Conduction and Convection Heat Transfer

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Lecture – 20
Heat Transfer From Extenede Surface.

Good morning and welcome you all to this session on Conduction and Convection Heat Transfer. Now, in this session we will start a new topic Extended Surface and Fins. You have seen in several essays practical scenario we use an extended surface. For example, in a wall, hot wall, if you have to enhance the heat transfer we add some extended surfaces you will see in many places the radiators of a car.

In heat exchange, as if you see the two fluids exchange heat between each other, one is hot another is cold and they pass through different passages one is for example in a shell and tube heat exchanger they are on the inside tube another is the outside annular area. And you often see that to enhance heat transfer there are extended surfaces attach to the inside tube to enhance the heat transfer there.

So, several applications that they are in buildings you will see, the fins at there to enhance the rate of heat transfer at the outside wall. Now the basic principle of using extended surfaces which are known as fins to enhance the heat transfer depends upon again the Fourier Law, whole life time, the entire steady of heat transfer I tell you those who of you who may make career heat transfer will remember this depend, that the pivotal point is the Fourier heat conduction law which is used in convection heat transfer.

Also, that is heat fluke, total heat transfer rate is K-thermal conductivity, times the area, times the temperature gradient. So, we can increase the heat transfer rate by increasing the thermal conductivity of the fluid or dissolute that is a property. We can increase the heat transfer rate again by increasing the area that is the geometry, more is the area more is the heat transfer, you know that when you want to cool or tea sometimes.

We pour it on a flat plate where it gets more surface area, it is cold being rate of heat transfer is enhanced? The third one is the dT/dX or dT/dR at temperature gradient. Now one is the property another is the geometry and dT/dR is the temperature gradient that depends upon the boundary condition and geometry for conduction and for convection heat transfer these

DT/DR depends upon the flow condition which will be dealt in more details in the convection class.

And the example of that is again, first we pour that tea on the plate but still we are not satisfied than we try to blow air over it or put it in the bottom of a fan. That means in that case we create a higher temperature gradient, we do not touch area, we do not touch the property. What do we touch? Change the flow field as I explained earlier temperature gradient.

So, therefore, in heat transfer this has to be remembered for whole life that these three parameters scientists are searching that these three parameters had to be dealt to it to change the heat transfer rate. Now in extended surface or fins are used to increase the heat transfer rate by increasing more surface area. How it is done?

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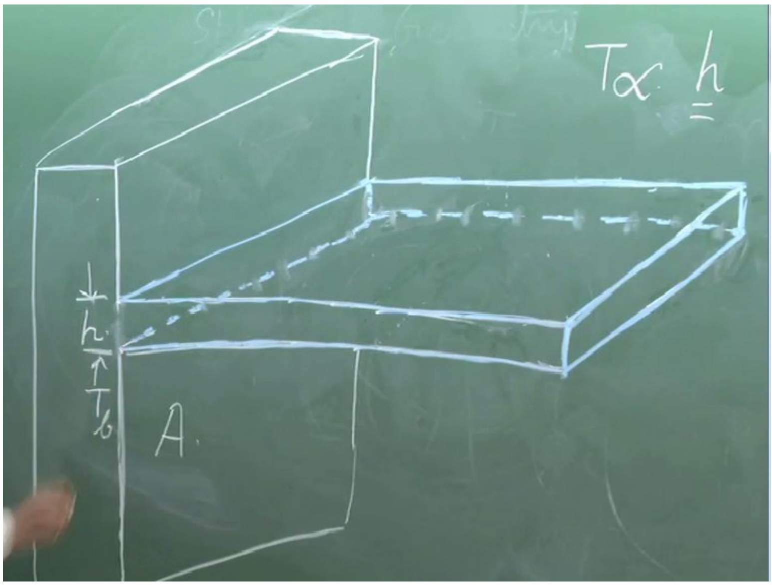

Let us consider a wall, this surface of wall, we are not interested what happens this side of the wall; just I have drawn the figure for your understanding. The problem is pour that this is the outer surface of the wall this surface which is kept at a temperature Tb in nomenclature b is used that is base, base temperature that means from this wall. And this wall is exposed to an ambient T infinitive it is hot wall this may be consider that something is generated yet in terms of energy.

It may be the outer wall of a building where too much energy is generated because of some action, huge people are there making noise, making some functions and all these things where this becomes hot, or it may be the wall of a farness where this energy is generated. These are the practical example that we get a hot exposed wall, this surface of the wall at some height in temperature Tb which is greater than the T infinitive.

Now immediately one person will tell, okay, then you have a heat transfer Q I tell you, it is h if you prescribe h as heat transfer coefficient which is again a very complicated thing, you will see can how complicated it is but it is the fun when you will deal with convection. But at present we are happy that h is given as a parameter which is constant. Then people will tell if A is your surface area of this wall then fine you have this heat transfer. Are you happy? No.

I am not satisfied with this heat transfer rate. But h is fixed I cannot change. The flow field is such of the outside there, no I cannot do. So, what you can do? I cannot change the T infinitive the ambient temperature. Then I have to set another refrigerator or air conditioning plant to reduce the temperature. So, this we have to accept. Okay, I will enhance the heat transfer rate by enhancing my area. How?

Wall is fixed, how you will increase the area? Or can I increase the outer wall area? So, what I do, I add additional surfaces like this in a plane geometry for simple understanding. Now let us consider this part of the wall I attach a rectangular surface like this a three-dimensional, my drawing is not that good, I think you can understand it, so I at this surface. Quiet a high thermal conductivity.

It is not insulating material it is a thermal conductivity high. K, let K is the thermal conductivity which is very important of this. Now what happens actually by adding this, if I tell that it provides additional area for heat transfer it is understood. How? Now when heat is flowing from this area – now this unexposed area remains safe they behave as the same thing h*A (Tb – T infinitive) area plus this area.

So, this area, let us now consider that h– it is very important I am telling you, few books I do not know explain this way, they simply write by providing more surface area, but how? You have understand surface area is being provided at the cost of what we are providing the surface area? “Professor - student conversation starts” -Hey, you please come to the first bench, you, hello, you yes. Yours next to you, you yes, hello you, yes come and sit here – not you in front of you, yes come and sit here please. “Professor - student conversation ends”

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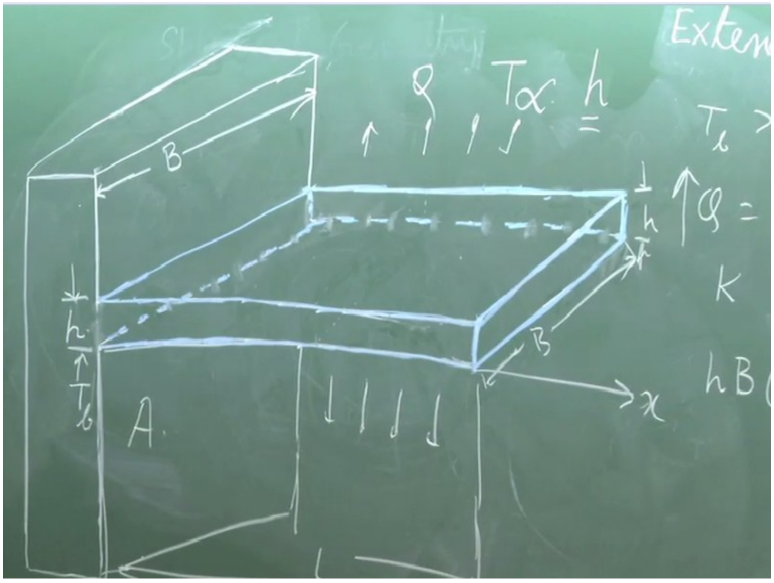

So, now, a thermal conductivity K. Now what happens if this is the height h and if I consider that this with understanding is our B, then from this part we have a heat transfer Tb minus T infinitive. H*Bb (Tb minus T infinitive), try to understand it this is very important I tell you, but the first part of deduction is may not be important but to understand the feel efficient it is very important. Now you can tell sir, I am generating the same area h*B here, so why you are bothered so much?

I am generating the same area, but you are adding materials that means the thermal resistance, what does it do? That this exposed area hB is not at a temperature Tb because there is a temperature dropped, that is why we tell this is the thermal resistance, that if you add more material and take the surface area somewhere here in the direction of heat flow means the temperature drop is there until and unless thermal conductivity is infinitive.

Less is the thermal conductivity more is the temperature, this way we realize that there is a reduction in the heat transfer, that means adding this material reduces heat loss from this part of the surface where it is attached. But, does not matter each and every portion that heat is being transmitted in this direction in this direction. Why? This is because we have a temperature gradient from Tb to T infinitive.

So, each portion at each length if we consider this as a distance x and let this be the length of this from here actually this length this is in isometric view this length L. So, along the length you have a temperature drop from Tb to some temperature here. But this is relatively higher but lower then Tb. So, at each and every point even if the temperature is lower than the base temperature but it exchanges heat to the surrounding because of the huge surface area.

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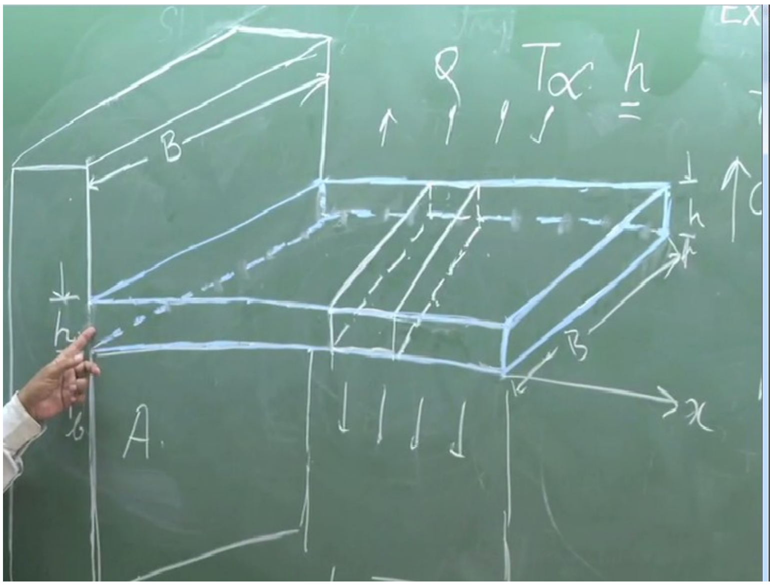

Let us consider a section here at a distance x. I draw a simple two-dimensional surface like this then it will be easier for us to understand that let us consider at a distance x, we take an element of length del x and heat is conducted from the base which is at a temperature Tb. From here it is conducted at position, let Qx is the heat coming in then there is heat transfer through the surfaces of this element.

And rest part of the heat is being conducted, that means this heat transfer through this lateral surfaces is huge. Because of this it draws more heat here. Finally, it is connected to here that means from the base it draws more heat that means what are those lateral surfaces, that means this one. I draw it here that is del x you can understand this, like this. That means, I think you can see it.

That means this lateral surfaces one is the top surface another is the bottom of surface that means del x into b, top and bottom then this surface that del x * 8, del x * 8. So, you will see there are four surfaces from which it transfers heat by convection. Here, if you cannot guess from this three-dimensional drawing that this surface, this surface top and the bottom that means one is this top surface perpendicular to this direction that means this top surface, this one that is B times delta x.

Similarly, the bottom on B time delta x from the bottom surface delta x and quiet in this direction perpendicular to the plane of the boat. And also, this surface that is this one whose 8 into delta x this area and the rare one that means it had this open or exposed surface to be ambience. Quite transfers heat and it has some temperature T at distance x which is less then Tb because of the thermal conductivity of this material that I accept it is not at Tb.

But even if that less temperature it has huge area to transfer a huge amount of heat and this why it is added up for the entire fin so that it helps to draw more heat from the base then that was being transferred without the attachment of this extended surface which is known as fin, this is a rectangular fin. Clear to everybody that how then the fin works that it pours an additional thermal resistance in conduction along the direction of flow.

But it gives huge heat transfer by convection by allowing a lateral, more lateral surface which are even being even less being even less then the base temperature but transfer the huge amount of it because of a surface area. So, now if this be the problem which we have understood then next one becomes so simple that why not we then find out the conservation of energy principle, no thermal energy generation.

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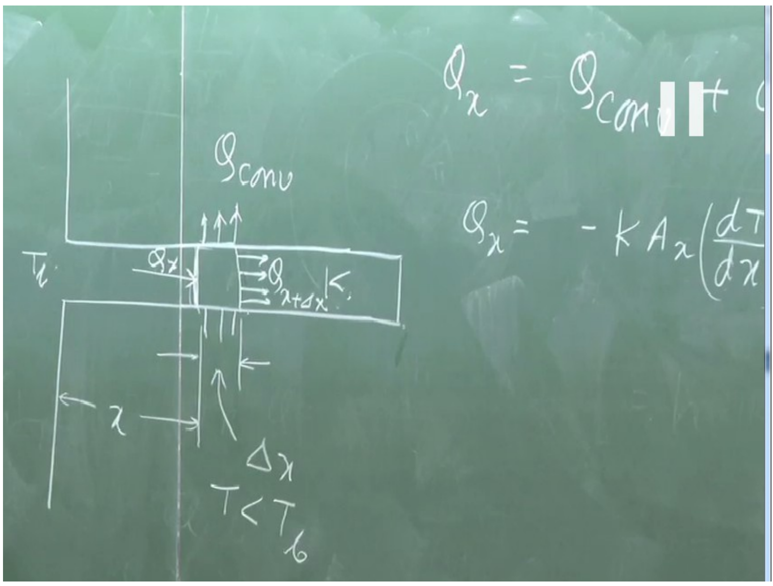

One rectangular fin if you consider let this is Q convection from the lateral surfaces. Then Qx which is coming is distributed as Q convection plus Qx plus delta x. Clear? Now Qx is –KAx dT/dX. What is Ax? Ax is the surface area here. Here you can say that it is uniform but I take a variable area. Let us consider a variable area.

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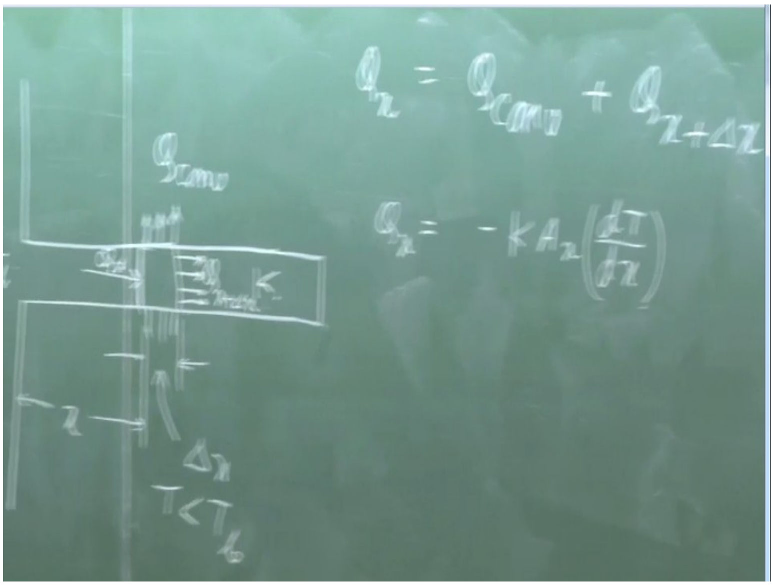

Let us consider a variable area. I have shown you here area which is straight but we can consider a variable area now for general purpose. But ultimately you will be solving this, this is variable area, this is a variable area, okay. So, this is Qx, this is Ax at a distance x then this is Qx plus del x and here this is del x, very simple, here Ax is a function it, I am not writing Ax plus del x not necessary.

Then Q convection that is the heat rate transfer. So, that is why minus Kh (dT/dX). Now if you solve this by expanding Taylor’s series and take it here then Qx plus del x is Qx plus dTX of Qx dX, that means minus K, take all sorts of Qx plus Q d/dX of Q is del x plus Q convection. What is Q convection?

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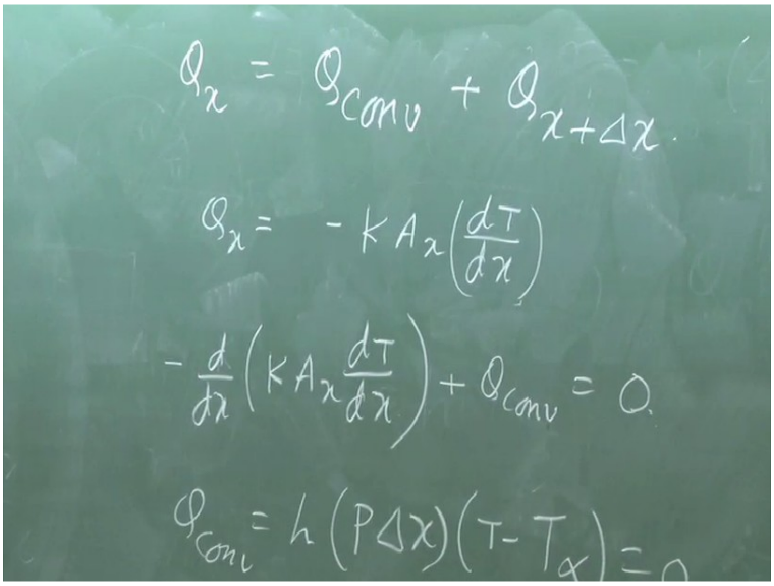

Now Q convection can be written like this, we can now rub this one, we will go by that. Now Q convection can be expressed as this. If you define a perimeter P, for example here you can understand better the perimeter in this case is the perimeter of this surface, that means here P plus 8 plus B plus 8 2b plus 8. So, therefore, if you consider P as the perimeter of this lateral surface which is perpendicular to the plane of the board then Q convection can be written as area 8 times the area, area become the perimeter into delta x.

In terms of perimeter, it is perimeter into delta into T minus T is the temperature there at that location x T is the temperature. So, in terms of the nomenclature perimeter I can write this equation as hP delta x T minus T infinitive.

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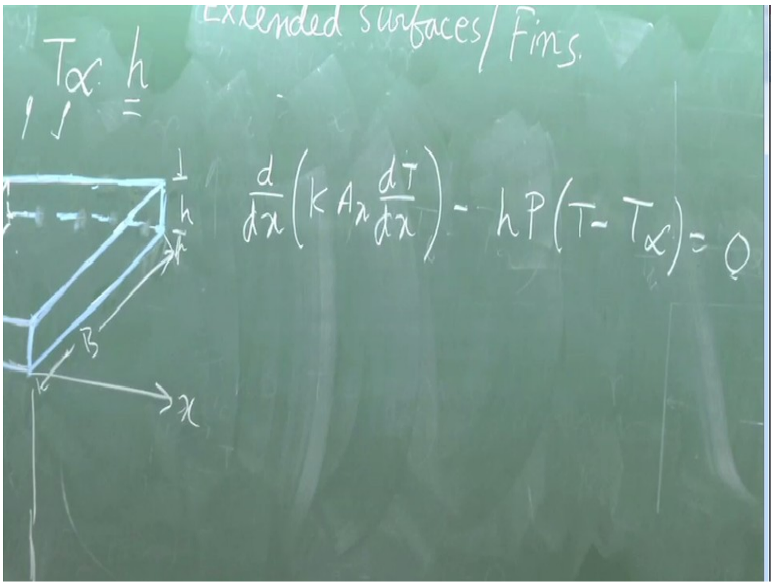

So, therefore I can write the equations in this fashion that d/dX of KAx, dT/dX minus hP, delta x you are very, very good delta x, I am sorry there will be delta x, so hP into T minus T infinitive very good there should be delta x, yes because this is Qx plus del x is Qx plus d/dX of Qx into delta x neglecting the higher order terms, very good. So, that delta x gets canceled. So, now this is the temperature distribution equation.

Difference is there, here heat is transferred simultaneously for each and every element both by conduction and conduction through lateral surfaces which is so far, we did not consider so that means it precisely boils down a conduction problem where lateral surfaces at each and every section of the conducting material along the length of the heat flow shares in the convective transfer. Okay that is the only the balance.

So, therefore, if you we know physics then we can find out the basic governing differential equation for the temperature distribution, let us consider a very simple case. Cost and thermal conductivity, no temperature dependences and this type of rectangular fin, that means A is constant. Then we get a very simple equation d square T/dX square minus hP/ KA (T minus T infinitive), okay.

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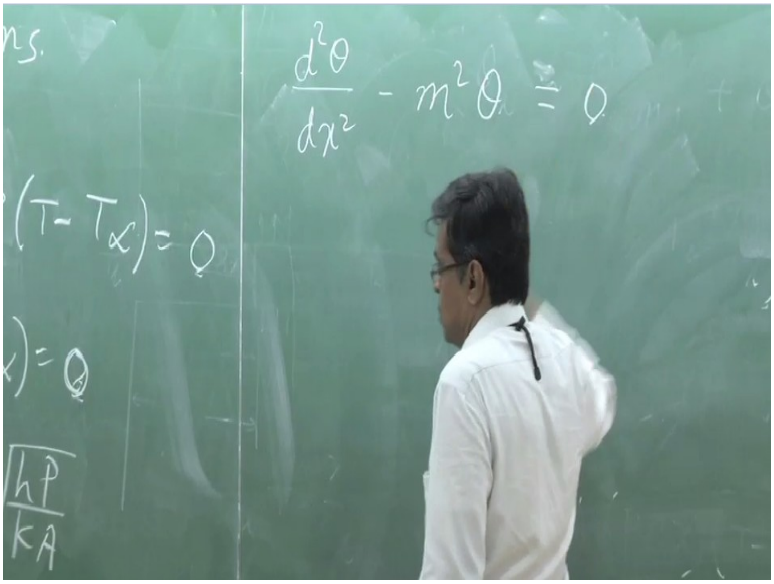

Now if we write by transforming the variable dependent variable temperature in the form of x temperature that means I change the temperature variable T transformation T minus T infinitive as the variable, that means it is the temperature over the ambient temperature. Then dT/dX is the theta dX, then dx square dX square is theta dX square but the advantage is there

here only we get theta that means this looks like a simple school level problem that this square.

By defining this and by defining m as root over hP by KA, m is a perimeter which is just defined by root over hP/KA, hp/KA is the m. Okay, if you define m as root over root over hP/KA then we get an expression, d square theta and dX square minus m square theta is equal to 0. If we define m as a dimensional parameter then we get an expression d square theta and dX square minus m square is equal to 0 whose solution is very simple. Now we define this as square root or rather m square is hP/KA this because hP K all are positive.

Okay so m is a real quantity that means hP/K can be expressed as a square of a real number m square, m is a real number because hP/KA cannot become negative, that is the logic otherwise we cannot define just square root of something as a real number, we have to investigate whether this is positive or not.

So, if we do so rest part the solution of this complimentary function you find out, there is no particular integral, that side is 0 this is a second there you differential equation it is an exponential solution e power mx plus C2 minus mx hyperbolic functions. C1 exponential function whatever you call e power mx. That is all.

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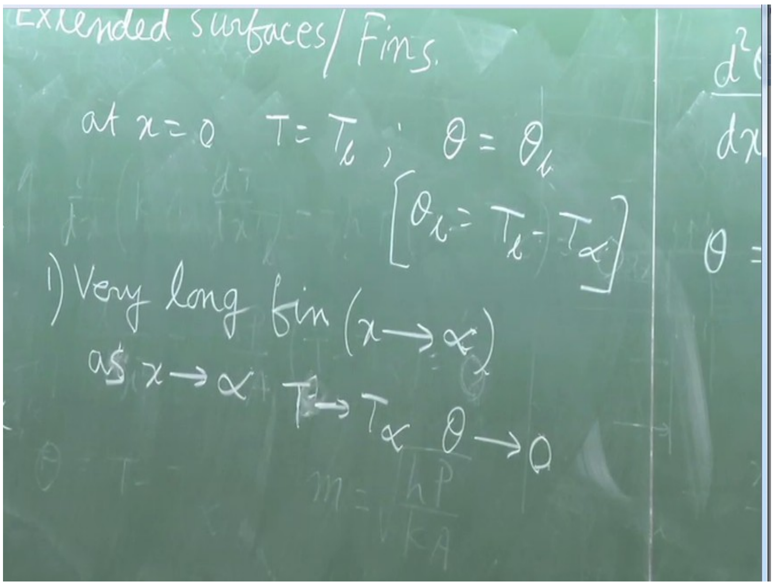

Now the boundary condition. Okay. Now the boundary conditions. What are the boundary conditions? Tell me, now boundary conditions then you ask me sir, how do I know? The governing differential equation comes from the principle of conservation of energy or anywhere principles of conservation principles. And the boundary conditions comes from the physical problem define to it boundary and other conditions that means the boundary conditions have to be physically defined.

One boundary condition that is defined or prescribed by the physics is that, base temperature is Tb where from x is measured. So, therefore that x=0, T=Tb which means theta is equal to theta b. Theta b means Tb minus T infinitive that mean Theta in terms of Theta it is Theta b. What is another boundary condition? One another boundary condition, you have to search for another boundary condition with respect to x that means you have to find out what is the boundary condition at x is equal to in.

If L is the length of the fin, so you have to know whether it is insulated or something else.