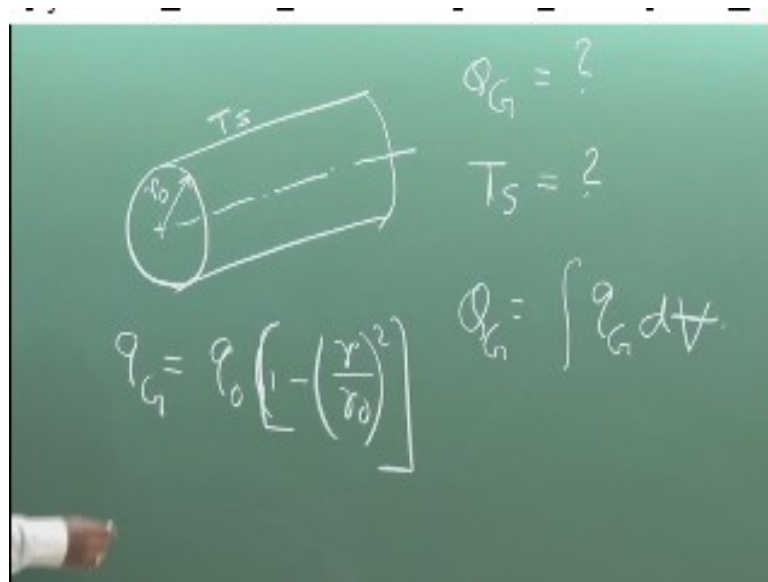


Conduction and Convection Heat Transfer
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Lecture 18

Now today, we will solve 1 or 2 problem relating to 1-dimensional steady state heat conduction in cylindrical geometry, cylindrical wall, that is in a pipe of circular cross section and a very interesting related problem is the critical thickness of insulation. So, let us solve 1 or 2 problems just to habits applications what yesterday also we solved a problem that is at the last class.

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Now the problem states like this, I am telling if there is a cylindrical container, this is the axis; cylindrical container of radius r_0 and the language of the problem is like that radioactive Os are packed in a long thin wall cylindrical container, that means the radioactive Os are packed that means ultimately works as a solid cylinder type of thing. The radioactive Os are packed in a long thin wall cylindrical container.

The waste generate thermal energy non uniformly according to the relation that the volumetric energy generation rate at a point is given by q_G , $q_0 \cdot (1 - r/r_0)^2$, where q_0 and r_0 are constant. That means, it is varying with r , which means there is a cylinder solid, because this thin wall container is packed with the radioactive Os, quick generates in such a way.

So, what they want to obtain expression for the total rate at which the energy is generated, total rate of energy generation and use this result to obtain an expression for this surface temperature T_s . So, this is simplest again problem. First part is the total energy generation from the volumetric energy generation rate so it is nothing but q_G is integral $q_G dV$. I usually represent volume as a V with a cart to distinguish it from velocity.

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$$\begin{aligned}
 Q_G &= ? \\
 T_s &= ? \\
 Q_G &= \int_V q_G dV \\
 &= \int_0^{r_0} q_G \cdot 2\pi r L dr \\
 &= \frac{\pi q_0 r_0^2}{2}
 \end{aligned}$$

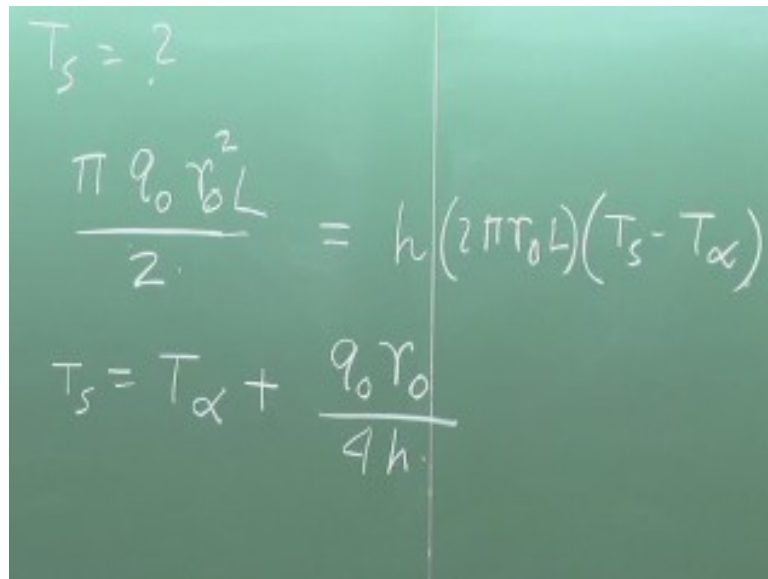
The convective heat transfer is associated with fluid mechanics fluid flow we use v as the velocity, that is why I use this big V , this V is the volume; dV is this small elemental volume, that means if we consider an elemental area like this in the container, thin cylindrical elemental area at a radius r and the thickness of this, is dr for example, then this will be nothing but q_G . What will be the volume? $2\pi rL \cdot dr$.

If you do that, you get the expression this as, $\pi q_0 r_0^2/2$, very simple. You put this $2\pi L$ will come out so $r, dr r^2/2$ that is $r_0^2/2$. Because this now will be; this is for the entire volume. Now when I put in terms of dr , it will be $0, 2r_0$. So, the first term will be dr, rdr that is $r_0^2/2$ and second term will be $r^2/r_0^2, r^2/r_0^2 dr$, that means r_0^4 and denominator r_0^2 , it will be again $r_0^2/2$.

So, ultimately you get $\pi q_0 r_0^2/2$. This thing, I am not doing in the class. So, this is the expression (()) (5:51) nothing, but what is the T_s ? Now to find T_s , expression at the surface temperature, you have to apply certain intelligence that always we do not want a routine (()) (6:06), that we have to find out a temperature distribution, heat generation; is not

required. Simply, the gross energy balance, the total heat which is being generated or total thermal energy which is being generated must go out from the surface.

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$$T_s = ?$$

$$\frac{\pi q_0 r_0^2 L}{2} = h (2\pi r_0 L) (T_s - T_\infty)$$

$$T_s = T_\infty + \frac{q_0 r_0}{4h}$$

Because the problem is prescribed by a heat transfer coefficient h and the T infinity. This problem is prescribed that the container is immersed in a liquid or fluid that provides a uniform convection coefficient h and at a temperature T infinity and expressed T_s in terms of T infinity, h and q_0 . So, therefore what we can write? We can write $\pi q_0 r$ square/2. This is per unit length. So, therefore, these as to be this. So, this is qG per unit length. I am sorry.

The result I have written since I have not worked it out here that is why this mistake is there that qG/L is $\pi q_0 r_0$ square/2. So, this L is there, so L is comes out, L will come out so qG/L . So, the total qG is this, this must be $=h \cdot \text{outer surface area}$ which will be twice by $r_0 \cdot L \cdot T_s - T$ infinity. So, therefore from here we get T_s , T_s will be $= T$ infinity that is the ambient temperature +, π , π will cancel, so 4 will be there that means $q_0 r_0 / 4h$, $\pi r_0 q_0 r_0$ square, $\pi r_0 q_0 r_0 / 4h$.

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Consider a pipe of inside radius $r_1 = 30\text{ mm}$, outside radius $r_2 = 60\text{ mm}$, and thermal conductivity $k_1 = 10\text{ W/(m}^\circ\text{C)}$. The inside surface is maintained at a uniform temperature $T_1 = 350^\circ\text{C}$, and the outside surface is to be insulated with an insulation material of thermal conductivity $k_2 = 0.1\text{ W/(m}^\circ\text{C)}$. The outside surface of the insulation material is exposed to an environment at $T_2 = 20^\circ\text{C}$ with a heat transfer coefficient $h_2 = 10\text{ W/(m}^2\text{ }^\circ\text{C)}$. Determine the thickness of insulation material required to reduce the heat loss by 25 percent of that of the un-insulated pipe exposed to the same environmental conditions.

I think it is okay. This is the problem.