Conduction and Convection Heat Transfer
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Lecture 17

We will discuss the cylinder, cylindrical geometry, internal thermal energy generation.

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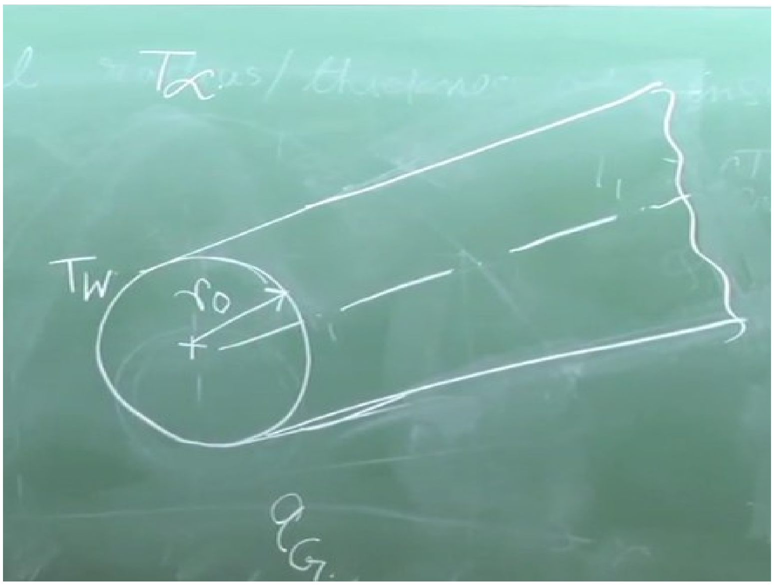

Let us consider a problem like this, we have a cylindrical rod, solid rod of radius r0, thermal energy is generated, which is specified by, the thermal energy generated per unit volume at a point, which may be a function of r. It is a one-dimension problem, so qG may be a function of r that is the radius of this solid cylinder, r may be constant. That means the volumetric heat generation rate is same at each and every point.

And this is the cylindrical element, and this is kept in a fluid or in an ambience, for example air, T infinity. Now energy is generated and this surface is kept at a temperature Tw, now our job is to find out the temperature distribution, within the element and what is the rate of heat transfer all these things. This particular problem originated from the fact that the nuclear power plant, the fuel rod.

That is the nuclear rod, nuclear fuel is used where the energy is generated within it by virtue of a chemical reaction, huge exothermic chemical reaction, to boil the water in which these rods are immersed. But the rate at which the energy is generated, it is very difficult to control the temperature of the water. So, cooling is extremely important, of course there are many more things.

I do not want to discuss these things here to moderators are used all these things are there to control the rate of reactions and all these things. But this originates from a problem like this, that there may be a solid cylindrical element where heat is generated one of that example, one of the example this the nuclear rod in a nuclear power plant. So, this is, my drawing is very bad, this looks like a divergent, sorry, this looks like a, sorry, let me draw it, this looks like a divergent, okay.

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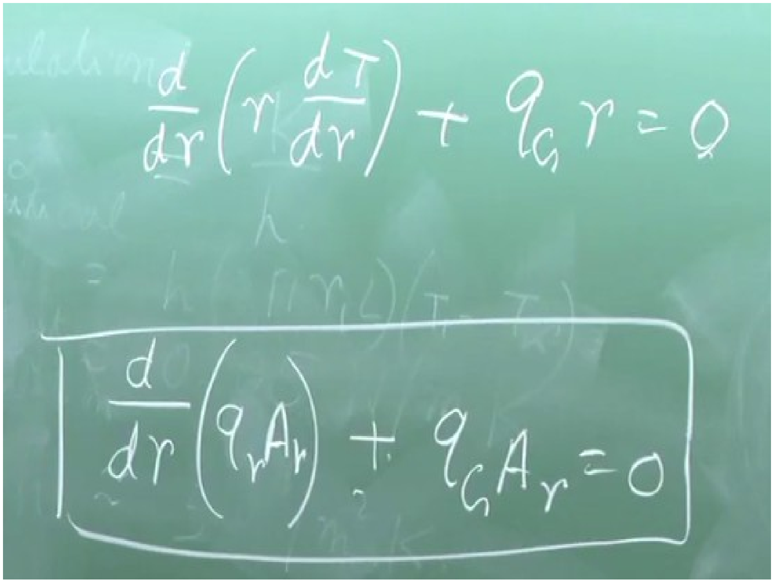

Now therefore we now compile this to our software language, that there is a cylindrical element, where from we will start. We will start from this, that we know d/dr of r dT/dr plus qG into r is equal to where from we get it, sir where from you have got it, you may have got it from the, you may get it from the general heat conduction equation temperature distribution.

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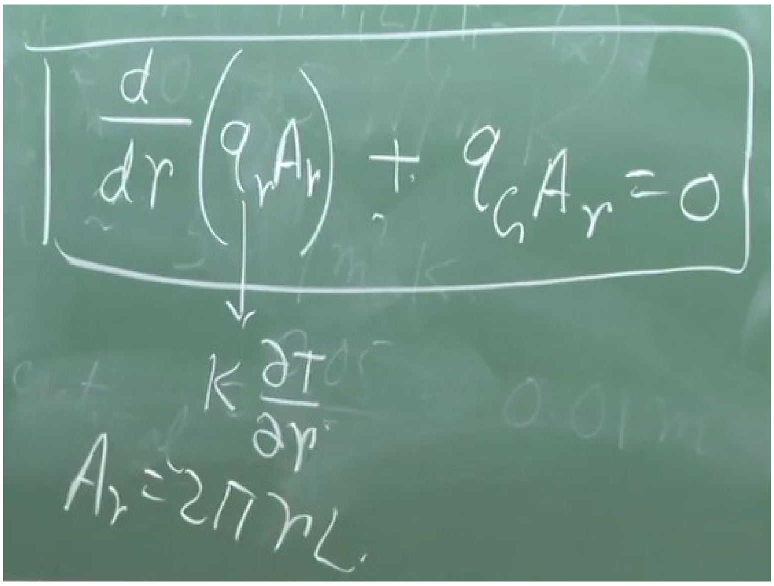

But I will recommend you, I am telling again and again, for a steady one-dimensional heat conduction you generate it form that. This may be cylindrical, this may be plane wall, very important for you people, that this is like this, d/dx or here I will write d/dr of Qr, okay, plus, Qr means, sorry I am extremely sorry, I am extremely sorry, qr Ar plus qG Ar zero, because this is very easy to remember.

I am telling you again, what is the nomenclature, this q is the heat flux for unit area, perpendicular to the direction of heat flow, here r is the direction of heat flow. If it is x direction in the plane geometry, it is d/dx of qx into Ax, Ar is the area normal to the r direction, similarly Ax is the normal to x direction, plus the energy generated per unit volume into the area, it may be Ax or Ar is equal to zero.

Where from you get it, you take a small elemental volume in plane surfaces is a rectangular small element, in a cylindrical surfaces, it is small cylindrical ring and you make an energy balance, that heat coming in, heat going out must balance the energy generated within it, because nothing can be accumulated, nothing can be absorbed, because this is at steady state. So, if you have this thing in your mind, so you can start from here, clear.

Now qr is K delT/delr or delT/delx, Ar accordingly in a plane area, with constant cross-sectional area A will come out, otherwise A may vary in a converging diverging passages. And Ar in case of twice pi rL, so if you put that this is L of heat conduction this is this and you consider K to be constant, then you get this equation, for cylindrical co-ordinate system. Hello why you are doing like this, this is very simple, you do not get it?

This was done in the class, emphatically you are not getting it, K is missing everything is missing, first expression K is missing, first expression where K is there. It is qG by K, okay, Kr, here K will be there, qG by K, okay. K is missing, I am sorry. This is okay, you tell me, sir K is missing at immediately. Here K will be there, qG by K, okay. K is missing, I am sorry. This is okay, tell me, sir K is missing that it will be qG by Kr that is why the mistake is there.

If you just try to remember from the equation, you see, if I do this mistake then it will be there for you. That is why I am telling why you will mug up the equation like this, very good, qG by K. It will be detected when you go on deriving this thing we will see that something is not coming, some K is missing. Where it is missing? The main equation. But better you start from here d/dr of qr into Ar, delta r will be cancelled qrAr is 0, so you get the equation, clear.

Now you differentiate it. Now, if I consider qG is constant or uniform.

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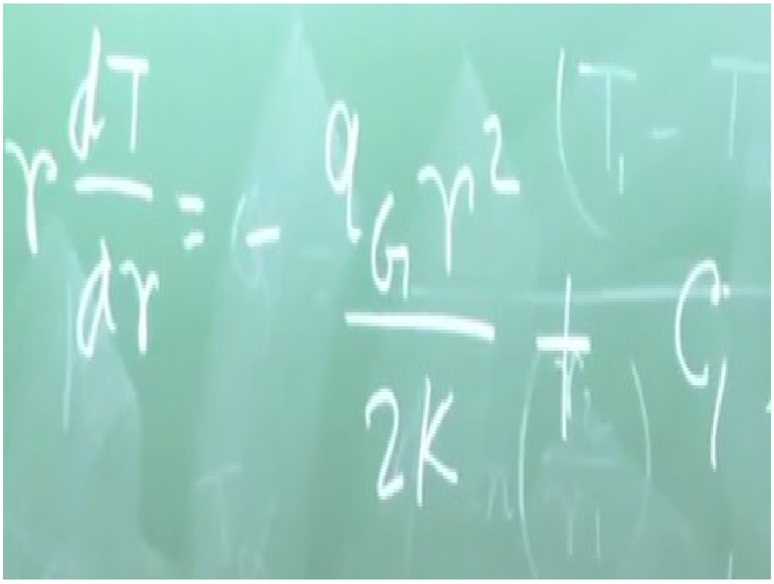

Now you differentiate it. r dT/dr is equal to minus qG r square by 2K plus C1, okay. Immediately you tell, sir your K is missing, okay. Otherwise there will be problem, why do you keep mum, sir K is missing. I will be very happy, okay. Then next part is that this will be C1 by r, so next one T will be minus qG, again r, r, r square by 4K. There are two constants, you require two boundary conditions. What are the two boundary conditions?

One is very simple at r is equal to R0, the problem is prescribed at R0, T is equal to Tw.

This boundary condition comes from the practical information that we have to keep this surface at a given temperature C1 lnr, so I have not integrated, good, so I will get 0, so that is an advantage of a teacher that he does not zero, but I am sorry, in the examination, but I will not give you zero.

I tell you even if you write C1 by r, why do you know, at least you are able to write the first step from the energy conservation, then I will consider that hurriedly you forgot to integrate, I will deduct the marks, but not I will give you zero, but however do not do that, sorry. C1 lnr plus C2. How do you get two constants, quick, quick. What is C1? Just without any boundary condition, what is C1? Zero because it will give a discontinuity at the center.

The problem is not physically discontinuous at the center. It cannot define the function, so C1 has to be zero, but the problem requests a mathematician. A mathematician will not leave you. Engineers are very smart. They tell many things, okay I understand, but it matches correctly, mathematician will tell, no you write the number and condition. Your mathematics teacher will tell that there are two constants you write, second order differential equation, two constants, two boundary conditions,

So, another boundary condition, I told you in the class earlier known as symmetry boundary condition. The problem is symmetry, why? Because the temperature is uniform azimuthally, qG is constant, uniform heat generation, not the one-half heat generation is higher another half lower, it is not like that, so therefore problem is entirely uniform azimuthally, means uniform with respect to the axis r is equal to 0, which means an symmetry boundary condition that at $r = 0$, the derivative is 0.

Function has a value, which is either maximum or minimum that means function is continuous with a 0 derivative exhibiting a maximum or minimum. That is mathematical statement. Physically, a smart boy will tell C1 cannot exist because lnr ka place nahi hai here, because this has to be 0, so now what will be the value C2 will T0 plus qG r0 square by 4K that means the expression T is equal to what?

T2 is Tw, T minus Tw rather you write like this is equal to qG by 4K r0 square, am I correct. C2 is Tw plus qGr0 square by 4K. It will be in the numerator, r 0 square will come on, it will be in the numerator, so qGr0 square by 4K minus qGr square by 4K. Now it is okay, so this is the distribution. Now where is the maximum temperature, it is very simple that r is equal to 0 is the maximum temperature.

You do not have to make this dT/dr zero, but dT/dr zero will tell you the same thing that r is equal to 0. Maximum value of r is r0, so therefore at r = 0, so this is maximum temperature that means T0 is the maximum temperature.

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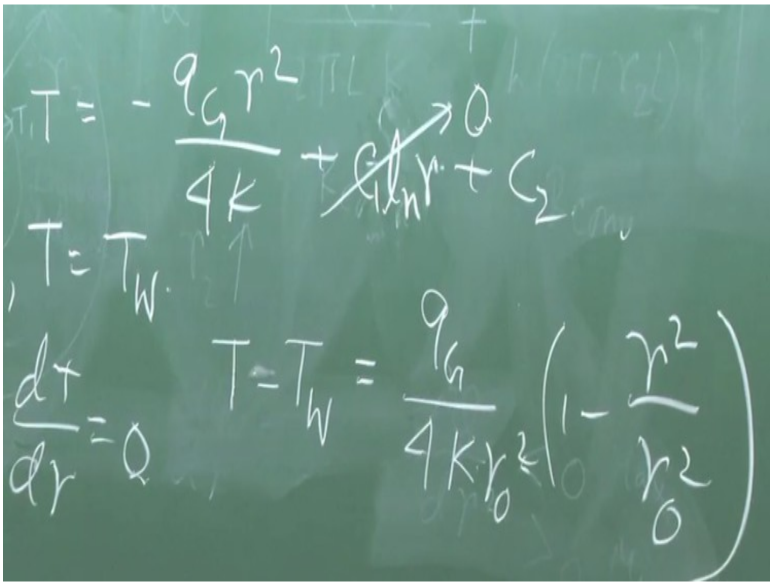

T0 minus Tw is qG r 0 square by 4K, so there is a convention that we write the non-dimensional form of the temperature distribution or dimensionless wall like this 1 minus r square, that means this is a parabola, so therefore, this we can show like this.

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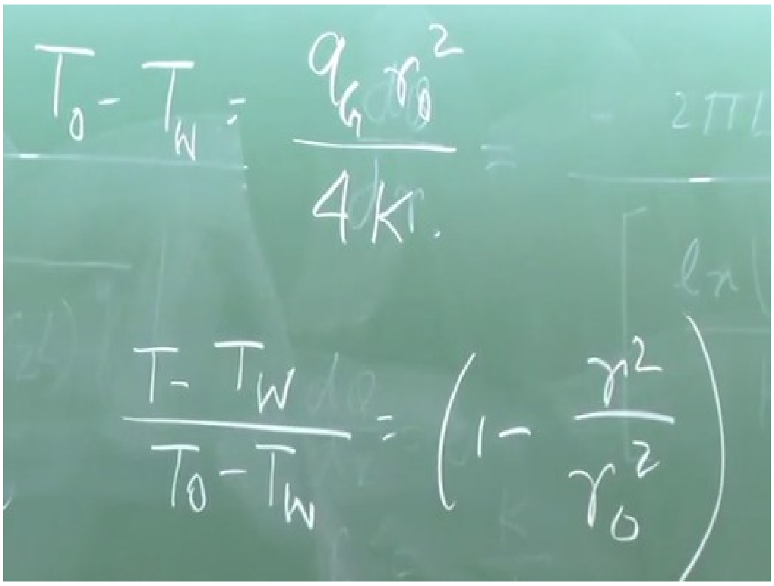

Here, this is the temperature scale, this is Tw and this is T, T as a function of r, so that means it is a parabolic distribution symmetry about r is equal to 0, very simple. You can find out dQ/dr at the surface.

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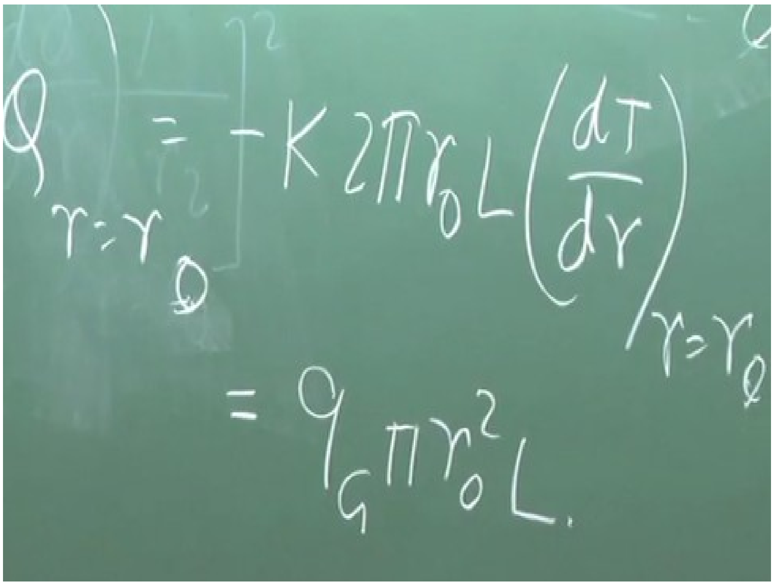

We are interested what is the dQ/dr at r = r0, what is that? It is minus K area twice pi r0 L into dT/dr at r = r0. Heat flux is not same at all radius, because heat generation is there, thermal energy generation, at steady state it is so. So, therefore if you ask me the general expression of, oh god, I am doing so much mistake today. Q at r is equal to r0, so Q at r is equal to r you can find out as an expression.

Then you can put r = r0 or you can write instead the area into dT/dr, r = 0. Now here if you find out the temperature gradient at r = 0 from this expression, you can find this as qG into pi r0 square L, okay, clear. You can find out the value Q at any r, that also you can find out that is minus K twice pi r L into dT/dr, which is qGr0 square by 4K into minus two r by r0 square, r0 square and r0 square will cancel.

One can find out Q this is minus K at any r, twice pi r L, at any radius r into dT/dr. What is dT/dr? dT/dr is twice r qG by 2K minus qG by 2K, r0 square and r0 square will cancel, 2Kr. That means minus pi, minus will cancel, K pi r square qG into L.

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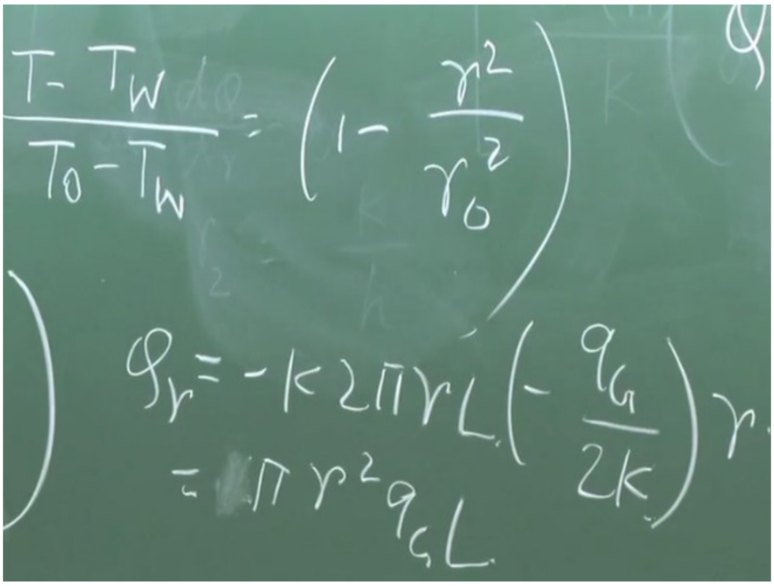

K will also be cancelled, so therefore this is a common expression for Q at any radius and it shows that it is a function of radius, obviously. Q at all r will not be the same because in a disgeneration, it is not a steady state with no generation, so all concepts will be matched. If you do like that that is also possible step for what you can find out that r = r0 if you are told to find out at the surface.

But you can find the general expression r = r0 square, which means that the entire thermal energy, which is generated is coming out from the surface. That is the steady state requirement. That is the requirement of the steady state heat transfer, okay, alright. Now after this, we will solve very simple problem in the class. Now we will solve a simple problem just to apply our equation.

The simple problem, let us consider this one: A tube wall of inner and outer radii ri and r0 that means.

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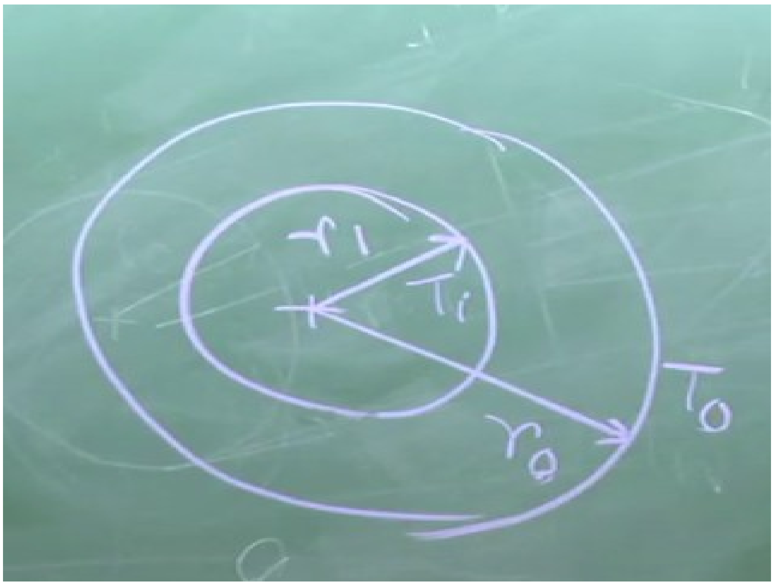

This is extremely simple problem. I do not know whether there can be any problem simpler than this, ri and r0 whose temperatures are maintained at Ti and To, o is outer, surface temperature. No ambience, nothing. You can see, can you this thing? Now cylinder conductivity, thermal conductivity is temperature dependent, so long in the class Prof. Som has talked about constant thermal conductivity.

But now the problem is a conductivity dependent thing, but does not matter, it is mathematics. K is K0 into 1 plus at, where a is a constant. K0 and a are constant, often an expression what the heat transfer per unit length. How to find out the expression for heat transfer per unit length. There are two ways, I am telling you. I told you earlier also.

You can find out the temperature distribution and you can find out from the temperature distribution of the heat flux, but I think better it will be better if you stop there, the heat transfer distribution, not temperature. For example, what is that?

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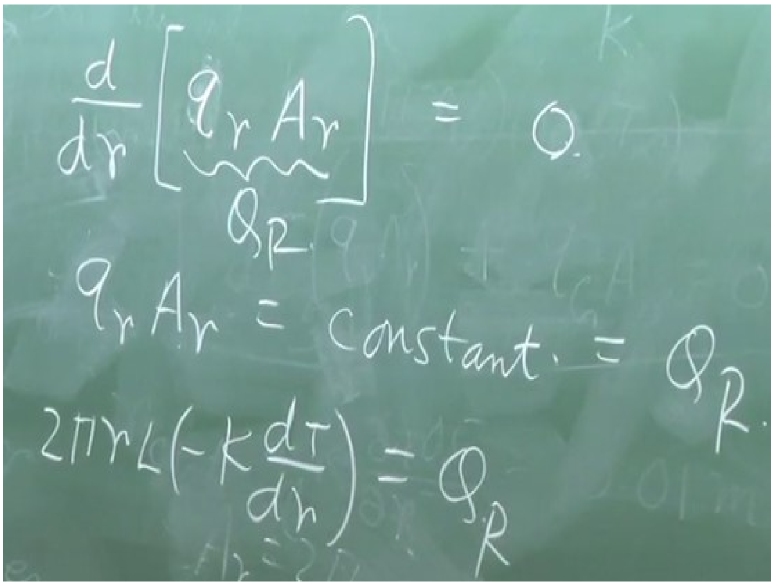

Again, I will go through in a simple unique way d/dr of qrAr, sorry, extremely sorry. d/dr of qrAr is what, there is no generation, 0. I will stop here. I will not go for temperature distribution. Stop here means, I have to put of course temperature because it has to be found out Ti, To, then d/dr, no I will not do like this, not temperature. What I will do, then qrAr is equal to constant and that constant equal to qr, that is the rate of heat transfer.

Total rate of heat transfer. Now what is qr? Ar is two pi rL and qr is minus K dT/dr and that is equal to constant and that constant I will use as QR, which is constant, total rate of heat transfer. Here total rate of heat transfer from any Ar is constant because this equation tells that. This is QR, this is nothing but QR. So, therefore QR is minus twice, now if we use that, if you are told to find out the expression of heat transfer QR.

Then it is greater to use that an integrate. Then, how to integrate?

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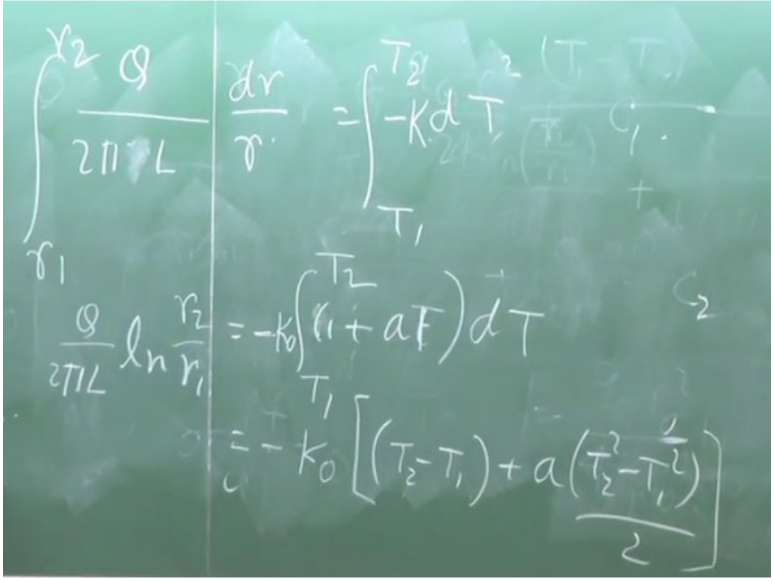

Q by twice pi rL dr/r. Now I am writing QR as Q, because this is constant. This is not dependent on r, Q by twice piL dr/r is minus dT. Since Q is constant, it is better I can integrate simple integration by taking, K is there, I am sorry, minus K dT, okay. Q is constant, so therefore Q by 2 pi L ln r2/r1 is equal to now K is K0 1 plus AT, so minus K0 will come here, minus K0 1 plus aT.

Here this aT is, this book nomenclature is this. It is function of temperature T, small t is for time, a plus T. So, this is minus K0 into T2 minus T1 plus a T2 square minus T1 square by 2. Okay, this is alright. So, you can find out the heat flux per unit length as a function of T2, T1, but if I define a thermal conductivity at the mean temperature T1 plus T2 by 2, then what will be the thermal conductivity at the mean temperature?

Km is equal to K0 1 plus a into Tm where Tm is equal to T1 plus T2 by 2. Why I am doing that? Because this can be written as extremely simple.

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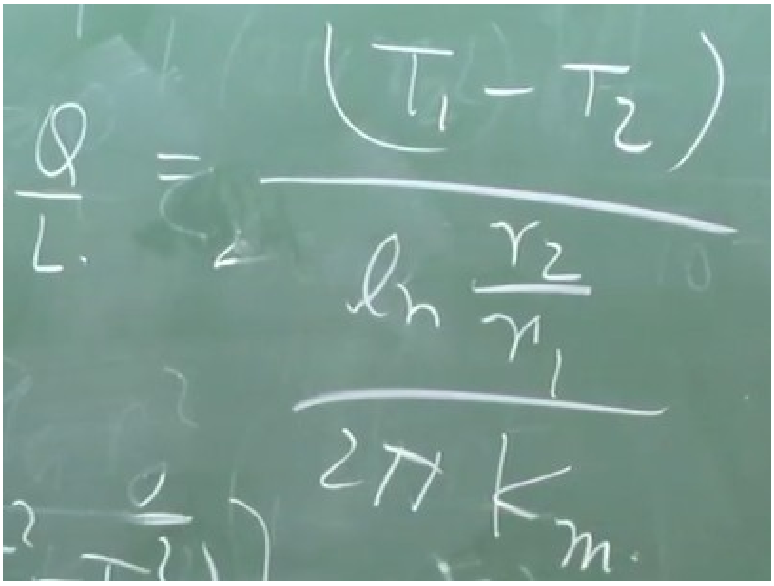

This can be, the right-hand side can be written as minus Km into T2 minus T1 that is T1 minus T2 or you can write the right-hand side now. Again, I write Q by twice pi L ln r2/r1 is equal to, I take here minus sign, K0 into T1 minus T2, not K0, Km. Why I am writing this mathematically? Then, I can tell that if we define, if this type of linear temperature dependence is there.

If we can define thermal conductivity at mean temperature, then it is similar to a constant thermal conductivity case that at the mean temperature. Because for constant thermal conductivity, this is the expression. That means Q is equal to the same expression, the potential difference divided by ln r2/r1 and twice pi Q/L rather twice pi Km, as if it is a constant thermal conductivity, but at the mean temperature.

This is nothing, this is simple class 9 level school mathematics, that means we just take care of the thermal conductivity as a function of temperature.