Conduction and Convection Heat Transfer
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Lecture 16

So, last class if you recapitulate, we have discussed the steady one-dimensional heat conduction through cylindrical geometry. We discussed the problem of a cylindrical wall, which in practice is conceived when the heat is transferred through the wall of a pipe, cylindrical pipe.

And we derived the expression for steady state temperature, then heat flux, everything.

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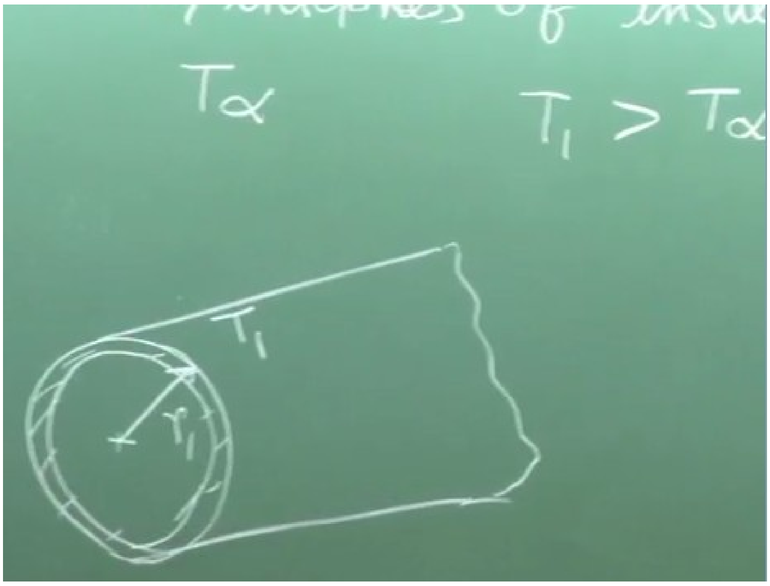

Now today as a corollary of that we will discuss an aspect which probably, I told you in the last class is the critical, critical radius or thickness of insulation, what is the meaning of it. Critical radius or thickness whatever you call, of insulation, what is the meaning of it. Let us consider a problem on the back drop of our knowledge which we had in the last class, that we have a pipe, cylindrical pipe, like this, with external, that is outer radius is r1.

The cylinder may have a thickness, which may be very small or large, but I am not bothered about this thickness. This is because the problem is prescribed like this we have a outer radius r1, at a temperature of T1. That means the problem is prescribed like this. That means somehow heat at the T1 is greater than the surrounding temperature T infinity, which means in practice.

We can consider like this, a hot fluid is going through the pipe, and from hot fluid by convection the heat is coming to the inner wall having some temperature by convection heat transfer, as we discussed earlier, causing the convection resistance, then it flows through the thickness of the pipe, which may be small or which may be large. If thickness is very small, the resistance is very small, temperature drop is less.

All these things we know, then ultimately it comes that the outer surface, whose temperature is T1. Now this problem have not discussed, if supposed like this we have an available outer surface of a cylindrical pipe, whose temperature is T1 and heat is lost from the outer surface to the ambience T infinity. And a convection coefficient is prescribed, what is ambience, here as h, heat transfer coefficient, try to recognise the problem from physical and practical point of view.

Then we will compile it to our own language of solution and we will solve it. So, practical problem is like this, we want to reduce the heat loss. Now if the hot fluid is going through it where we want to preserve the heat, we do not want that heat should be lost. Then what we will desire, that this outer surface will become hot. If this pipe is a conducting material, made of conducting material.

Then we will put insulation that is our common knowledge at the outer surface to reduce the heat loss. As we have seen in practice, now the question comes, that if we put insulation over the outer surface.

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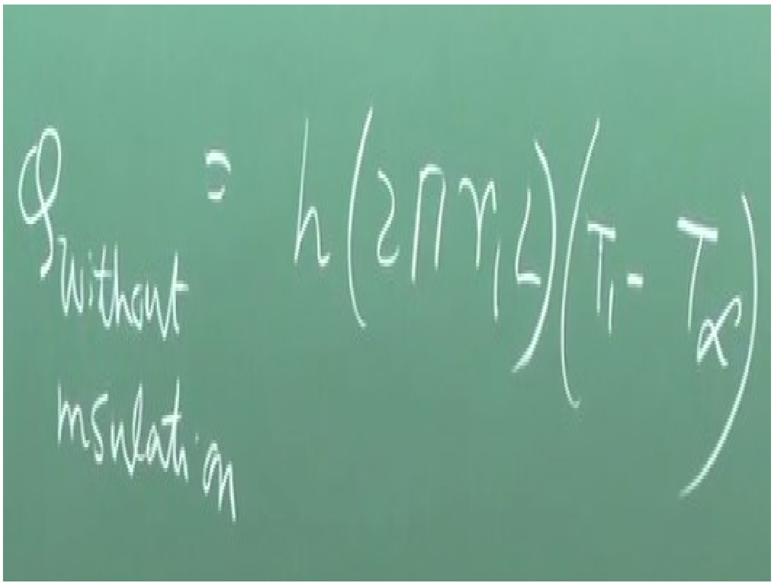

Now if we do not put the insulation what is the heat loss from the outer surface Q is equal to, without insulation, without insulation, without insulation, equals to h into area 2 pi r1 into L into T1 minus T2. Now I want to reduce this heat loss, common sense says that you add insulating material. That is material of lower thermal conductivity, okay.

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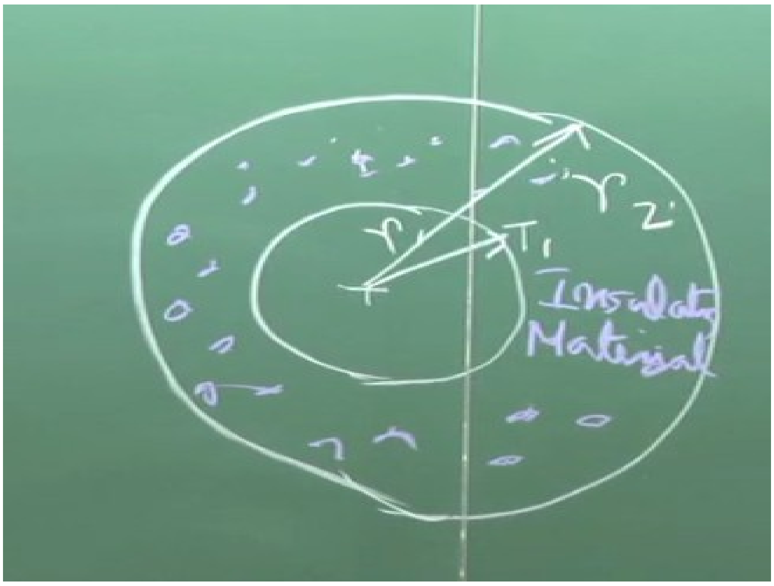

Now if you do so, let us draw this here again, front view that is r theta plane, let us have this insulation. I am only drawing the outer surface. The problem is forced with outer surface at T1. And now you put insulating, this is insulating material, this will look good, this is insulating material. And let us describe this radius r1 and let us consider the insulation is given up to a radius r2.

Then I am drawing this outer surface, and the outer surface of the cylinder and this is the outer surface of the insulating material, which is in the form of another concentric cylindrical surface over it, okay.

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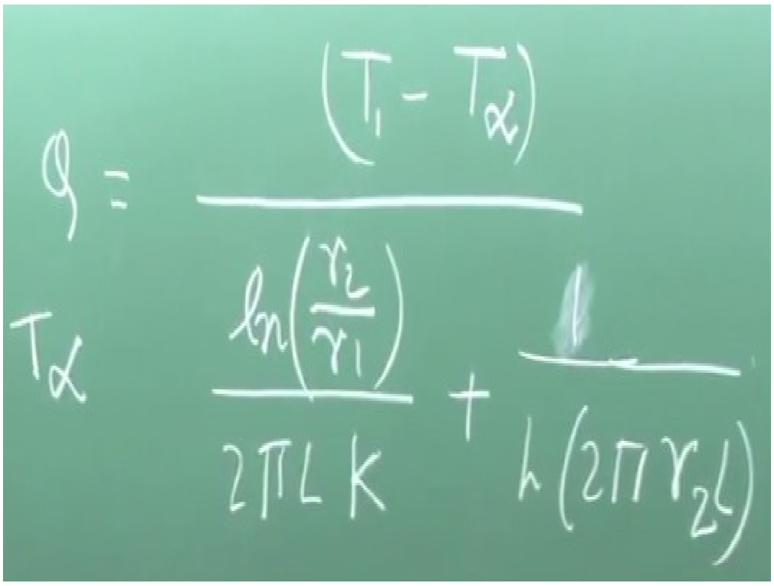

Now if you write the heat transfer equation Q, as you know that now this becomes a series problem as you have already done, from T1 it goes via conduction through the path, r2 minus r1. Then from here by convection to the surrounding T infinity with a heat transfer coefficient h. Now I can write the rate of heat transfer connecting the extreme potential that T1 by which the problem is prescribed.

And the ambient temperature T1 minus T, and in the denominator, there will be two resistance in series. They are conduction resistance, first one, that is ln r2 by r1 divided by twice pi LK plus another resistance, which is the convective resistance from the outer surface to the air, convective resistance when we will discuss convection you will know in detail what this convective resistance is meant for.

When you will understand the mechanism of convection. But so far, we know the convection heat transfer is determined as h to area into the temperature difference, so therefore this is one by this is the convective resistance. That means one by h into r, that is now here this r is the outer radius, that means the radius of the insulation, twice pi r2 into l. So, this is simply the expression.

Now you see that what is the, now if you consider a problem that r1 is fixed, the pipe radius is fixed. We have an insulating material of giving thermal conductivity, length of the pipe is fixed. The problem is, again I am telling prescribed with the given T1 that is the temperature of the outer surface of the cylinder and the ambient temperature is given. So, therefore the problem becomes at how I will vary r2 the radius of the insulation, to minimize Qr to have as less Q as possible.

That means in other way we like to know what is the influence of r2 on Q. Now if you write the mathematical expression, now student keen on mathematics only, they can, he can immediately tell that if we increase r2, if we increase r2, then this increases, this part, that is conduction resistance, R conduction. Obvious physically, because you are increasing the path of heat flow, path of flow of heat conduction.

So, more you give the conduction resistance thickness of the path, increase the length of the path, then your conduction resistance is increased. But what happens to the convection resistance, convection resistance is decreased, convection resistance is decreased. That means there is, there are two countering parameters. That means with the increasing R one way we are increasing the conduction resistance, which will reduce the heat transfer.

But on the other way it increases the, sorry decreases the convection resistance to increase the heat transfer. That is to physically realize, in which way as we go on increasing this R this surface area is going to increase. It does not happened on a plane wall, if one is told that can you decrease by putting he will, it is a monotonically decreasing function, Q will always decrease, or di distance is always increase.

If you go on adding the insulating material, because surface area is independent of the length or direction along which heat flows. But here if we increase the length of the heat flow path, surface area also increases, so conductive transfer from the surface, outer surface increases. So, physically also we see there are two countering parameters which are observed mathematics, which means Q is not a monotonic function of r2, decreasing or increasing.

That means Q has either a maximum or a minimum. So, check it all is school level thing, whatever I am teaching, so Q is having either a maximum or a minimum.

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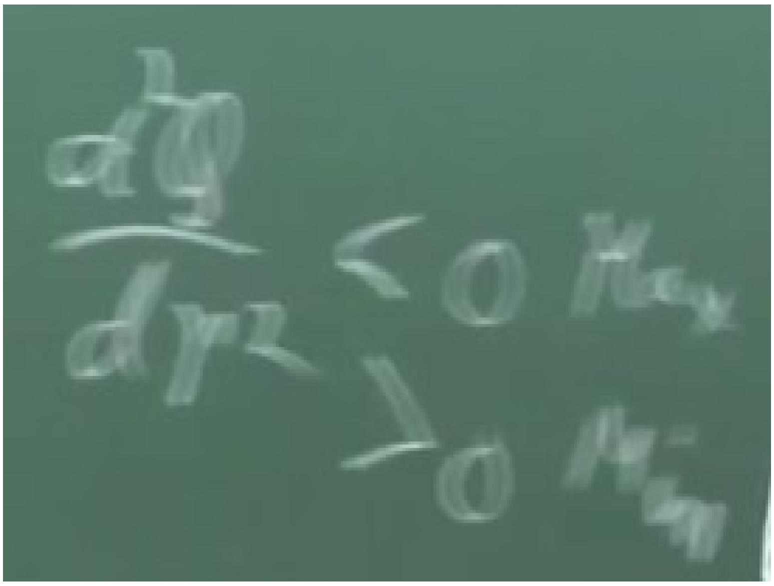

Now how to find out, we find out dQ/di and set it to zero. So, therefore we find out the value of r2, for a given r1 KL, where dQ/dr is zero. So, we set dQ/dr zero for extreme condition, then we will check for minimum or maximum of the function depending upon dQ, d square Q dr square less than zero it is maximum and greater than zero it has a minimum.

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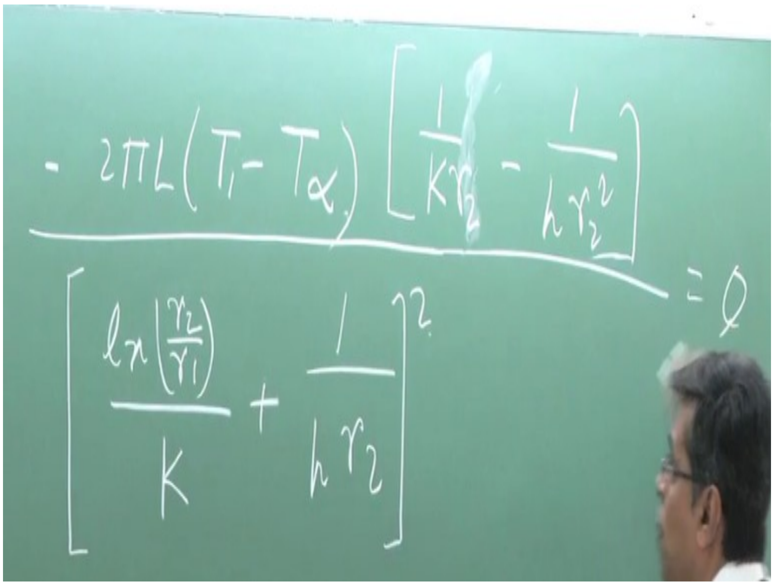

So, this is our routine task at school level. So, if you now differentiate this with respect to r2, let us take this, this is constant that is differentiate with respect to this as a whole then differentiation of one by X minus X square. That means, minus let us consider the twice pi L is constant, twice pi L goes here. Please check it if I do any mistake you tell me. And then ln r2 by r1 divided by K plus 1 by hr2 whole square into here itself.

I will write like this into now, we have to differentiate this expression in terms of r2. That will be first term is 1 by K, now differentiation of ln r2 by r1 with respect to r2 is 1 by r2, Kr2 minus 1 by hr2 square. Now this is set to zero. The mathematics that certain stage becomes only logic not much of operations, when all operations are done then logic remains what is the logic. Logic is that this cannot be zero, T1 cannot be T infinity.

The problem is not defined, okay. This cannot be infinity, problem is not forced, r2 cannot be infinity h cannot be zero.

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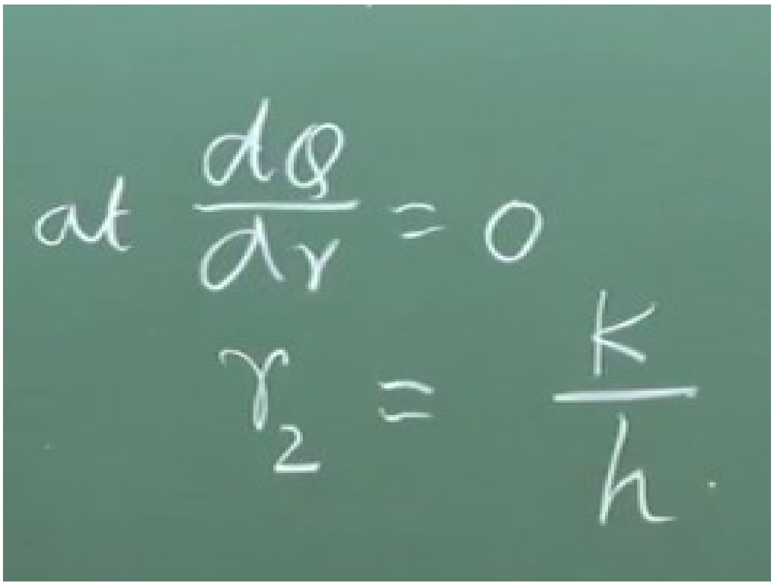

So, for the problem to be defined the only alternative is that this quantity in the brackets will be zero, which tell you that at dQ/ dr zero r2, at dQ/dr zero r2 is equal to K by h. That means K by h value is the value where Q is an extremum, Q attains an extremum value, and if you makes a second derivative and you take that r2 is greater than r1, you can prove that is left to you, that d square Q dr2 square is always negative.

That is a task for you that d square Q dr2 square for this function is always negative. If r2 is greater than r1, so therefore mathematically it can be told, that this function has a maxima at a value of r2 is K by h, okay.

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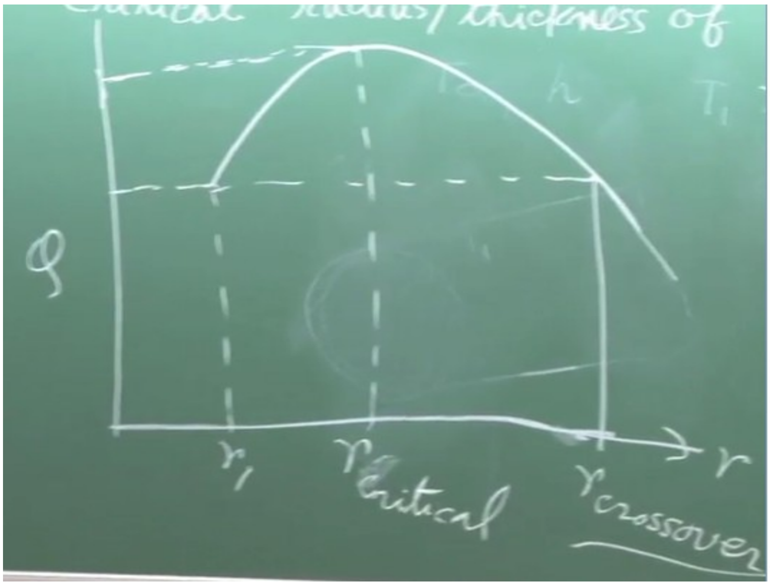

Then immediately we can make a draw a figure like this at r2 is K by h, it has a maximum. That means if I now draw a figure, that heat transfer rate Q verses r2, sorry, it has a, then it decreases. This is r1, this is this value K by h and this is known as r critical, or sometimes rc is equal to K by h. So, this is r critical, this is r critical. This axis is r2 rather r you can tell, r critical.

That means at critical radius this is the, now this is the heat, this is the heat loss from the bare pipe, this is the heat loss from the when the insulation radius is r critical. That means if I go on increasing the radius, adding insulating material to increases radius up to r critical, we are rather not serving the purpose of insulation, we are serving the opposite purpose. We are increasing the heat loss.

That means we have to ensure that r critical thick, our thickness, actual thickness of insulation in practice must be lower than the critical thickness or critical radius rather thickness what is used in the heat transfer. So, thickness is r2 minus r1, it is colloquially used but people tell thickness, but use the nomenclature radius. So, therefore I will tell the radius. So, it should be higher than the critical radius of insulation.

So, it is very interesting and very simple, that there must be a radius where the heat transfer is same as that of the bare pipe, because that is the maximum then it decreases and come. And probably it is not given in any undergraduate heat transfer book, that radius is known as crossover radius, r crossover radius. That means the radius after at which the heat transfer, for any insulating material of any given K, depends upon this value K, K and h, becomes same as that of bare pipe.

So, therefore actual insulation radius should be higher than this crossover radius, clear, hello you, clear. But now the question is that, this also came in our mind when we were students, why teacher is not so much bothered about crossover radius, why the book does not tell anything about that, only gives the expression for critical thickness of insulation K by h, K by h then stops there.

But it is not K by h which is very important to me, because I know that K by h it is maximum. Then after K by h it falls, I am interested to that value where again it comes to the heat transfer from the bare pipe, and I must know that what is that value, so that my insulation radius should be more than that, because if I know only this value and I am satisfied at my work and tell sir, my insulation resistance is this much, a radius is this much.

Okay, this is higher than r critical, but still I get a value higher than the heat loss from the bare pipe. So, therefore I do not gain, it is understood or not. This is answered, this you will not get in any book, I tell you, be attentive in the class, this answer is like this. This is a conceptual information for engineers. Engineers do not go so point to point mathematical information, what is that information for engineers.

Now they calculate two things that what may be the maximum value of r critical, in practice.

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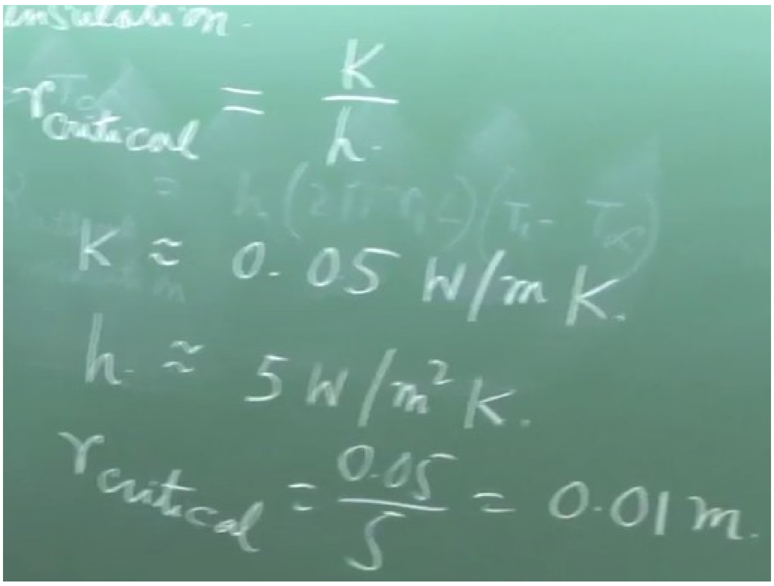

So, in practice the insulating material usually has a range of thermal conductivity like this. It just a scaling type of thing, approximate value, watt per meter K. And the lowest value of h which can be achieved, at a quotient air, having only one significant flow having natural convection is in the order of 5 watt per meter square K. So, if I use this minimum value, because if h is lower, critical radius of insulation is higher.

Then I get a value of r critical, 0.05 by 5. It comes about 0.01 meter. So, why we are only interested with r critical, not crossover, crossover radius is an academic information. We solve problems, but in practice it has got not much value. Now therefore, we see that if the critical radius of insulation is more than 1 centimetre, then it solves the purpose. So, we only know the maximum value.

By scaling analysis, that what can be the maximum value. Afterwards we will see when we will do the scaling analysis, we have already done it in fluid mechanics class, how do you scale the things, you can understand the order of magnitude analysis. A parameter can attain maximum these values. So, therefore if I ensure that my critical radius of insulation much more than 1 centimetre.

I am happy that it is definitely gone more than the crossover radius, clear. So, therefore it is the critical radius of insulation, which is more important. And it is directly dependent on K obviously, if K is very high, it will be conducting. So, insulating material K should be low, so if K is lower we require less radius. Similarly, if h is lower you require higher radius. And if h is higher you require, h is high you require lower radius, okay, clear, okay.