Conduction and Convection Heat Transfer
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Lecture 15

Now next similar to the plane wall, we can have a convective boundary conduction, means that this if I draw this conview that will be in arctic or plane.

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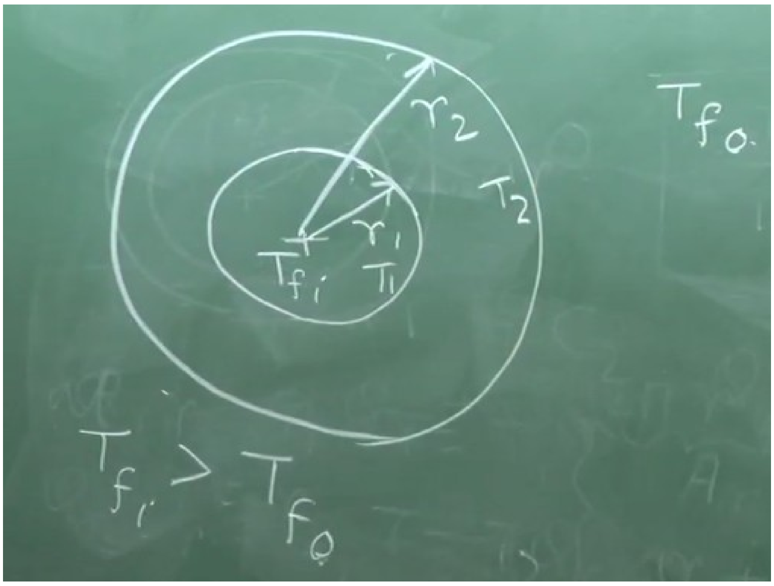

Instead of prescribing the inner surface than outer surface temperature, we prescribe the fluid temperature, inner fluid temperature which may be Tfi and outer fluid temperature which may be Tfo. That means the heat comes from the inner fluid through this, let us consider Tfi fluid temperature inner is greater than Tfo. Then first by convection heat will come to the inner surface and it will have a temperature T1, which is not prescribed.

And heat will flow by the conduction to the outer surface, which will attain some temperature T2, and from T2 by convection it will go to outside fluid Tfo. For example, heat is lost from a hot fluid flowing through the pipe, flowing through the pipe. So, in that case T1, if T1 and T2 are not prescribed only Tfi Tfo, then what we can do we know that conduction heat transfer between T1 and T2 are the inner and outer surface temperature, which may not be prescribed but I take as temperature denoted as T1, T2.

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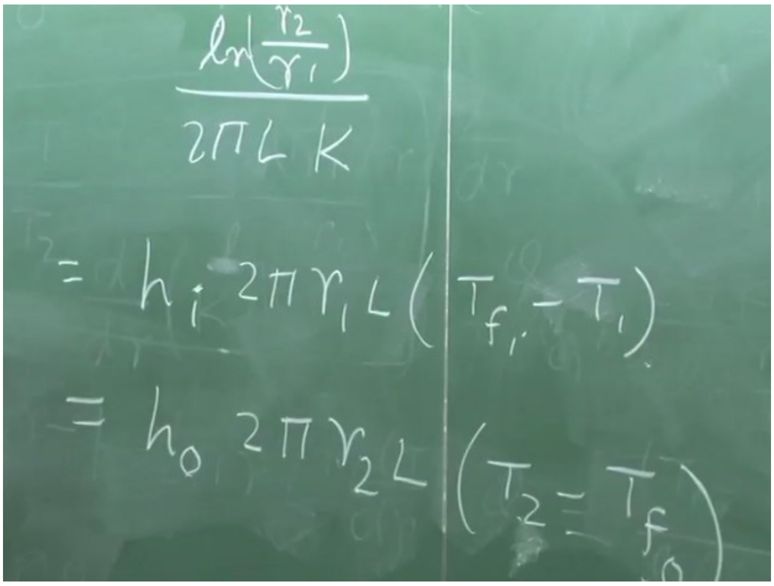

I already know that this is equal to ln r2 by r1 divided by 2 pi LK, and same heat Q is coming through, because of steady state the same thing is flowing by convection from the inner hot fluid to the surface, and if I prescribe HI as the heat transfer coefficient of the inner fluid and HO that of the outer fluid, then by our definition of the heat transfer coefficient in convection heat transfer.

It can be written as heat transfer coefficient into area, twice pi r1 L, inner surface area into Tfi minus T1 and that is same as that those out from the outer surface to the outer fluid environment, which is taken to the cold here less, therefore this will be twice pi r2 L into T2 minus Tfo. So, in the similar way as we did earlier, we can express the T1, T2, Q times this. Tf minus Ti is Q time this, Q divided by this, and T2 minus Tfo is Q divided by this, and this will sum it up.

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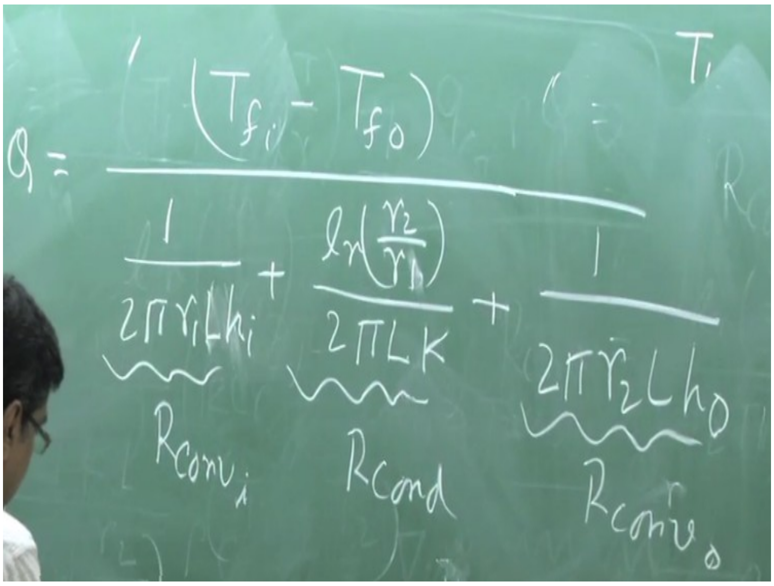

Then we get Q is equal to in terms of the extreme temperature difference, we here prescibe that is Tfi minus Tfo divided by 1 by 2 pi r1 h1 plus ln r2 by r3 divided by twice pi LK, r1 l, sorry twice pi, I am sorry, twice pi r1 L hi plus 1 by twice pi r2 L ho, extremely simple. So, it is the r1 and it is r2, ln r2 by r1, I have written r3, sorry, very good. Now this is same as the flat plane thing.

That means it is the sum of these three-series resistance. This is convection resistance, heat convection, inner surface i, this is R conduction, and this is R convection outer surface.

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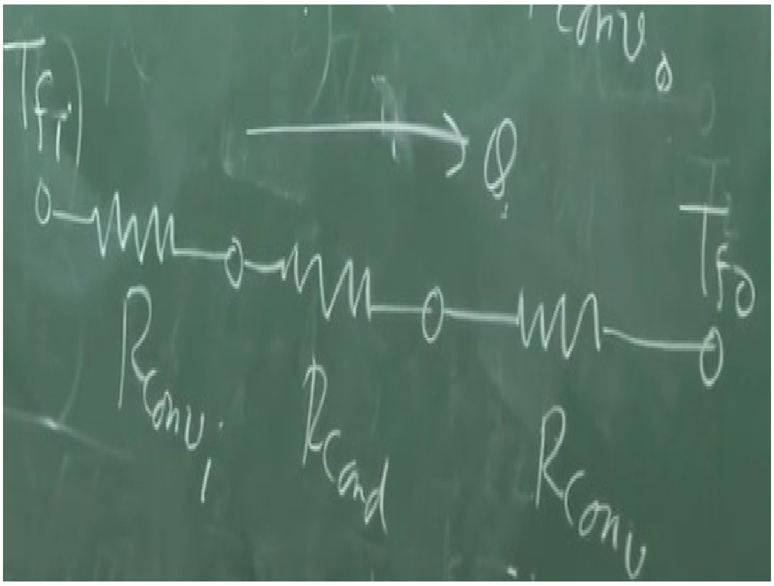

That means extremely simple, series resistance are in parallel, through which the same heat is flowing, and the electrical analogous circuit is like this, Tfi and Tfo. The same heat is flowing Q, with this is R convection i, whose value is 1 by 2 pi r1 L h1, and this is conduction and this is R convection, same thing. We can also think of composite cylinder or composite cylindrical wall, which may be composed of different thermal conductivity.

Then, we will have the different conduction resistance in series, different conduction resistance in series. So, it will be nothing great if you solve problems, then you will see the application of this. That means instead of one cylindrical wall, we may have a composite cylindrical wall. That means we have another cylindrical layer of material, of different thermal conductivity.

Then we will be adding another conduction resistance like this, that will be from r2 to r3. That means if we have another, just like this I give you a picture like this and it will be very simple to deduce.

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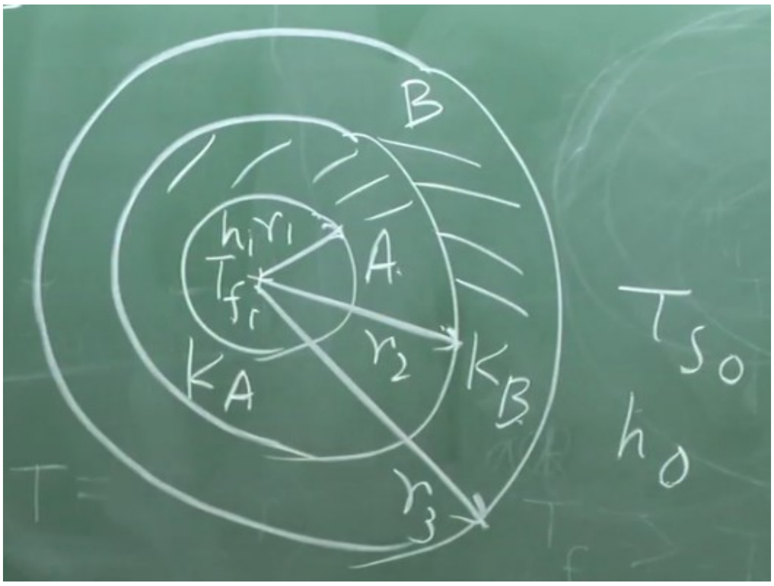

It will be very simple to deduce, like this if we have two cylindrical wall, the radius is r1, r2 and r3. Then r1 to r2, this material, this is A, thermal conductivity KA, and this r2 to r3, this material is B with thermal conductivity KB and if we have similar Tfi and Tfo, with ho and hi, heat transfer coefficient, then your circuit will be the same with another added resistance, which will be very simple to conceive and we can draw the network like this.

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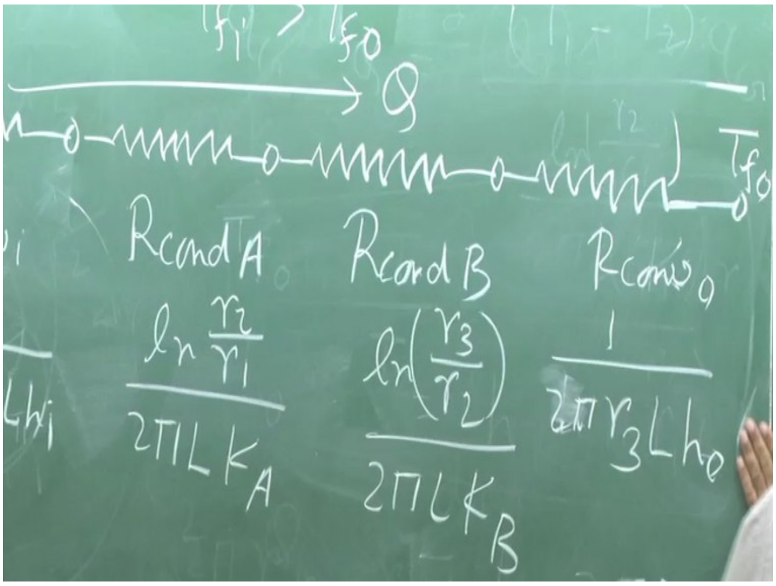

One convective resistance, then two conduction resistance and another one convective resistance, so that the two terminals have the extreme potentials Tfi, Q flows like this. Tfi is greater than Tfo and we get this convection. What is that? 1 by twice pi r1Lhi then this is r conduction for material A, that is Ln r2 by r1, outer to inner radius divided by twice pi LKA. Similarly, this is R conduction B, which is ln r3 by r2 outer to inner radius of this cylindrical wall divided twice pi LKB.

And this is finally R convection o, that is outer 1 by twice pi, this I write first r3, the area, L into ho, so this is so simple. Therefore, this is the thing, that means composite cylindrical wall with convective boundary condition are tackled like that. How do we get this distribution? We solve the temperature distribution by developing an expression from the energy balance that is by heat balance, energy balance in the conducting medium by taking a small elemental volume.

Or you can start with the general heat conduction equation as I have told, then you solve it for the special case. If there is heat generation, then heat generation format have to be taken is dependent with the spatial coordinate have to be taken if thermal conductivity depends on temperature, then the dependent has to be taken, and the problem becomes mathematical that means whatever is involved is mathematics.

So, there is no other heat transfer concept, but one very important case, which sometimes many books forget to tell that for a variable area, plane area problem, if you start from the general energy, general heat conduction equation, you will be lost. You have to develop that equation by taking the, that I told in the last class, energy balance. Because the concept of variable area is not there.

Because it is integrated over a cross section to get a new temperature effect or the temperature being uniform, but area is varying that part will not be manifested. If you step forward take from the heat conduction equation, this is a very general mistake the student does, I tell you. A teacher always tells his experience from students end. The students always jump to the general energy equation.

He finds that okay, we make steady state, del T/del t 0, we make Q 0, we make everything 0, then you get that d square T/dx square 0, T has to be linear in x. Unfortunately, in a one-dimensional heat conduction is an approximation of four, for a varying area T is never linear. It is A into dT/dx, it is constant, so dT/dx is inversely proportional to A, that constant has to be cleared, but in cylindrical coordinate system.

Because of the coordinate system itself that area, it is inherent to the coordinate system that the area normal to heat flow, for example in the r direction heat is varying with that, it is directly proportional with r, so therefore from the general heat conduction equation, it is x a special, it will be the same as we derived by taking a simple element one dimensional Qr and Qr plus del L like that. Both the things are same.

I think for you people these two things have to be kept in mind, otherwise you will be in problem. That even a variable area, you will draw a linear temperature profile, then come to the teacher and tell why in a steady state one dimensional constant thermal conductivity, temperature is linear, why I have not got the marks, so this is very important. So, today I will stop here.

Well, this is little before the time, but the next thing is the critical thickness of insulation. That I will take in the next class. Now, I tell you, just wait. I give you a clue now before starting the next topic that when this heat flux is given by this, one thing you see that if you increase the outer radius, then two things happen in contrasting nature. We increase the outer radius, the conduction resistance increases, but the convection resistance decreases.

This is because the area, surface area of heat transfer increases. You understand, which is not the case for a plane area. If there is a heat transfer from a wall, if you go on adding material to increase the wall thickness in the direction of heat flow, you are sure that you are putting more conduction resistance and the heat flux will be arrested. That means, it will be reduced, obviously, because the dT/dx is getting reduced, but in a cylindrical geometry to increase the area.

If you increase the radius by more material, that means you are increasing the radial pump for conduction and conduction resistance increases, but at the same time mathematically the convection resistance decreases means, the surface area, from which the convection heat transfer takes place increases and convection is directly proportional to surface area. Do you understand me?

So, therefore the two counteracting heat results in a very interesting problem as critical thickness of insulation, that means if you instill it a cylindrical pipe by giving insulating material, adding insulating material, the thermal conductivity is relatively much lower than common conducting material, does not always mean that we are going to reduce the heat loss. Ironically, you will see that you are increasing the heat loss.

Because they are two contradictory things. Conduction resistance increases, but the convection resistance decreases