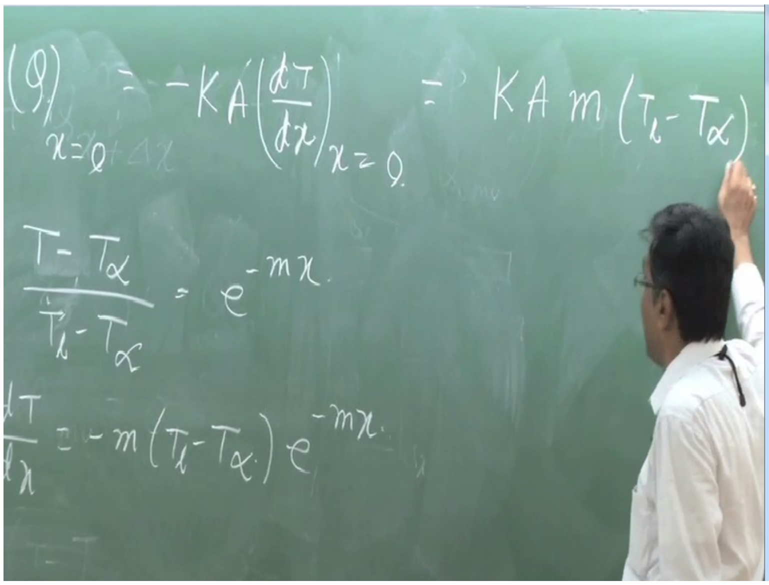
(Refer Slide Time: 31:18)



Okay I will tell you very simple minus K area dT/dX, this area now I am using as the area of the fin, earlier when I talk to you to introduce the problem I told this is the area of the wall A but now whenever I am using area A this is for the cross-sectional area of the fin. In the equations also I derived that, do not get confuse with this area. So, this is the cross-sectional area of the fine and in this problem, we are considering a rectangular area having constant cross-sectional area.

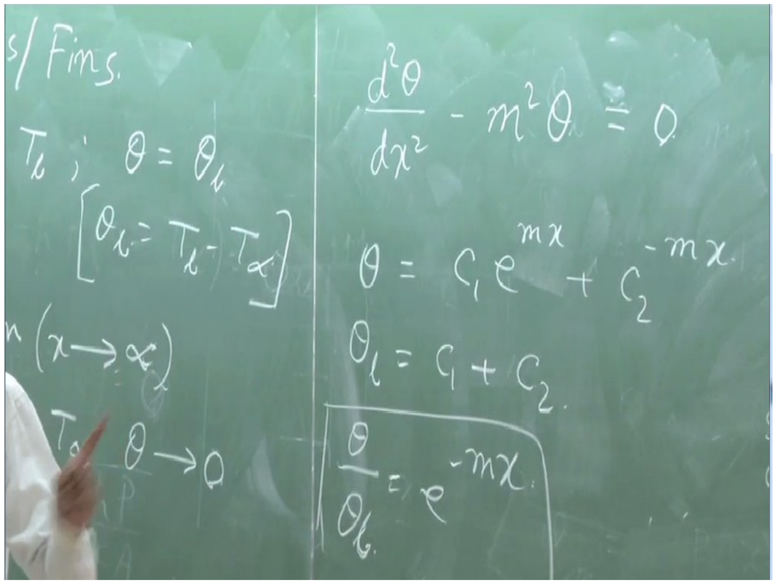
So, therefore this is the cross-sectional area A, so therefore –KA and dT/dX at x=0.

(Refer Slide Time: 32:17)



So, what is dT/dX at x=0? Theta by Theta b means I write yet T minus T infinitive divided by Tb in terms of temperature decoding the variable transform, variable into the actual temperature is e power minus mx, so therefore it is very simple that dT/dX is equal to – m (Tb minus T infinitive) e power minus mx and that x=0 the exponential function will be 1 so it will be –m Tb minus, minus will plus that is KAm into Tb minus T infinitive.

(Refer Slide Time: 33:10)



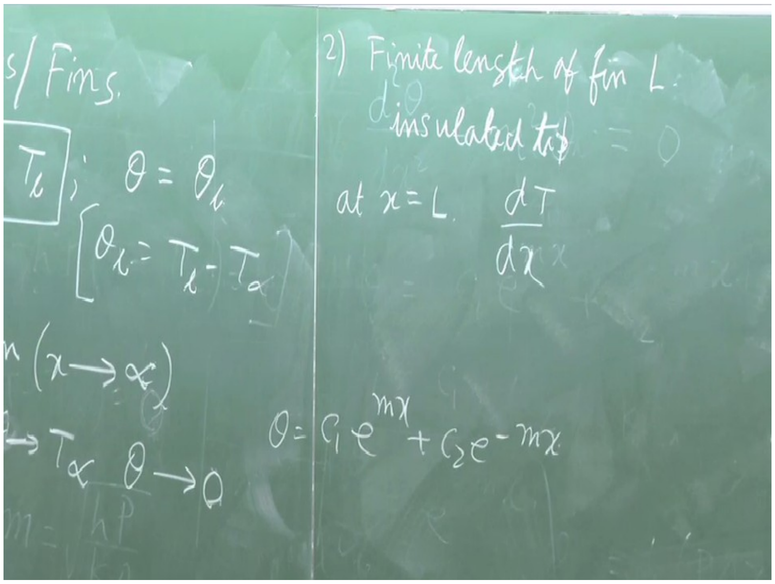
Now m is root over hP/KA I defined earlier, again I write m was defined as root over hP/KA. And if we put that then Q at x=0 becomes root over hPKA * Tb minus T infinitive which can be written as Theta. So, this is the heat transfer from the base and this is the temperature distribution. You do not have to remember any formula but you have to know how it has been derived.

Again, and again I am telling you, you have to be capable of generating the governing differential equation by taking an element with the understanding, physical understanding of the conservation of energy value, what is happening, something is coming, it will go whether generation inside, whether there is a lateral convection altogether you have to develop the basic equation.

And then slowly you have to think that which are constants given in the problem or everything is varying then it becomes a problem of mathematics, how complicated it will be. And if you do it meticulously you will arrive at any equation. In the examination, also if any problem is there we have to derive the equations. So, do not mug up that Theta by Theta b to the power of minus mx, Q is equal to root over Hp K, Theta b for a long fin, not required.

But sequentially, how the deduction is made understand a problem both physics and the mathematics.

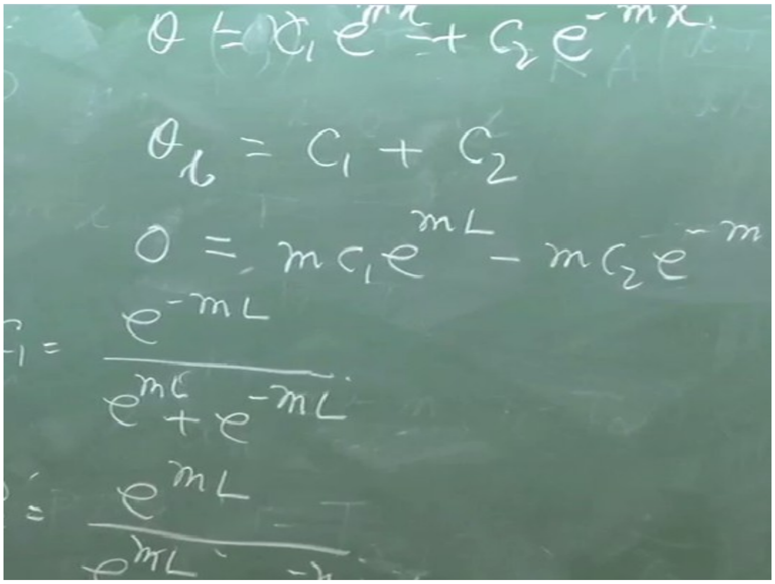
(Refer Slide Time: 35:30)



Now the second group, second boundary condition is another type of fins, secondary boundary condition means another type of fin. We may consider. Note long fin, Finite length of fin, let it be L. But, insulated tip that means tip is insulated; insulating material is pasted on it that means no heat will be transfer. So, this boundary condition is common to all, this temperature Tb. So, what is the second boundary condition, in this case will please tell at x = L.

What is the boundary condition if it is insulated? In which form of boundary, it will come if it is insulated means no heat transfer, please anybody flux 0 means I have to solve the temperature equation. dT/dX 0 who has told it? dT dX is 0 you tell that because I have to solve the, I have to solve which equation? Theta is C1 e power mx plus C2 e power minus mx. I have to solve this equation, let me write this here.

(Refer Slide Time: 37:31)

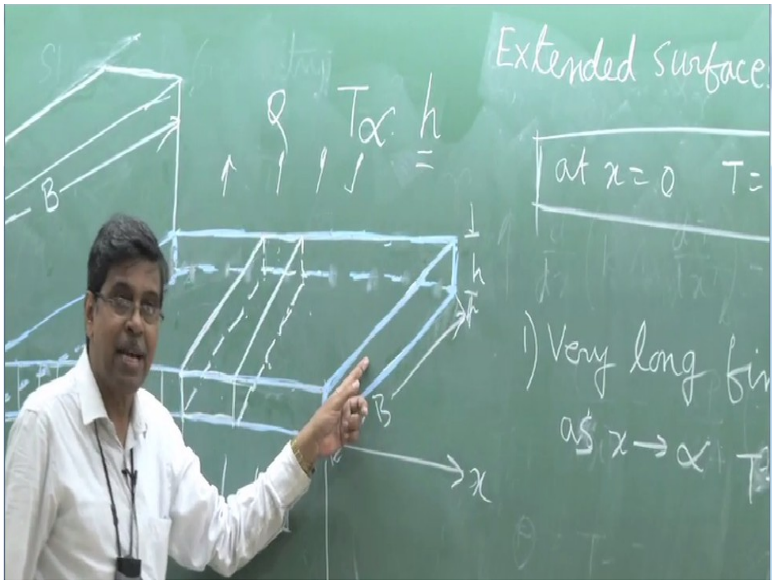


So, I have to generate from – you are correct, you are also correct but ultimately you have to translate in this form because I have to solve this C1 C2, Theta is C1 e power mx plus C2 e power minus mx. And one condition is that Theta b at x=0 that means Theta b is C1plusC2 another condition is use, heat transfer 0, heat fluke 0 means –K dT/dX is 0 that mean dT/dX is 0 that means dT/dX at x= L is 0, what is that?

That means 0= dT/dX 0 means, the Theta dX is 0 same thing because Theta is T minus T infinitive. 0 is mC1e power mL- mC2e power -mL. If you solve it, you will get C1 is equal to e power -mL/e power mL plus e power -ml and you will get C2 is e power mL/e power mL plus e power -mL and if you substitute this from the second case you get the expression Theta by Theta b in terms of hyperbolic function cos m (L- x) this is the argument divided by cos hyperbolic mL.

You know the hyperbolic function that cos x is equal to exponential x plus exponential plus exponential -x by 2, okay. So, this is the final expression. Now it becomes a routine job. Only thing is that you have write the boundary condition correctly. Then things are done.

(Refer Slide Time: 40:25)



Now third category of problem is fine of finite length L but no insulation. It is the most practical problem. If there are long fin means what? What how much long? How do you take that T is T infinitive, T maybe very low, if there is a very huge drop from Tb depends upon thermal conductivity also. Now insulated sometimes the fin surfaces maybe not be insulated and even with insulation there maybe heat loss so the third one is the most practical condition.

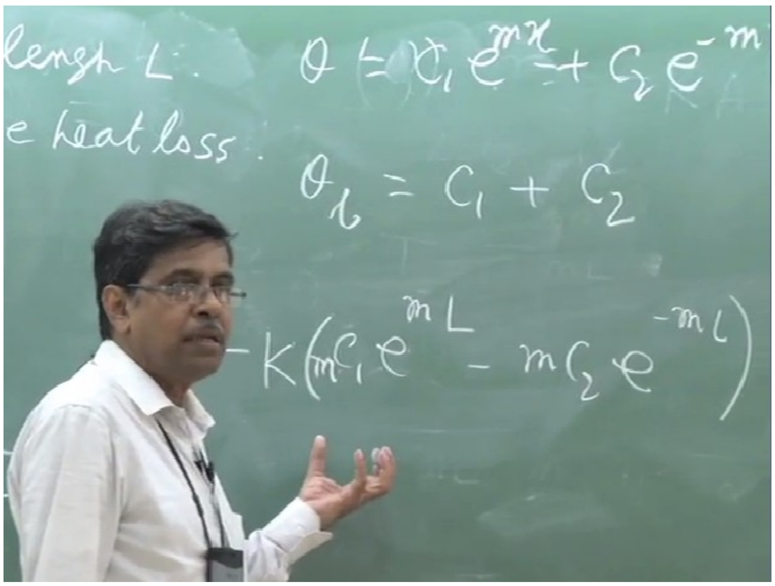
That Fin of finite length L but with convective heat loss at the tip. What is meant by that? That means at x = L what do you have? This is a conjugate heat transfer problem, K (dT/dX) L, for unit area I am writing or you can write area –K into area that means the heat which is coming to this surface by conduction -K dT/KA dT/dX which we did earlier combined conduction convection problem in series the same it is being transferred to the h*A (T- T) T at L minus that means T L is the temperature at L.

So, if you can understand this and correctly write this then things are okay that means -K dT/DX at x=L must be equal to h into TL - T finite. TL means T at this is clear to everybody? That means this is the conduction heat at the tip which is being conducted that is what. I told you that ultimately what is happening if you go on dividing into number of elements each element this takes heat some is going by lateral surface then some is going by lateral surface then it is conducted next.

So, this will be conduction heat transfer is getting reduced when it come here most of the heat which has been taken from the base has been convicted by the lateral surface. So, the rest part of the heat which is conducted finally to the extreme expose surface is being lost to the surrounding ambience my convection from this part of the expose surface, which is the same area because we are considering constant area rectangular fin.

So, with this thing, this concept in mind one can write the differential – sorry boundary conditions this. Clear?

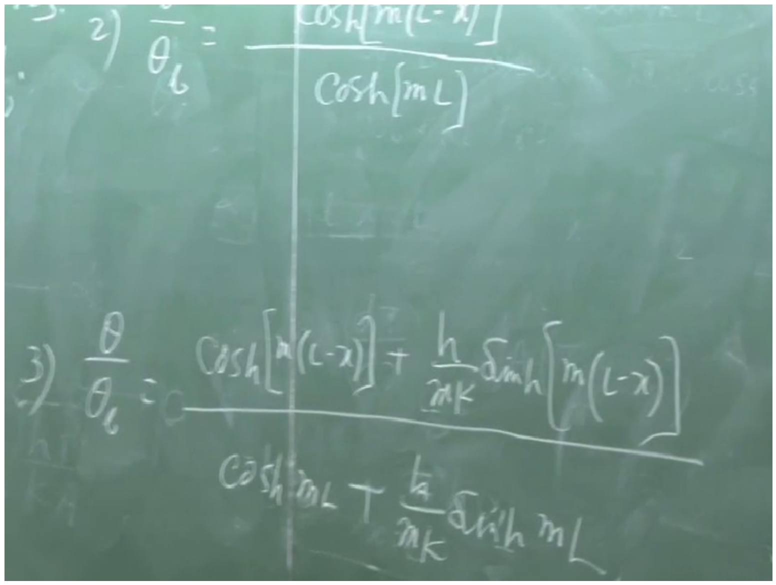
(Refer Slide Time: 43:57)



Step two is, now is convicted boundary condition in terms of our Theta and all these things. How to do it? Now dT/dX the Theta dX that means –K dT/dX means the Theta dX that means I can write now this as the Theta dX and this has Theta L in my nomenclature because my equation is in terms of the transform variable Theta that is excess temperature Theta L. Now what is -K d Theta dX, what is d Theta dX?

C1 e power mL - mC2e power -mL. h into Theta L that is C1 e power mL plus C2 e power -mL. No, this side C1 mC1, this side is simply C1. Sorry mC1 d Theta dX is mC1 e power mL plus mC2 e power -mL. So, this is another equation. You can rearrange it some coefficient into c1 plus some another coefficient into c2 equals to something that means these are two equations for c1, c2 in terms of m and L. And if you substitute this then you get a solution like this.

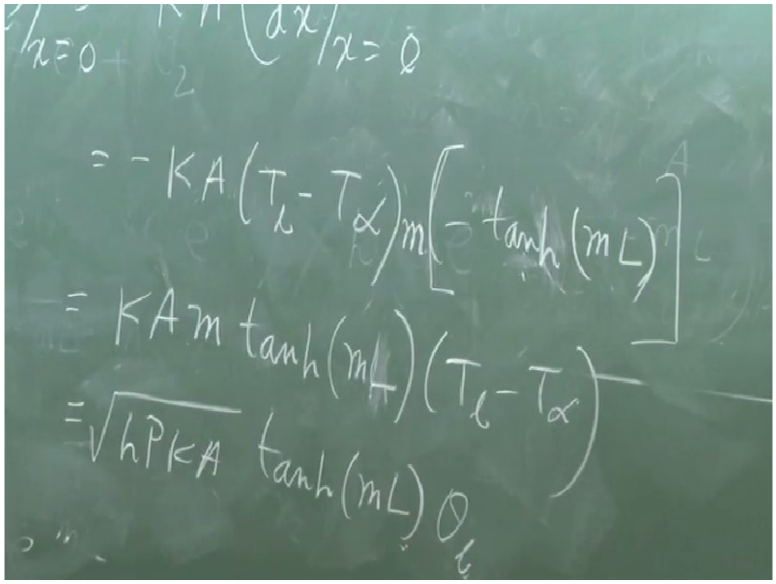
(Refer Slide Time: 46:47)



Now you get a solution like this Theta by Theta b. Now earlier case we got a solution Theta by Theta b is Cosh hyperbolic m(L-X) that is the earlier case insulated tip divided by cos hyperbolic [mL]. Now I get a solution Theta by Theta b for this third case, this is the second case this is the third case, Theta by Theta b is Cosh hyperbolic mL- x plus h/mK sine mL-x divided by Cosh hyperbolic mL plus h/mK sine mL.

This is a routine matter but tedious job to find out the especially the rearrangement things that means you have to find out c1, c2 and then you can find it. Other two things I have forgotten to tell you that even in the earlier case this is insulating tip, this one of finite length L, difference is that this is non-insulating tip of finite length L which transfers heat with the ambient in terms of convection where h is the convection coefficient, okay. Now what is the heat transfer?

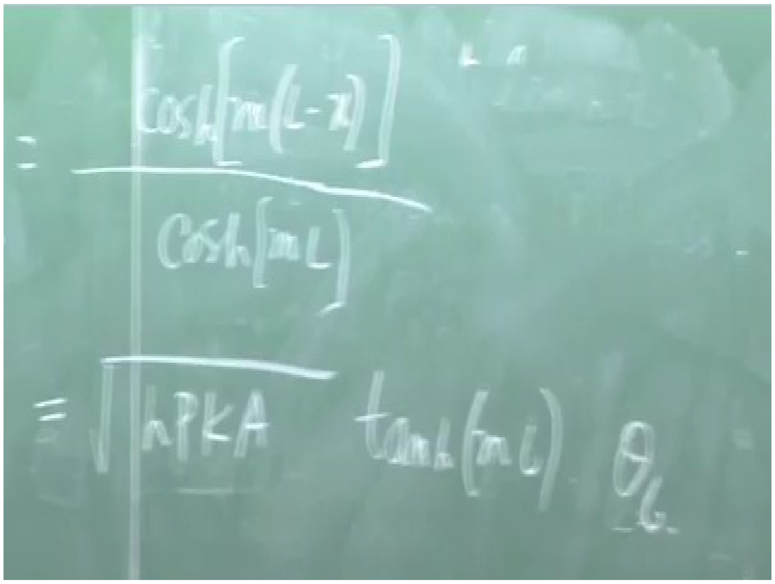
(Refer Slide Time: 48:38)



Earlier I did it for long fin, (Q)x=0 = -KA (dT/dX) at x=0. Now if you do it from here what is dT/dX? This Theta by Theta b means T minus, Theta is T minus T infinitive, Theta b is Tb - T infinitive. So, if you do it then you get it is equal to -KA, T - T infinitive, so therefore Tb - T infinitive * Cosh hyperbolic mL - x that is sine mL-x with a minus sign and Cosh ml at x=0 Tanh mL.

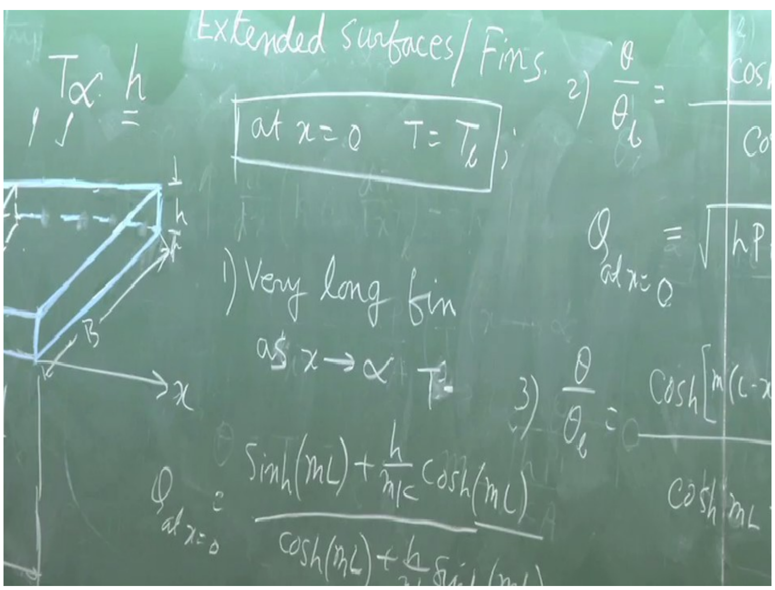
That means minus –tan(hmL). That means this becomes is equal to KA, sorry m, m will be there m. m Tanh mL Tb - T infinitive and we recall that m is root over hP/KA, so it is root over hPKA Tanh mL * Theta b. This is an expression. Now these become all routine task.

(Refer Slide Time: 50:45)



It is root over hPKA tanh mL into Theta. This is the heat flux at x=0. Similar you can find out the heat flux expression for this. It is a lengthy expression that I can tell you, you can see in the book also.

(Refer Slide Time: 51:21)



For this case, number three, the heat flux at x=0 will be let me see that it is difficult that which function will come. With insulated fin tip sine mL plus h/ mK cos mL that means it will sine (mL)plus h/mK cos (mL)/ cos (mL) plus h/mK sine (mL) this is very tedious I know also bore you but you have to afford to do this because without this it is not complete. Concept is very interesting that how the fin enhances the heat transfer, then by setting up the governing differential equation.

You have to solve it with the boundary condition. And for more practical cases the boundary conditions are such things are little complicated. So, complicated means tedious in equation in solving the things. So, that finally we solved for temperature distribution in terms of the excess temperature and the heat transfer from the base which is enhanced and temperature distribution heat transfer from the base for different cases.

So, these three cases are most important boundary conditions. Another boundary condition is there that fin is a finite length and its end surface is kept at a temperature T_L that means T_L is specified. So, everything has to be found in terms of T_L that is given as a task for you to do it. Then we will be solving few problems on this fins extended surfaces so that we know that how we can apply our knowledge that means these equations which we have derived along with our knowledge to those practical problems in the next class.

And I will finally give you a generalized approach mathematical approach for any one-dimensional steady heat conduction problem which will act as a fin; which will act as a simple geometry without extended surface, lateral surface all those things combined. So, therefore next class, we will be most probably the concluding class for the one-dimensional steady state heat conduction.

I will be solving few problems and then we will start the two-dimensional heat conduction steady state and then we will go for unsteady heat conduction, okay. So, next class means tomorrow. Okay thank you.