

Conduction and Convection Heat Transfer
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Lecture 19

Next problem, consider a pipe of inside radius r_1 , 30mm, outside radius r_2 , 60mm. This is the pipe of inside and outside radius and the thermal conductivity K_1 , that is the pipe material is 10watt/meter degree Celsius. The inside surface is maintained at a uniform temperature 350 degree Celsius but some (()) (9:32), there may be some hot fluid going inside but the problem is prescribed this way.

The inside surface temperature is 350 degree Celsius and the outside surface is to be insulated with an insulation material of thermal conductivity, that means from this outside surface temperature, heat loss will be more to the ambient. Because of that insulation is required, then the problem states that an insulation is made with a material, insulating material of thermal conductivity K_2 , 0.1watt/meter degree Celsius.

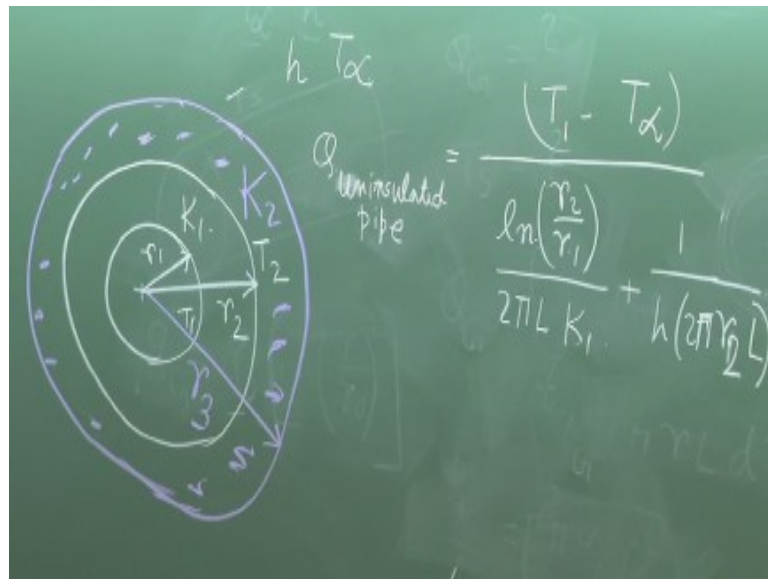
Now the outside surface of the insulation material is exposed to an environment. This is an environment temperature T_2 which is a 20 degree Celsius. After the insulation, the insulating material is exposed with a heat transfer coefficient h_2 , that means the environment is at a temperature 20 degree Celsius and it has a heat transfer coefficient 10 watt/meter degree Celsius.

Now we have to determine the thickness of insulation material required to reduce the heat loss by 25% of that of an insulated pipe exposed to the same environmental conditions. This is the problem, problem is very simple. That means first we have to find out without the insulating material, what will be the heat loss from a pipe of 30mm inside radius and 60mm outside radius? where the inside surface temperature is 350 and the ambient temperature is 20 degree going through conduction resistance and the convection resistance.

Then we have to calculate the thickness of the insulation, we have to consider an another radius of insulation after putting the insulating material and then we have to find out again the rate of heat transfer considering that insulating material. That means, adding another thermal

resistance, conduction thermal resistance and we will find out another convection resistance, which will be reduced, because of the increase in the surface area as I have told.

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That will be 0.75 of the without insulation because it will be reduced by 25% so, this is the problem. So, there is heat transfer is 20% in this problem, so the rest part is the mathematics. Okay. So, let us solve this, so this is a problem, have a look, you can write in your own language not exactly all the watts have to be written. Okay. First, I find out the, this is the concentric cylinder this is r_1 , this is r_2 , this is T_1 , this is T_2 which exposed to the same environment condition $h T$ infinity.

Now if this be the cylinder the first now, heat transfer without the insulation Q without insulated pipe = we know that this is the potential difference which we have done, so we do not want to deduce it \ln . If you want to deduce it, you can deduce it but if you want to remember it, you can write a straight forward \ln . Sometime certain thing have to be remember, I tell you because the time is short.

We will try to read problems in the examination were very very less amount of thing you have to remember almost but sometimes what these problems again to generate this, then this is the conduction resistance twice $\pi L K_1$. I tell you K_1 , in Nomenclature is like that K_1 is this, in the problem which is the thermal conductivity of the cylinder material + the convective resistance.

When the pipe, is the bare pipe without any insulation then it is $h \cdot \text{twice pi } r_1, r_2$ very good, $\text{twice pi } r_2$, tell loudly, $\text{twice pi } r_2 \cdot L$. Very good. Okay. Now we have the insulating material, this is the insulating material with the thermal conductivity K_2 and we consider which we have to find out the, we take this radius to be r_2 , that means insulating material is provided of to a radius of r_3 and we have to find out this r_2

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$$Q = \frac{(T_1 - T_\infty)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L K_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L K_2} + \frac{1}{h(2\pi r_3 L)}}$$

insulated = 0.75 Q uninsulated pipe

What is the Q insulated pipe = their subjective to same potential difference because environment remains same and the inside surface remains same, $Q_1 - T$ infinity, very simple problem, heat transfer until and unless you come to convection, heat transfer things are very simple as far as this your under-graduate level material is concerned, how about conduction contents, high level mathematics after watts 3-dimensional conduction unsteady state.

$T_1 - T$ infinity. Now in this case, there are 2 conduction raise them, one due to cylinder r_2/r_1 , cylinder material $\text{twice pi } L K_1$, another one \ln due to, this is also a solid material conduction through this thick insulation which is extending from r_2 to r_3 , so therefore it is $\ln r_3/r_2$ and here it is $\text{twice pi } L K_2$ and finally the convection resistance $1/h \text{ twice pi } R_3 \cdot L$. Okay. $1/h \text{ twice pi } R_3 \cdot L$, I am sorry, R_3 small r . Okay.

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$$\begin{aligned}
 r_2 &= 0.06 \text{ m} \\
 r_1 &= 0.03 \text{ m} \\
 K_1 &= 10 \text{ W/m}^\circ\text{C} \\
 (3L) \quad K_2 &= 0.1 \text{ W/m}^\circ\text{C} \\
 T_1 &= 350^\circ\text{C} \\
 T_\infty &= 20^\circ\text{C} \\
 h &= 10 \text{ W/m}^2\text{C}
 \end{aligned}$$

Now according to the problem Q insulated 0.75 Q uninsulated, (Q insulated pipe). So, if you make this, that this = to 3/4th of this Q1 uninsulated T1-T infinity will cancel then you can write this after putting, substituting the values, now I am writing the values here T1, what are value? r2 is; what is that 60mm that means 0.06m, r1 is 30mm, what is r1? 30mm, 60mm 0.6; I am sorry., 6m and this is 0.3m. 0.6 or 0.06, are you., 06.

It was all right, 0.06 and 0.03m, 0.06 as I am writing the value 0.06m, 60mm / 10 to the power of 3,0.38m, what is K1? K1 is 10watt/m degree Celsius and K2 is what?0.1watt/m degree Celsius, what is T1? T1 is 350 degree Celsius. What is T infinity? T infinity is 20 degree Celsius but incidentally this T1 and T infinity will not be require to solve the problem, but the problem gives as like that and h which is very very important which is 10watt/m square that is its units.

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$$0.0693 + 10 \ln\left(\frac{r}{0.06}\right) + \frac{0.1}{r} = \frac{4}{3} \times 1.736$$

$$r = 64.3 \text{ mm.}$$

$$\text{Thickness} = 4.3 \text{ mm}$$

At the same time, this is given that $Q_{\text{insulated}}$ is 0.75. Now if you put these value, finally you have a relationship like this, this denominator is totally calculated, because $r^2 L K_1$ everything I know, so this becomes a finally a shape like this. I am telling the final step $0.0693 + 10 \ln(r/0.06) + 0.1/r = 4/3 * 1.736$, actually this 1.736 is these value.

I can tell you that, $1/2 \pi L *$, that I have give you a hint, that these value if you calculate, actually $2 \pi L$ will cancel from both the sides, 1.76. So, these denominators, these resistance is $1/2 \pi L * 1.76$, so this is a tedious calculation. So, after writing this, $Q_{\text{insulated}} = 0.75$ this and if you write this is = 0.75, then cancel it, 80% credit is there but do not take these advantage. This is because I am telling you, it is not only the marks that matters.

I can give you marks but you have to develop your ability of completing the work, finally when you will go to the shop or actual field, then you have to deliver things, deliver the final products so if you say sir these. Because at certain stage you cannot tell okay my assistance will do it. If you grow, go up then it will be possible but at some stage you have to work so definitely I am telling both the things.

I am not telling that final answer is be the only thing that you will get marks otherwise you will get 0, No. You may get 80% credit, if you just write this, even up to this, even you do not solve it but this type of problem may not come where the iterative solution is there. So, there is an iterative solution, because r this fashion. So, r comes out to be 64.3mm, that means the $t_{\text{insulation thickness}} = 4.3 \text{ mm}$.

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$$Q_{r+\Delta r} = Q_r + q_G A_r \Delta r$$
$$\frac{d}{dr}(Q_r) \Delta r - q_G A_r \Delta r = 0$$
$$\frac{d}{dr}(Q_r) - q_G A_r = 0$$
$$\frac{d}{dr}(q_r A_r) - q_G A_r = 0$$

Now my intention to tell it that, up to this okay, you can get 80% credit but you have to complete the work, that will help you in developing this T2 complete a thing. Okay. So, this is the simple problem just application of this principle of critical thickness of insulation. Now I will discuss about this steady one-dimensional heat transfer through spherical geometry. Spherical wall.

Now discussing different types of geometry gives you additional information at the same time brushing up again and again the fundamental equations which we are using, so this will built be in your memory. Now spherical geometry, the most simple example is the simple spherical shell, just like this a reactor, spherical reactor, you consider a spherical reactor. All these deductions are also the problem.

Spherical reactor which is spherical in nature with radius r_1 and r_2 . Now the same thing that is why I am telling that recapitulating; how to write the heat balance here again? you take the cell, spherical shell, only difference is that it is spherical in nature, in case of cylinder, it is cylindrical in nature, in case of plane wall, it is plane in nature, that is the only difference, but our problem, tackling the problem is same.

Let us consider at a radius r , we have this shell, which is having a thickness Δr and the same thing that the heat which is coming here Q_r and the heat which is going out, this is, here I will not draw this will be overlapping here, for example, Q_r this is going from the outer

surface of the cell $Q_{r+\Delta r}$ and again, small $r+\Delta r$ and again, we consider a volumetric heat generation per unit volume defined at each and every point in the spherical cell.

This material, which extends from r_1 to r_2 , this is the final r_2 . Okay. r_1 to r_2 , this is r_2 and we take an element at radius r of Δr and Q_r is the heat, total heat coming into this elemental volume and $Q_{r+\Delta r}$ going out and q_G is the volumetric internal thermal energy generation. The same equation again and again $Q_{r+\Delta r}$ must be equal to $Q_r + q_G \cdot \text{what is } A_r$? A_r is the area and Δr .

Always A_r is the area, which is perpendicular to the direction of heat flow, perpendicular to the r direction, that is all and now we expand in Taylor series, $Q_{r+\Delta r}$, then we get d/dr of, then this will be $Q_r + d/dr$ of $Q_r \cdot \Delta r$ neglecting the higher order terms, that we had done enough in plane and cylindrical geometry that means $Q_r \Delta r - q_G A_r \Delta r = 0$, again and again you are practice, so d/dr of Q_r r direction $-q_G A_r$ is 0.

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The image shows a chalkboard with the following handwritten equations:

$$\frac{d}{dr} \left(-k A_r \frac{dT}{dr} \right) - q_G A_r = 0$$

$$\boxed{\frac{d}{dr} \left(k A_r \frac{dT}{dr} \right) + q_G A_r = 0}$$

Below the boxed equation, it says $A_r = 4\pi r^2$. Then the equation is further simplified to:

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{q_G r^2}{k} = 0$$

Now Fourier heat conduction equation d/dr of, that is heat flux, heat transfer at any direction is heat flux into the area $-q_G A_r$ is 0, Again and again I am doing the same thing. Now this is not the Fourier heat conduction equation sorry, I am just splitting it heat flux into area, then Fourier heat conduction equation here, heat flux is $-K A_r (dT/dr)$ because heat flux is $-K dT/dr$. $-q_G A_r$ is 0, take this $-$ sign out, so that d/dr of $K A_r dT/dr + q_G A_r$.

Why I am writing this again and again, Prof. Som is writing, this is because if a student can do this thing well, he cannot use problem in 1 dimensional steady state heat conduction, so

far up to this equation, this is the most generalised equation, I have not imposed any geometric constants, this is the direction of heat flow. If it is a plane area, then it will be A_x , $qG A_x$, plane area may vary with, normal area may not be vary with normal area.

We have done it earlier. It may be cylindrical area, where r is the direction of heat flow, so therefore if you can remember this equation or you can immediately derive this equation from the conservation of energy, things are over. Now for any condition, if you consider K is independent of, now what or they are generalised, K is temperature dependent, may be A_r may be dependent on r , it is the 1 dimension, direction of heat flow.

qG may be constant, may be a function of r or may be 0, that means there may not be any internal energy generation. So, if you take the most simple case where K is not dependent on temperature, A does not depend on the path of the heat flow, this happens only in the plane area, okay, and there is no heat generation, then it give simple solution that $d^2 T / dr^2$ is 0, that means dT/dr is constant.

So, if you can had hide this thing, everything is done, so spherical geometry now it becomes application, A_r is $4 \pi r^2$, put that and take K constant, so what will be the result? $4 \pi r^2$ and K constant. This becomes d/dr of $r^2 dT/dr + qG r^2 / K$ is 0. Simply, so from here I can go anywhere and in any problem, if everything varies with their independent variable, okay, then it is a problem of little complicated mathematics, nothing else.

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The image shows a chalkboard with the following handwritten mathematical derivation:

$$q_G = 0$$

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = 0 \quad \frac{T_1 - T_2}{T_1 - T_2} = \frac{\frac{1}{r_1} - \frac{1}{r}}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T = -\frac{C_1}{r} + C_2$$

$$\begin{aligned} \text{at } r=r_1, \quad T=T_1 \\ r=r_2 \quad T=T_2 \end{aligned}$$

So, now if I solve this for a simple case without heat generation what do I get? Please tell me what do I get? Without the heat generation, do it, this is the problem, tutorial problem type of T. without the heat generation, when $q_G=0$, then d/dr of r square dT/dr , did that means dT/dr is some constant by r square, first integration. Second one is T is, so if you integrate this – c_1r+c_2 , now it becomes routine and to some extent boring.

But as a student you have to go through this part also, so it is not challenging at this moment at boundary condition $r=r_1, T=T_1$ at $r=r_2, T=T_2$ and if you do it meticulously then what you get this. If you solve this with the boundary condition, you get the temperature distribution like this, T_1-T/T_1-T_2 , straight forward you get, $1/r_1-1/r$, r is the current radius where the temperature is T , $1/r_1-1/r_2$, so this type of function will come.

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$$\begin{aligned}
 Q_r &= -K(4\pi r^2) \left(\frac{dT}{dr} \right) \\
 &= -K(4\pi r^2) \left[\frac{-\frac{1}{r^2}}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right)} \right] (T_1 - T_2) \\
 &= \frac{4\pi K}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right)} (T_1 - T_2)
 \end{aligned}$$

Next part is to find out the heat transfer rate at any radius r , it is $=-K \cdot \text{Area}$, $(4\pi r^2)$ $\cdot dT/dr$ at $r=$, sorry not r , at any r , because I am finding out at any current radius which is r , $4\pi r^2 \cdot K(dT/dr)$, Now what is dT/dr ? $4\pi r^2$, now dT/dr is, there is a – sign, $1/r_1-1/r_2$, so there will be a – sign, so $-$, dT/dr is $1/r$, $-1/r$, that it $1/r^2$, $-$ will be there, sorry, that means these r^2 and r^2 will cancel out, $4\pi r^2$.

dT/dr okay, $-4\pi K/$ and T_1-T_2 , yes I have done a mistake, T_1-T_2 , meticulously you do, dT/dr is $-$, this is the – sign, $T_1-T_2 \cdot -1/r$ will be $1/r^2$, so therefore this $-$, $-$, will be, therefore, $4\pi K/ (1/r_1-1/r_2) \cdot T_1-T_2$, you can check whether you are doing a mistake or not, if you remember the final formula, that it will be again the potential difference, $-$, divided by a resistance, whose formula is this one $r_2-r_1/4\pi K (r_1 r_2)$.

That means this is the conduction resistance, which is $\ln r_2/r_1$ by twice $\pi L K$, sometimes the memory helps this way, that you can check Oh, this is not coming, somewhere I have done some mistake. Another thing by your concept, when you are finding Q at any cross-sectional area, where the area is the function of r or function of x , that will be cancelled finally with the dT/dr expression, because Q total heat transfer has to be independent of the direction.

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The image shows a chalkboard with the following handwritten equations:

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{q_G r^2}{K} = 0$$

$$r^2 \frac{dT}{dr} = - \frac{q_G r^3}{3K} + C_1$$

$$T = - \frac{q_G r^2}{6K} + \frac{C_1}{r} + C_2$$

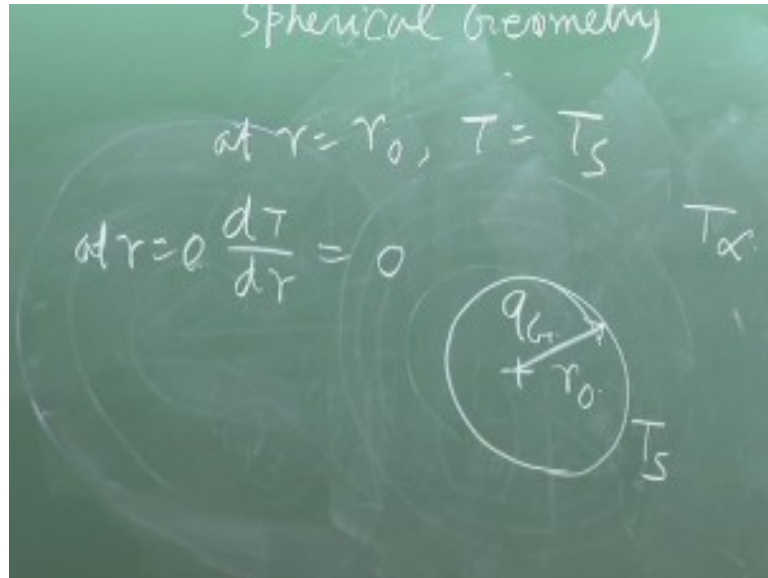
Because at all section same, clear. With heat generation, how to find out with heat generation? To find out the temperature distribution, you have to solve this equation, d/dr of r square (dT/dr) which we arrived earlier, we discarded this term considering q_G to be 0, $q_G r$ square/ K . Now with heat generation, but q_G to be uniform, that means constant, volumetric heat generation at any location is independent of the coordinate not dependent on the direction along the heat flow.

That means, q_G is constant, not be the function of r . In that case it will be again a very simple integration considering q_G constant and you can find out, should I do it or you will be able to find it out? What is the expression you can find it out? that d/dr of r square (dT/dr), that means you take this that side, then you, first integration r square (dT/dr) is $-q_G r^3/3K$, Am I right? then dT/dr is $q_G (r/3K)$.

Next is + some integration constant will come. The next will be $T = -q_G$, this will be again r , so this will be ultimately r square/ $6K + c_1/r + c_2$, before writing the boundary condition, now this problem is not a spherical cell, yes, just I am solving as a mathematician, a differential

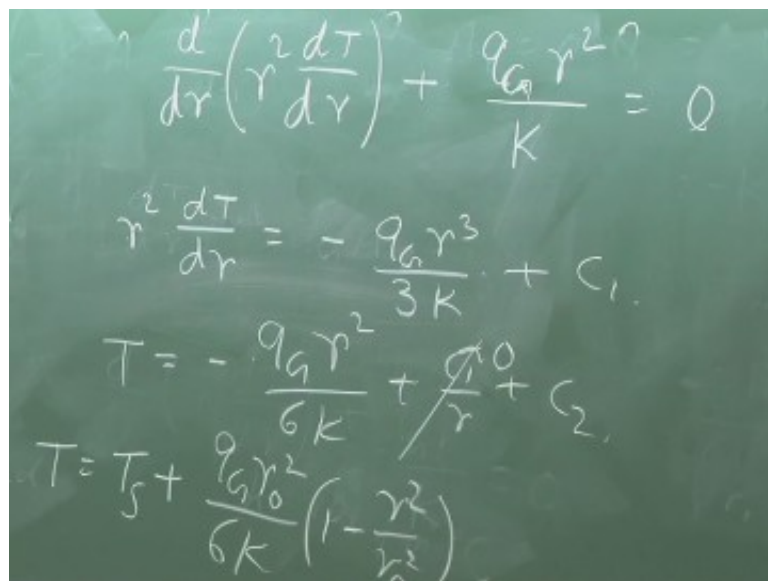
equation I am provoke, tempted to solve it, but what is the practical problem? There is a spherical container packed with material, that means this is a solid spherical container, whose radius is r_0 .

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The problem is prescribed that thermal energy is generated at a rate of q_G per unit volume which is constant throughout and I have to keep this surface temperature T_s constant, well there is an ambient T_∞ which is not very important in determining the temperature distribution, so this is a solid sphere, so the boundary condition requires that at same thing, at $r=r_0, T=T_s$.

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Because of the symmetry dT/dr is 0, at $r=0$, centre, which means that C_1 has to be 0. C_1 cannot exist because there should not be any term $1/r$, which will give a discontinuity at the

centre and moreover dT/dr is 0, at $r=0$ and first boundary condition T_s gives $c_2 = T_s + q_G r_0^2 / 6K$, so therefore we can write the expression as $T = T_s + q_G r^2 / 6K$, this is so simple that is the repetition.

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Handwritten equations on a chalkboard:

$$Q_r = -K(4\pi r^2) \frac{q_G}{3K} (-r)$$

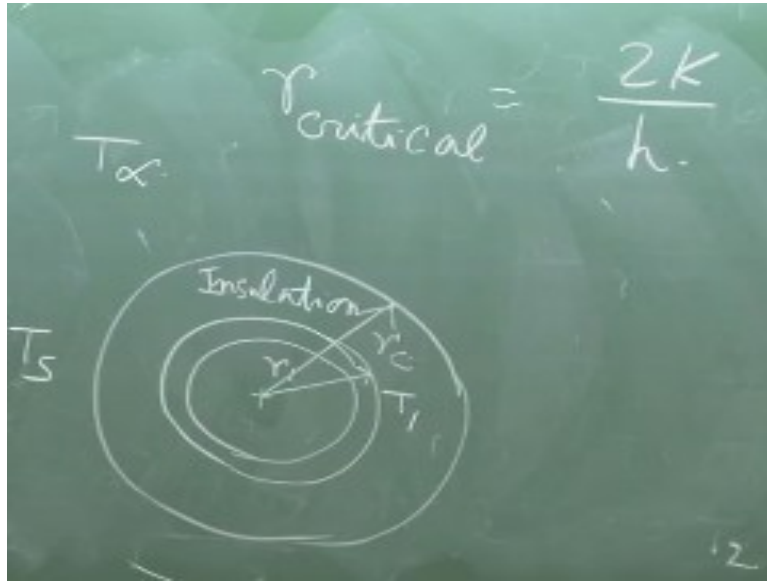
$$Q_r = K\left(\frac{4}{3}\pi r^3\right) q_G$$

$$Q_{r=r_0} = \frac{4}{3}\pi r_0^3 q_G$$

If you find out heat transfer at any section Q , what the expression is? $Q = 4\pi r^2 - K A$, again and again doing means, this will be memorised. $Q =$ at any r , $-K \cdot 4\pi r^2$, are into dT/dr , what is dT/dr ? $2r$ or r_0^2 square, r_0^2 square will cancel, that means $q_G/3K \cdot 2r$, $q_G/3K \cdot 2r$, that means 2 , 2 will cancel $3K$ and this will be $\cdot r$. that means and there will be a $-$ sign, so $-r$. So, if you take care of this, it will be $K \cdot 4/3 \pi r^3 \cdot q_G$ and this K , K will cancel, sorry.

This K will not come, So, why I have written that? I want to show you here with heat generation at any radial location, heat transfer rate is the function of r , because it is changing, because of the generation of local generation of thermal energy. It is not constant, without q_G , it is constant, but that $r=r_0$, this value must match with the total gross energy balance, total heat generation. This is total heat generation.

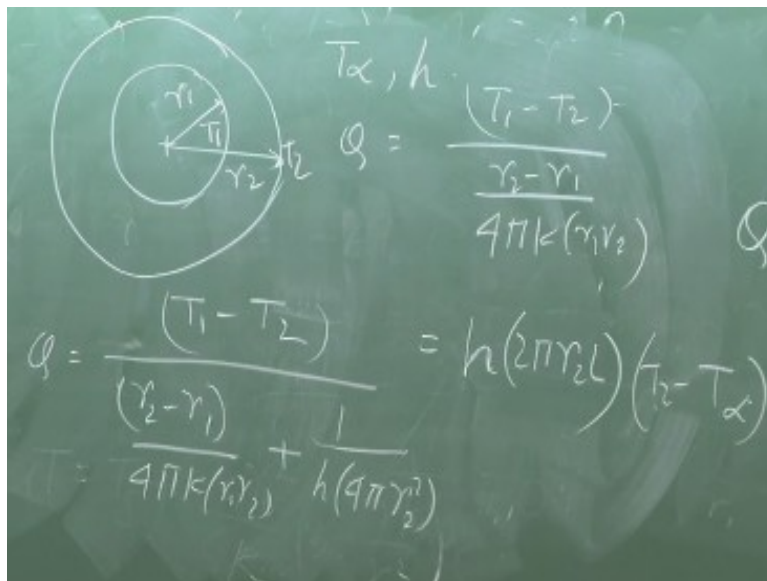
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That is all, here also if you add insulating material in the spherical cell, the concept of critical radius of insulation will come and you derive it, it is your task that I will not do, the critical radius of insulation for spherical wall is r_{critical} is $2K/h$, that means if I have a spherical cell or a spherical ball, whatever you call, spherical cell and the problem is prescribe like this, this is r_1 , the outer radius and heat at a temperature of T_1 .

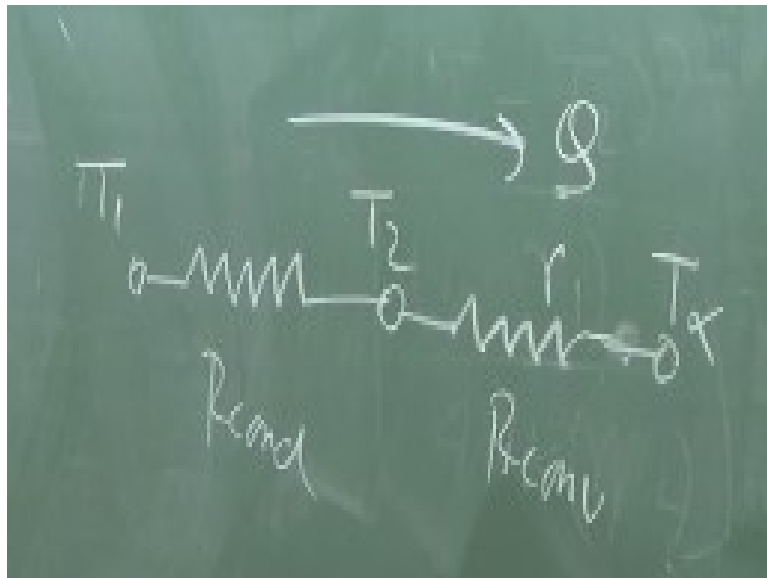
Then if I put insulation, where this is the radius r_c , the critical radius, the insulation. In a similar way, we derive for a cylindrical coordinate, we can derive the heat transfer rate will maximize, initially increase and then maximize where r_{critical} is $2K/h$, this is the conduction resistance, you will have to consider the convection resistance, that means if this sphere, a spherical cell is subjected to convection.

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The temperature is given of the ambient, I am giving you hints that how to solve this problem, so this is the conduction resistance, but you have to give, in the convection resistance that if a, before solving this problem, you must have this thing in your mind that if a spherical that like the same cylindrical one. If this is r_1 , this is r_2 and we are provided at a temperature at T_1 and T_∞ and h , then $Q = (T_1 - T_\infty) / \text{the conduction resistance}$.

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That is $r_2 - r_1$ as we are generated that is $4 \pi K(r_1 r_2) + i/h (4 \pi r_2^2)$, this is because heat is, Q is $T_1 - T_\infty$, if I consider this surface temperature is T_2 , which is not prescribed by, but I can write in these way $r_2 - r_1 / 4 \pi K (r_1 r_2)$ and the same heat is flowing in series under steady state with $T_2 - T_\infty / h$, rather it is better to write this fashion, it will be understandable by the definition of h , $h * (2 \pi r_2 L) * (T_2 - T_\infty)$.

That means again it is a series problem that means series network is like this, conduction resistance similar to that cylindrical and convection resistance T_1 , T_2 , T_∞ , so this is $R_{\text{conduction}}$, this is $R_{\text{convection}}$ this term, so therefore this is R_{series} this series is the similar to that of cylindrical geometry and then you have to take care of and then differentiate to find out these, critical radius, very good, $T_2 - T_\infty$, correct anything okay, any question? any question? okay all right.