Conduction and Convection Heat Transfer

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Lecture 2

Good morning and welcome you all to this session of the course conduction and convection heat transfer. Last class we were discussing the Fourier laws of heat conduction, which defines the heat flux is proportional to the temperature gradient and the proportionality constant is called as or defined as thermal conductivity, which is a property of this system.

Now in this regard again I repeat that the two modes of heat transfer conduction and convection is this way that conduction is the basic mode of heat transfer and when there is a flow in a medium, which is present in liquids and gases, then this conduction mode is affected by the flow and we call it as a convection mode. So therefore, convection mode cannot appear in solids, the heat transported in the solid is always by conduction mode purely without being affected by flow because there is no flow within the solids.

Solid as a whole can be translated, but within the solid there is no flow and under some special conditions when the fluid, the liquid or gases are stationary, there is no flow then the heat transfer takes place purely by conduction and when we will talk about conduction heat transfer, we will consider purely conduction mode without being affected by any flow that means there is no flow in the medium, which mostly happens in solids.

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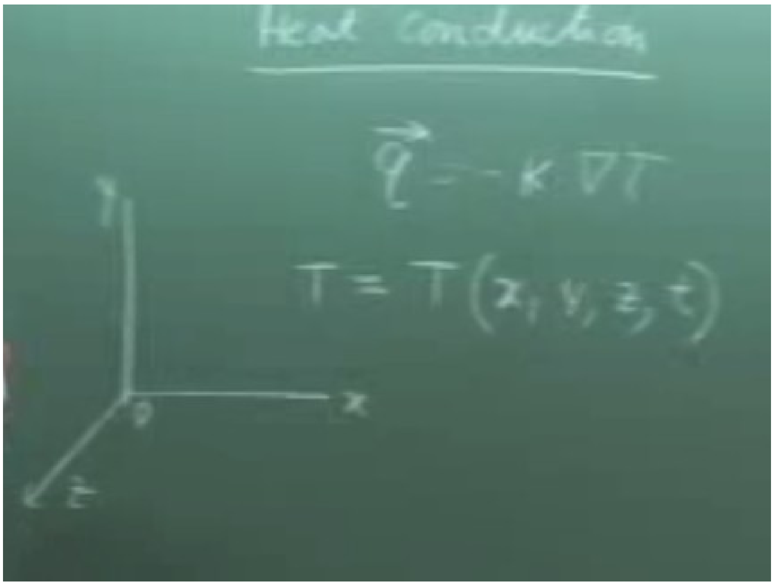

And if we, now think that the heat conduction, heat conduction again then we have seen that the heat flux that is the heat transfer per unit area normal to the direction of heat transfer is minus K gradient of T, where the heat flux is taken as effect. Now therefore, there has to be a temperature variation in this space coordinates. It is a continuous temperature variation for which we can define the gradients that means in the medium.

The temperature has to be a function of space coordinates if you consider a Cartesian coordinate system x, y, z simple that T is mathematically expressed as a function of space coordinates x, y, z and also it can vary with time that means that any point the temperature can also vary with time that means temperature is varying with the space coordinates. So there is a temperature distribution and this distribution is also varying with time that means the function of time.

So, in general, one can express temperature in a medium of heat conduction or in any medium, but here now we are discussing the heat conduction in a medium of heat conduction is a function of x, y, z space coordinate and time.

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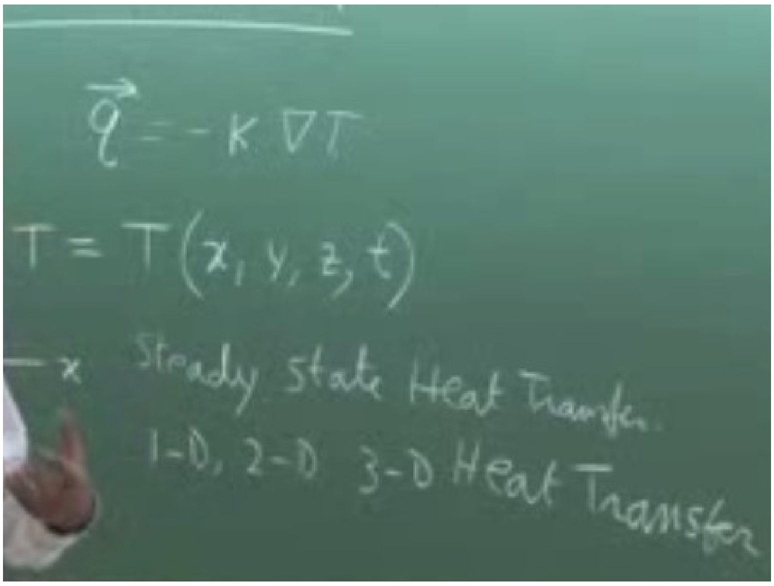

Now in this regard we should think of two states that one is steady state heat transfer and is one dimensional, two dimensional or three-dimensional heat transfer. Now in steady state transfer as you know what steady is associated with invariance with time that means something which does change with time. For example, you know the study flow were the flow velocity pressure, these quantities and the derived quantities like mass flow rate, volume flow rate are all in variant with time.

So, for a system at steady state, do you know the properties are invariant with time except for the control mass system, which is defined as a fixed quantity of specified mass, mass is excluded other properties are invariant with time because mass is always invariant with time, so we do not bring mass same in steady condition mass is constant. But for a control volume system or control volume inclusive of mass everything should be constant with time that means all parameters, all properties will be constant with time.

So therefore, temperature being a parameter, temperature is or a property of a system remains constant with time under steady state situation and we are concerned with the heat transfer processes at the moment.

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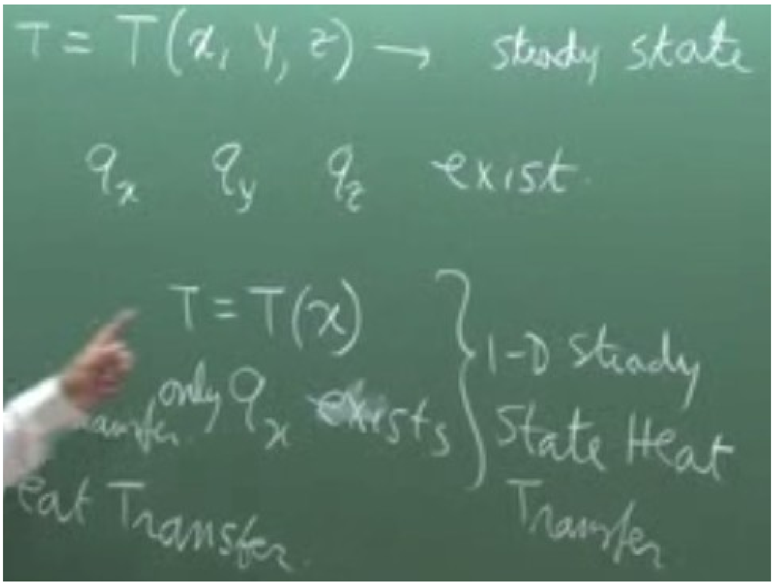

So therefore, when the temperature ceases to be a function of time under all steady state situations, we call it at steady state that means in that case temperature becomes a function of space coordinates that means that at steady state transfer, temperature is a function of space coordinates only that means if we get a temperature distribution within a medium that remains invariant with time. Now when temperature is a function of three space coordinates, the heat flux in all these space coordinates exists.

Because of the respective temperature gradient and these heat transfer situation is known as three dimensional. If this is not a function of time, then it is a steady state heat transfer process. Similarly, if the temperature becomes a function of only two coordinate systems ceases to be a function of z, then it is a two dimensional or any two x, z depending upon the way you define your coordinate axis.

And if the temperature is a function of only one space coordinates that means temperature varies only in one direction, so heat flux exists in that direction only that is a one-dimensional heat transfer that means only qx exists that is one dimensional steady state heat transfer. So, this is the mathematical definition of steady state transfer one dimensional, two dimensional or three-dimensional heat transfer.

Let us concentrate on one dimensional steady state transfer a temperature ceases to be function of time and it is a function of only one space coordinates along which there is a heat flux.

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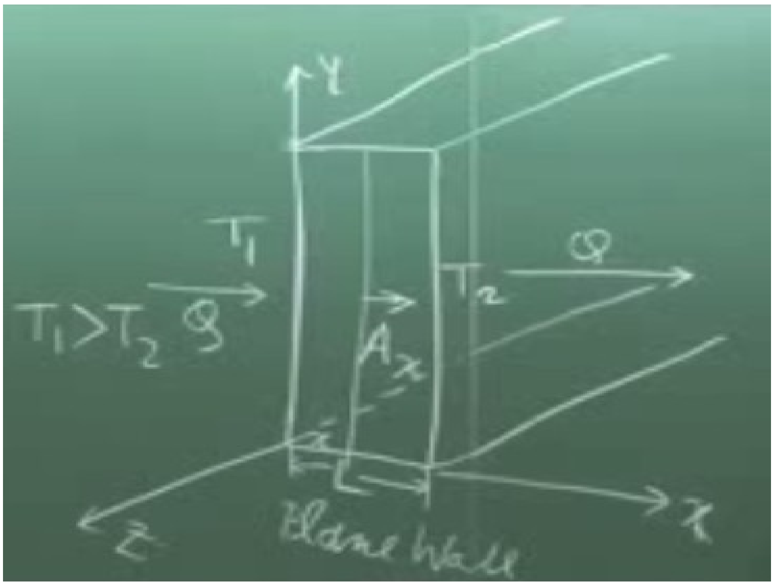

So, to realise this in practise let us consider a wall like this whose dimension in this direction, which we call length or width is much small L compared to this dimension in this and this direction that means if you consider this to be x, this direction to be y and this to be z that means this dimension in y direction, which you can call height its dimension in z direction, which you can call as the breadth or whatever may be is much higher as compared to this width or length L.

If you consider such a situation of a wall or a slab, which has a constant cross-sectional area with respect to the axis x, that means the plane wall of this dimension this is known as plane wall and if we consider a situation that the left-hand surface is kept that a temperature T1 constant, the surface temperature is constant. That means at each and every point on the surface that temperature is T1 that means which is independent of y and z direction.

And if this right-hand surface is kept that a constant temperature T2, this we assume this is the prescription of the problem, that means problem is specified like this T2 and if T1 is greater than T2 by your basic definition of heat transfer, heat will flow in this direction from high temperature to low temperature. Now this case assumes a one-dimensional heat transfer why because temperature varies only in the x direction.

It is by intuition the temperature at any point will be a function of x only because in y and z they are constant. The boundaries are such the boundary conditions have been made such that the problem is made one dimensional because of the boundary condition and the geometry. The temperatures are constant in two phases and moreover this dimension, the length or width, is much smaller compared to the dimensions in other direction.

So, by the definition of the problem by its geometry and boundary condition it is one dimension, now how to conceive of a steady state. Now by basic understanding of the steady state, all points that temperature has to be invariant with time, there may be a distribution but invariant with time, so to meet up that we have to approve physical requirement. One is that definitely the boundary temperatures has to be invariant with time, which I have prescribed for this problem.

And at the same time there should not be any energy generation internally by this wall by any action, there may be an internal reaction or anything else. At the same time, there should not be a heat absorption by any action within the wall and wall should allow the heat to flow without interruptions that means without adding anything to it or without taking anything from it, which means that the heat flows at any section will be constant.

And I can tell the amount of heat, which is coming in to this wall from the left side is going out from the wall at the right side, the same heat.

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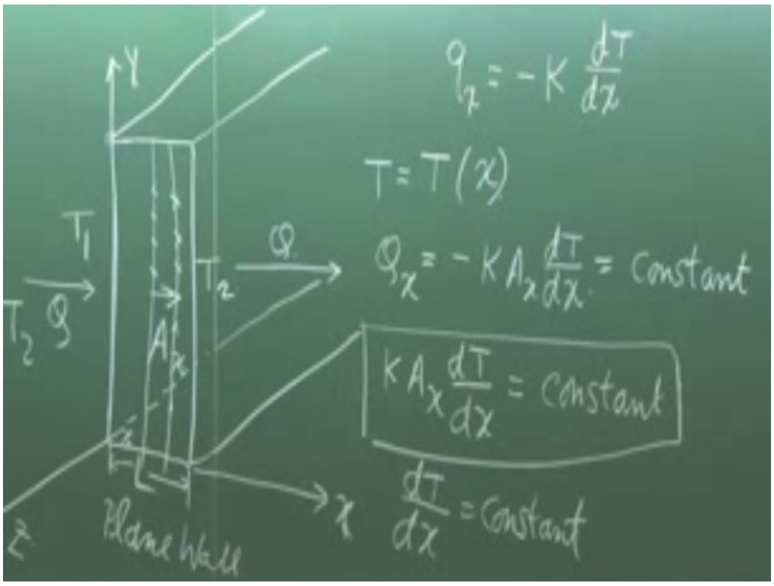

Now if I take a section at x and let the cross-sectional area is Ax, then what is the heat flux qx is minus K dT/dx. Now T is a function of x, so therefore T Tx can be either a function of x or constant. It can never be a function of y or z, which means that qx is not a function of y or z provided, K is independent of the directions. Well, then we can tell that heat flux is constant at all points on this area, so that I can write qx is equal to minus K Ax dT/dx.

And if this is constant at all planes because of the requirement of the steady state then one can write that this is equal to constant that means simply K Ax dT/dx is constant and that is the requirement of a steady stand one dimensional heat conduction. Now if we assume additionally that K is independent of the temperature and the direction that means the material is isotropic and its thermal conductivity is independent of the temperature or does not vary much within these range of temperature studied.

Then at the same time if we consider Ax constant, consider means the problem is defined for a plane wall that means the cross-sectional area Ax is independent of the direction of heat flow x. In that case, we can write dT/dx is equal to constant that means we get a linear temperature profile from T1 to T2 within this slab. So, therefore a steady state transfer with constant thermal conductivity with constant cross-sectional area, which happens in a plane wall, the temperature gradient is constant that means the temperature is a linear function.

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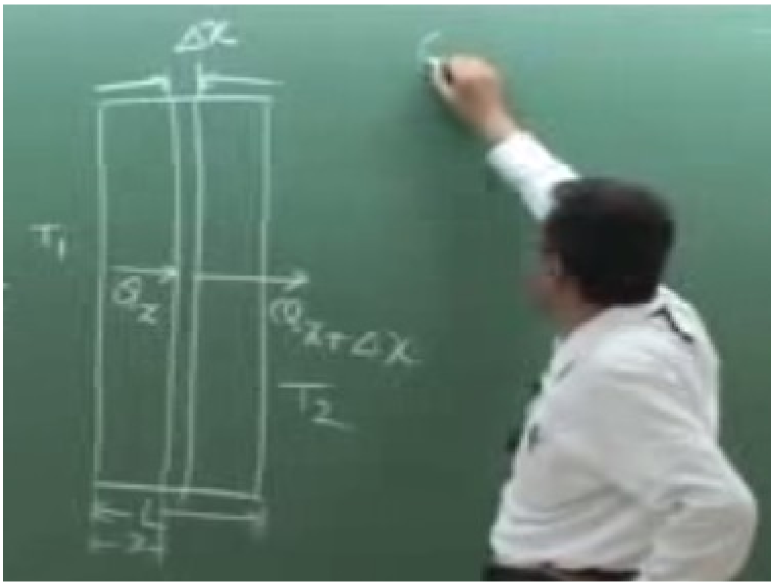

This you can also do it in this fashion in a little elaborate way I think that will help you afterwards in the deduction that you can do in this way in a separate I am showing you that if you take a small elemental volume or small element within the wall which is kept here as T1 and it is temperature T2 and at x, at a distance x, I take a small element of length delta x. Then we can it a little elaborate.

This is the most simple way of looking into it that if the total heat flux at this left side of this elemental volume is Qx and if I designate the heat out from the right phase at x plus delta x as Qx plus delta x.

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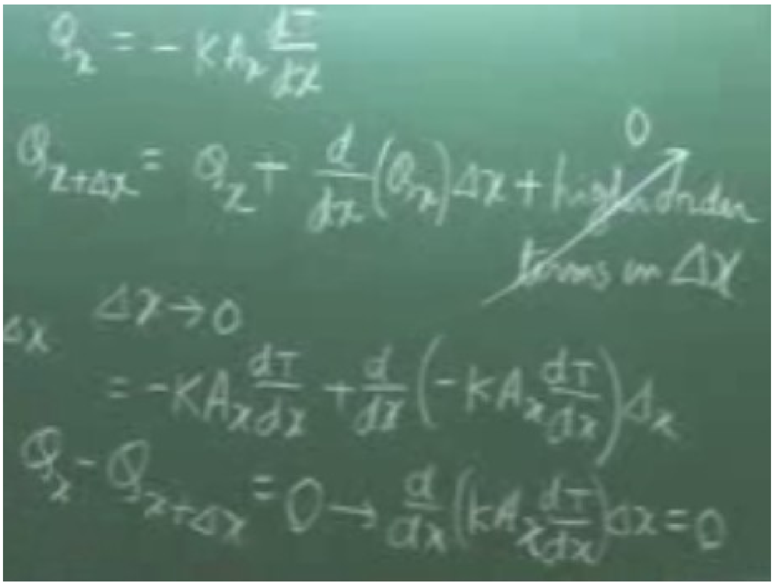

Then I can write the same thing just in a different way Qx in a elaborate way is nothing but minus K Ax because dT/dx is not a function of (()) (16:57) T is a function of x. So therefore, the same logic and Qx plus delta x, we can expand in a Taylor-series at x in the neighbourhood of x plus delta x by this way d/dx of Qx delta x plus higher order terms in delta x that means next term will be double derivative delta x 2 by 2 as you know plus going on an if delta x is very small in the limit delta x tends to zero.

In the limit delta x tends to zero, otherwise we do that neglect the higher order terms and therefore Qx plus del x is Qx that means this can be written as with the help of this KAx dT/dx plus d/dx minus KAx dT/dx into delta x. Here I have not taken into consideration that K is constant and Ax is constant. This come as a whole within the bracket d/dx that means this can change to it x.

Now if, it has to be at steady state there should not be any generation of energy within the element or there should not be any absorption of thermal energy within the element that means element will act as a transparent element whatever it receives it will allow to go out that means a steady state control volume. Influx of any material where that is mass or energy equals to the flux, so therefore, one can write Qx minus Qx plus del x equal to zero.

And if you make this thing you arrive at this same equation that this is constant means d/dx, this minus this means d/dx of KAx dt/dx del x is zero. So therefore, we get the same result that KAx dt/dx is constant, is not dependent on x though if you make this you get the result that d/dx of KAx dt/dx into del x is zero, which means that KAx dt/dx is constant okay. Therefore, the requirement for a steady one-dimensional flow is this and if Ax is constant if it is a plane wall then dt/dx is constant.

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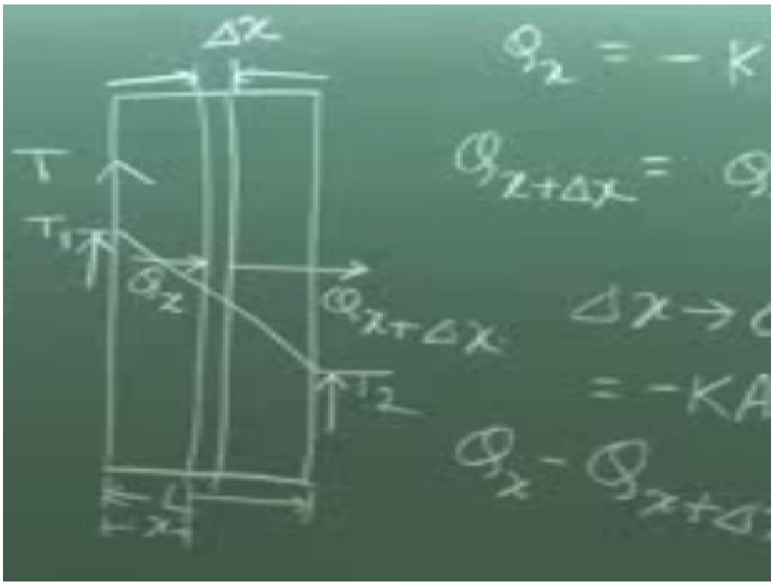

So therefore, for a plane wall with constant thermal conductivity we have a temperature, if you show the temperature scale here this is T1 and this is T2 from any reference datum so we give a linear profile. The slope of which will depend upon the thermal conductivity okay. This is clear, when we have a linear temperature profile. Now in this context, I will tell you certain things okay.

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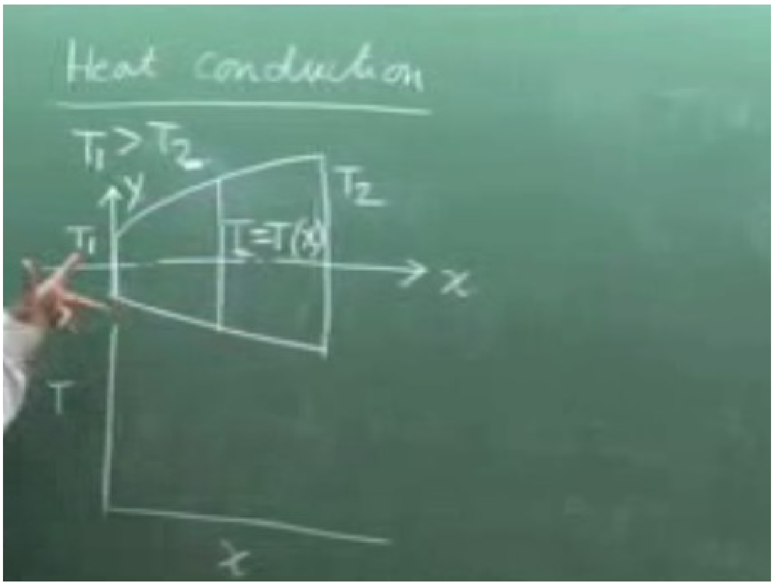

Now if we have a solid like this, whose area is not constant and if we try to conceive one dimensional steady state heat transfer by making this surface that a temperature T1, this is at a temperature T2 so that T1 is greater than T2. Then what will be the temperature variation, will it be linear, will the temperature variation at steady state one dimensional will it be linear no because area is not constant.

But one catch is here that this type of geometry which is not a plane wall that the area is not constant strictly speaking does not give a one-dimensional heat transfer picture. Usually, this type of geometry, the temperature is a function of both x and y but under certain cases the variation across this section is less as compared to the variation along the length depending upon the boundary condition.

So, what we will do, we take almost a cross-sectional average temperature T and tell it as a function of x. This is cross sectional average temperature that means it is in other word visualisation of a really truly speaking two-dimensional heat conduction as one dimensional when the variation in other direction is more. So, this happens in this type of geometry purely strictly speaking one dimensional heat conduction happens in a plane area flow.

This I will connect afterwards, after the derivation of general heat conduction equation.

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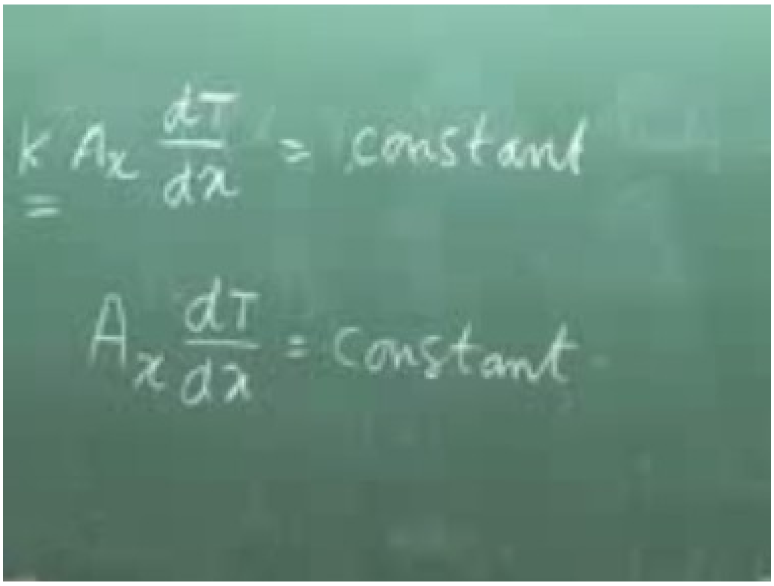

See in this case, I know my basic KAx equation for steady state is dT/dx constant not dT/dx constant. Now if K is constant that we will assume here Ax dT/dx is constant. That means dT/dx is inversely proportional to A. So, such a diverging medium a bar like this, a conical bar so that means temperature variation will be like this. This is T1, (sorry this will go like this) so this is T2.

That means the slope goes on decreasing. It is just that reverse if this is a converging bar. So, this is clear. So therefore, the essence is that KAx dT/dx is constant for one dimensional steady state transfer. K may vary, A may vary. K may be a function of temperature, since T is a function of x so K varies with temperature it will be also function of x. A may be a function of x.

So therefore, to find out analytically, the temperature as a function of x this all depends now on mathematics. So, concept of heat transfer ends here, K A dT/dx is constant. Now rest is mathematics. A very simple of the simplest case is that K constant, A constant plane wall then the mathematics becomes with the simplest thing that dT/dx is constant in a linear temperature profile okay.