Introduction to Fluid Mechanics
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Lecture – 09
Non – Newtonian Fluids

We continue with our discussions on viscosity that we had in the previous class. So, we were discussing about the non dimensional number Reynolds number.

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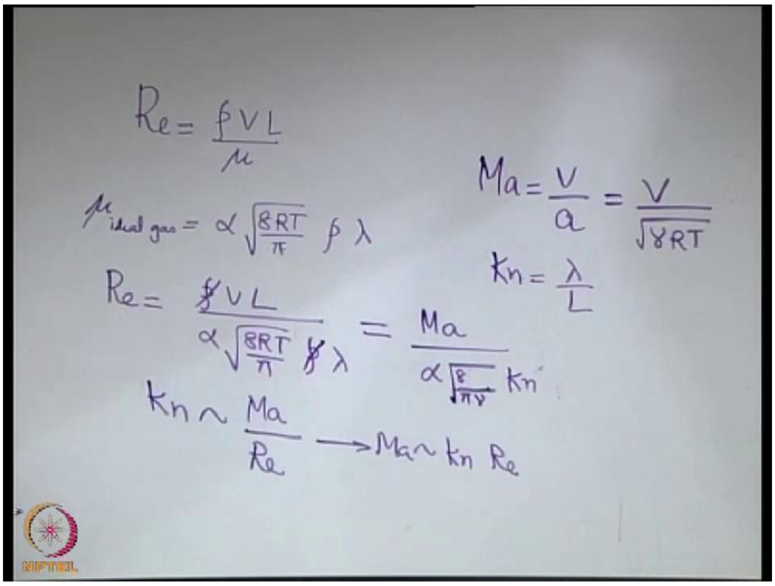

$\mathrm{Re} = \dfrac{\rho V L}{\mu}$ is the expression that we got while sort of trying to relate the inertia force with the viscous force. And this is what the expression that we got while sort of trying to relate the inertia force with the viscous force. In whatever problem we are trying to see whether there is a relative dominance of inertia force over viscous force or not this number is useful. Even in other contexts when the inertia force as such is not there that is the fluid is not accelerating, but it has some energy which could have been utilized to accelerate it to an extent even in those contexts that may be compared with the viscous effects, which are present in the flow through this non dimensional number and this is a very important number we will see later on that this will sort of dictate that what is the nature of the flow is it laminar is it turbulent and so on these terminologies we will understand and appreciate later.

There are many other non dimensional numbers till now we have seen maybe 3 non dimensional numbers Knudsen number Mach number and now Reynolds number. Now let us try to see that can we develop a kind of interrelationship between these 3, of course it is not possible to do it for the most general case, but perhaps for the most simple case that is like the ideal gas.

Let us see that whether we can develop a kind of relationship between these. For that we recall that what was the viscosity for the ideal gas that we derived $\mu_{ideal\_gas} = \alpha\sqrt{\frac{8RT}{\pi}}\rho\lambda$.

This alpha is typically fractional number like 1 by 6, so for these type of scaling estimation it is exact value is not so important, we are just trying to see sort of the nature of the functional relationship between these, $\mathrm{Re} = \frac{\rho VL}{\alpha\sqrt{\frac{8RT}{\pi}}\rho\lambda}$. So, we can cancel out the pho from the numerator and the denominator, then you can see that there is a group $\frac{L}{\lambda}$ which is $\frac{1}{Knudsen\_number}$ because, l is the characteristic system length scale and lambda is the molecular mean free path.

So, if you recall the definition of the mach number it is $Ma = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}}$ where, gamma is the ratio of the specific heat Cp and Cv for gases. Of course, this expression is not valid for all types of gases only for ideal gases.

Exactly whether it is square root of gamma or that may not be so important, but at least the form is it is scaling with $\frac{V}{\sqrt{T}}$. So, here if you try to substitute that form so in place of $\frac{V}{\sqrt{RT}}$ we write the mach number then it should be adjusted with the coefficient α. So, $\mathrm{Re} = \frac{Ma}{\alpha\sqrt{\frac{8}{\pi\gamma}}K_n}$ because $K_n = \frac{\lambda}{L}$. So, we can see that Knudsen number for an ideal gas will roughly scale with Mach number.

So, Mach number is roughly scaling with Knudsen number into Reynolds number. So, if the Reynolds number is high; that means, the fluid is having a high inertia, on the top of that if the Knudsen number is high then there is a high probability that it is having a great compressibility effect, because it is trying to enhance the mach number also and we have seen already that mach number is a sort of indicator of how compressible the fluid is.

So, this is regarding gases, of course again this is not regarding all gases this is just a special case of ideal gases. If we go to real gases the situation may be more complicated, but at least what we can appreciate is that the viscosity of gases should try to increase with increase in temperature. The relationship is not as simple or straightforward as this one for real gases because, for the difference between the real gas and the ideal gas in this context is straightforward. For real gases you also have intermolecular forces of interaction and that needs to be considered for estimating the viscosity, it is not just transfer of molecular momentum.

So, a combination of transfer of molecular momentum and the intermolecular forces of attraction for any real substance will determine that what should be the viscous nature of the fluid. If you come to liquids as we have discussed earlier that for liquids, the viscous behavior is predominantly due to intermolecular forces of interaction at the same time we need to appreciate that whenever, we are qualifying something as viscosity for a liquid that so-called viscosity even might not exist for a liquid. Because that is defined for a liquid only if it is a Newtonian fluid, but there are many liquids which are not Newtonian fluids those are called as non-Newtonian fluids.

The obvious connection with the name is that they are not obeying the Newton's law of viscosity and whenever you are having a fluid which is not following the Newton's law of viscosity, then its constitutive behavior that is how the shear stress is related to the rate of deformation it may be very complicated it may be a involved non-linear function. We will not go into the details of how these non-linear functions are derived or how these forms look like.

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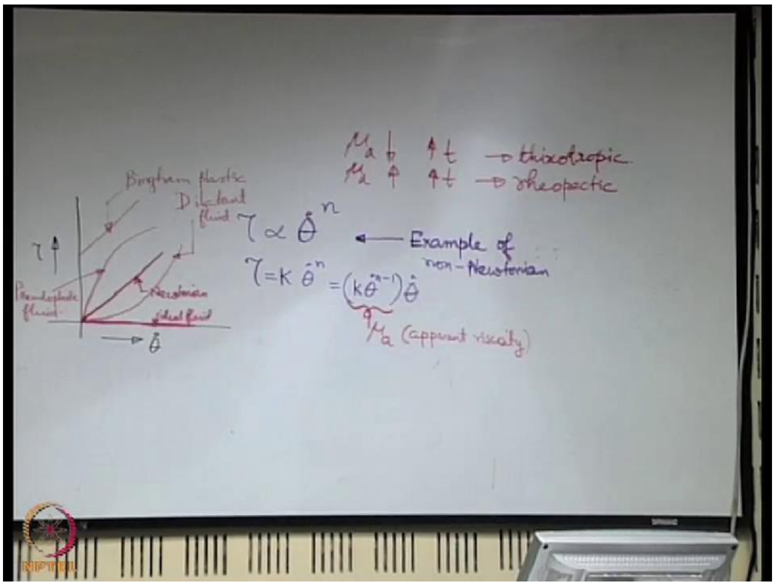

One of the examples is like you may have the shear stress is related to a power of the rate of deformation that is $\tau \alpha \dot{\theta}^n$.

So, this is an example of a non Newtonian behavior and this type of specific example this is known as a power law fluid. That means, the shear stress is related to a power or index of the rate of deformation, this index may be anything I will give you an example if you think of blood typically this index is close to 0.7. So, this index is something which dictates the so called viscous nature of this particular type of fluid again which obeys the power law not all Newtonian fluids, non Newtonian fluids will obey the power law neither blood is exactly power law fluid blood is not exactly a power law fluid.

It has it is constitutive behavior much more complicated than that, but if you just simplify as an approximation and try to express the stress and rate of deformation behavior in that way then that index will roughly be 0.7 for blood, again it is not a constant it is dependent on the composition of the blood and many other things.

So, this type of behavior if it is existing so you can write say $\tau = K\dot{\theta}^n$, you can also express this as $(K\dot{\theta}^{n-1})\dot{\theta}$. The whole idea is a desperate effort to cast it in the form of Newton's law of viscosity, so if that is cast in this form then if you know some local rate of deformation from that this could be apparently like a viscosity.

Of course, this is not really the viscosity because it is not a Newtonian fluid. So, these sometimes is called as apparent viscosity μ_a. Apparent viscosity is not really the viscosity in the proper definition sense, but of course it has the same dimension of viscosity and it has a sort of similar physical sense as that of viscosity. The whole idea is that if you have such a definition, it sometimes helps you to relate the behavior of that complex fluid with that of equivalent Newtonian fluid that is the whole idea and that is why the name apparent viscosity is not never a true viscosity.

Now, if you try to make a sketch of how the shear stress relates with the rate of deformation for different types of fluids. Let us take some examples let me first draw 2 trivial examples which should be understandable to you very easily. Let us say we have this as one example and this as another example. So, what special cases or special types of fluids these 2 represent?

Ideal fluid is having 0 viscosity, but in general fluids may have behavior which are different from this. So, let us try to draw some examples here so one case may be something like this, one may be something like this even you may have something like this. So, that is why if you take an example like this really puts a serious question into the definition of the fluid because, it shows that there is something which is of fluid type but it requires a threshold shear to deform and there could be many such types of substances. Think about a case that you have a toothpaste in a tube right you need to press it to apply some shear before it starts moving out and with negligible shear it will not flow, it will require a threshold amount of press before it starts moving.

So, this is also a fluid because it flows and it is many of it is characteristics are explained nicely with the equations of motion of fluids, but it is not that classical definition that it will start deforming with infinitesimal shear. So, this type of example is a very interesting example. But there are other examples which are more classical as fluids, so for example, if you consider this particular case this is known as a dilatant fluid and this example is known as a pseudo plastic fluid. This by the way this name is known as Bingham plastic fluid, these are names which have originated from the detailed studies of rheologies of substances and it is not so important that you have to remember these names, but these are just to give you ideas that there are different categories of so called fluids based on the shear stress and rate of deformation behavior.

So, some of examples of these types of fluids let us say dilatant fluid. So, like water or printing ink these types of fluids these are known as dilatants fluids. So, what are the characteristics of these if you see that as we increase the rate of deformation that is the shear increases with the rate of deformation and if you see that for a pseudo plastic fluid then also if you increase the rate of deformation the shear increases right. But it comes to a sort of state like this so the name pseudo plastic comes from the fact that you may relate this type of behavior with the plastic deformation of a solid so to say.

So, it is as if like there is some substance which is undergoing a plastic deformation as a solid goes. So, if you have, say a solid piece and you have heated it with a hammer, good old forging process, so then the material will be soft and that may be easily deformable. So, a solid type of material will start flowing and in mechanics of solids it is sometimes known as plastic flow.

So, it is not that a fluid is flowing, but a solid is moving as if it is a fluid. So, the critical demarcation between the fluid and the solid is often not there so to say and there are many examples of these types of fluids like pseudo plastic fluids as say polymer suspensions, so if you have suspensions of polymers. So, if say there is a system in which there are aggregates or there are chains.

Now, it is possible that there are 2 things which are possible one is if you are applying a shear then the aggregates or the chains of particulates they may be broken. So, you are having a system in which you have some liquid type of substrate in which you may have some particulate inclusions and they have formed aggregates so to say.

Now, you are applying a strong shear, if you are applying a strong shear those aggregates may be broken and it may give rise to the availability of new flow passages. So, it may so sort of help the fluidic motion. So, viscosity is sort of opposite to fluidity that means if it allows it to flow more easily we intuitively understand it is less viscous. On the other hand sometimes it may be possible that because of formation of local aggregates and so on, the movement of the fluid ensures that fluid elements are entrapped within the local aggregates and then they are in great entanglement and they cannot come out of those entrapment and flow. So, different substances are therefore different, in some cases it happens that because of this type of entanglement the flow cannot take place so easily.

In certain cases these aggregates or chains may be broken and it may help the fluid flow to take place in a much easier way. Obviously, we will not go into the details of the non Newtonian fluids because, that is not within the scope of what we are going to study as a part of this elementary course. But at least we understand that there may be different types of the shear stress versus rate of deformation behavior depending on the constitutive nature of the material of the fluid that we are looking for.

These expressions like the expression for the apparent viscosity will hide another important thing, sometimes this may be functions of time that is you may have apparent viscosity increasing with time or you may have apparent viscosity decreasing with time and whether they are going to increase or decrease with time that is going to be strongly dependent on again that how the rheological distribution or how the rheological property of the material is going to influence that.

So, there are cases when the apparent viscosity will decrease with increase in time. So, you may have the apparent viscosity decreases with increase in time that type of fluid is known as thixotropic fluid and if the apparent viscosity increases with increase in time that is known as a Rheopectic fluid. At the end it is a physical field that how the fluid will look like and how the fluid behaves in presence of a strain. So, let us look into maybe 1 or 2 examples with movies to understand that what type of flow behavior we are going to expect for different fluids. So, let us see maybe one first very qualitative example.

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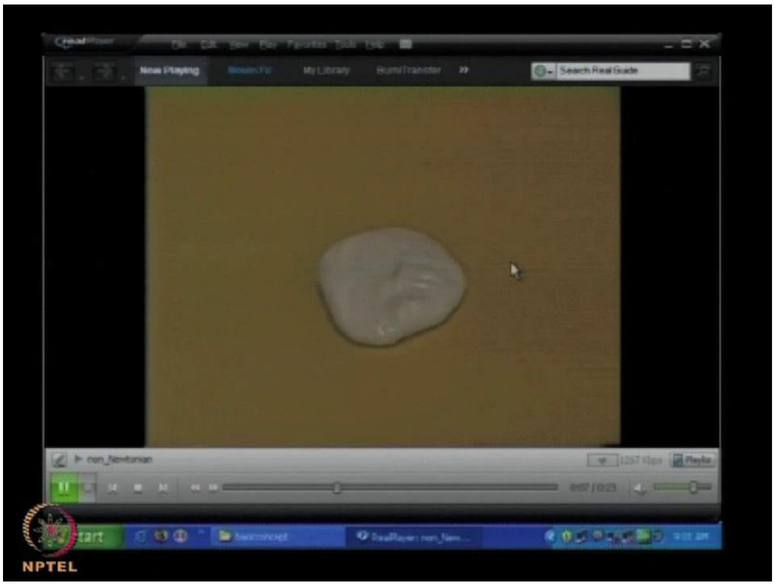

So, we will see some examples which will tell us that like I mean these are not very classical fluids, but these are like it is deforming with a shear and sort of with as if it is a thin film that is being spread on a surface, there are many fluids which are also like thin films which are spread on a surface and there is no reason to disbelieve that this is also a fluid.

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Of course we can clearly understand that this is an example of a non Newtonian fluid it does not obey the Newton's law of viscosity.

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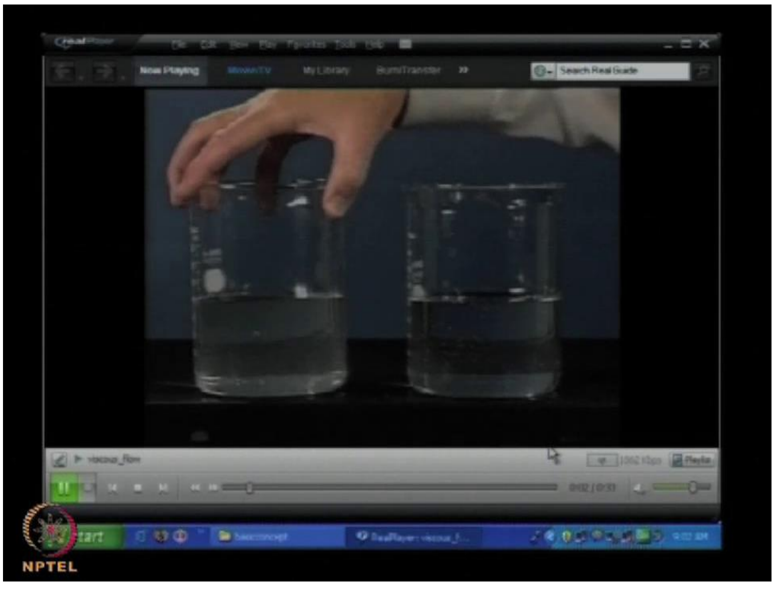

Now, to get a qualitative idea of what is the difference between a highly viscous flow and not so much of a viscous flow? So, let us look into these examples.

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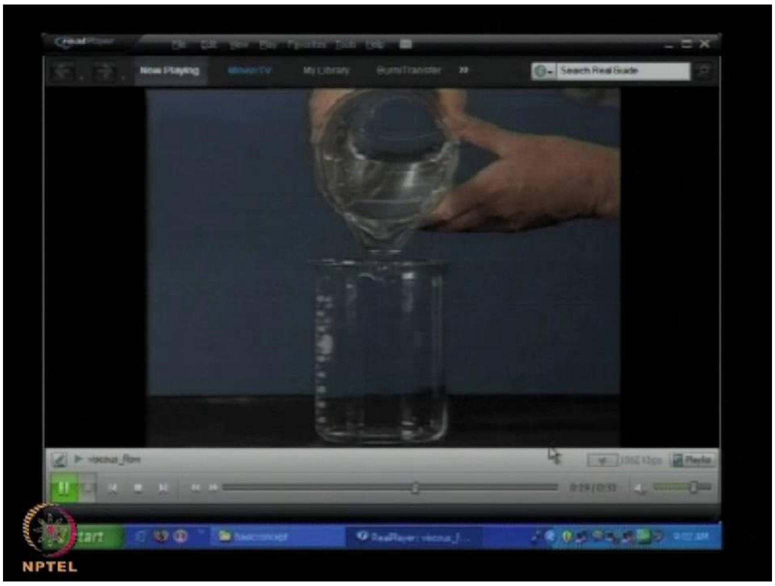

So, in one case it is like water is being put or poured on in one of the beakers and see the other case. So, this is also a fluid that is being poured and you can clearly appreciate that this is something which is of high apparent viscosity. So, this type of qualitative field is somewhat important and we can clearly recognize that the statement that it has something to do with like inverse of fluidity or flow ability so to say. Now, we will look into some bit more scientific way of looking into these effects of viscosity.

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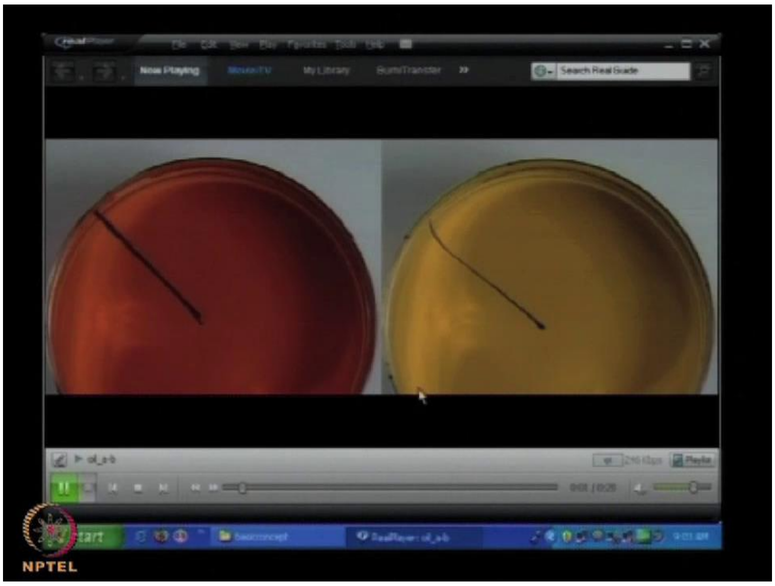

So, let us look into one example or rather 2 examples shown side by side, these are 2 fluids with 2 different viscosities there is a line colored line which is moving with the shear and that is a line which sort of represents the deformation and we just see it again hopefully it works. Now, can you tell from these 2 examples just qualitatively whether the red one has more viscosity or the yellow one has more viscosity?

The red one has more viscosity; how do you understand that?

Say you are still applying the similar type of influence or similar type of motion actuator for these 2. But if you see the red one let us play it again and then understand, see in the red one it responds to the change of momentum almost throughout right and it is sort of like it is. So, particular about responding to the change of momentum that it follows the change of momentum exactly how it is almost like very rigidly.

On the other hand in the second case it is not doing like that and there is a region where it actually shows a kind of lead or lag depending on whether you are describing the fluid or describing the bounding solid which is making the fluid move. So, it may be either a lead or a lag it depends on how fast or how slow these movements are taking place.

The first case there is no such lead or lag and it is like the entire fluid is feeling the effect of the disturbance and getting adjusted to that. So, with a high viscosity only that is possible and when you have high viscosity or low viscosity we have the characterizations to the Reynolds number and let us see a particular case say a low Reynolds number case.

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So, we are just trying to now give a bit of quantification to what we have observed till now, say it is a very low Reynolds number see what happens.

This type of arrangement we will work out maybe one example to illustrate that quantity sometimes this type of arrangement is used to measure viscosity of fluids, this is called as rotating type viscometer just like any meter is for measuring things. So, this type of device may be utilized for measuring the viscosity of a fluid kept between 2 concentric cylinders in the annular space.

Of course, this example is not for illustrating that measurement, but if you see now the direction of rotation has got reversed and because of a low Reynolds number what it ensures is that viscous effects are very strong. So, it adjusts to the change, so nicely again if you see that the marker Reynolds number less than one has come back to it is original shape. So, it is a perfect adjustment to the change let us look into another example.

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Student: what we had inertia force come into play like viscous force is greater called Reynolds number is less.

See inertia force as I told you that it is not exactly always the inertia force as such, if it is not accelerating it is not inertia force but it is having some kinetic energy which if could have been utilized to accelerate it, it could have been it could have given rise to some inertia force. So, when we are having low Reynolds number keeping that effect unaltered the lower the Reynolds number viscous effects are stronger and stronger. So, when we are comparing 2 cases the other effect which is present in the numerator is almost something which we are not disturbing, but what we are trying to see is that what is the relative change in the viscous effect in the 2 cases.

So, if you see this example now it is being reversed and it does not come back to the same situation if we play it again we can see, that it does not come back to the same state with which it started. So, we are having a particular deformation the fluid is trying its best to adjust to the deformation by propagating the momentum disturbance, that propagation of momentum disturbance is something which is not as efficient as in highly viscous fluid. So, when it comes back there is always a lag in that propagation of the disturbance and imposition of the disturbance and therefore it is not possible that it entirely reverses back it state. So, this is a qualitative feel of 2 different cases in one case you have a highly viscous flow and, in another case, not such a highly viscous flow.

Always whenever we are talking about forces we are comparing, so when we say that viscous effects are important we have to see that important in the context are in comparison to what. So, as we have mentioned in our initial discussion then when we talk about the smallness of a dimension, we always ask a question it is small with respect to what? Like if you have a 1 millimeter dimension to me it may be small to somebody it may be very large because it is very large in comparison to the atomic length scale, but maybe to me I am more happy in thinking about kilometers and then I will say that is 1 millimeter is very small in comparison to 1 kilometer.

So, whenever we are talking about smallness or largeness, we are really making a comparative assessment and therefore it is not just like a viscous force that is there, maybe there is some other force also which is competing with it. And always whenever there is a system in a sort of dynamic equilibrium type, then there are competing forces like otherwise it will purpose the effect will perpetually grow.

So, there are competitions and the understanding of mechanics is just to understand how these competitions are working in a system. So, there are always competitive forces and we will see that how these competitive forces are important. So, this we will not go too far and looking into these examples and now let us move on to the some of the typical examples in terms of problems, not just qualitative examples but where we try to illustrate the concept that we have learned through example problems.