Introduction to Fluid Mechanics

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Lecture – 60
Principle of Similarity and Dynamical Analysis-Part-II

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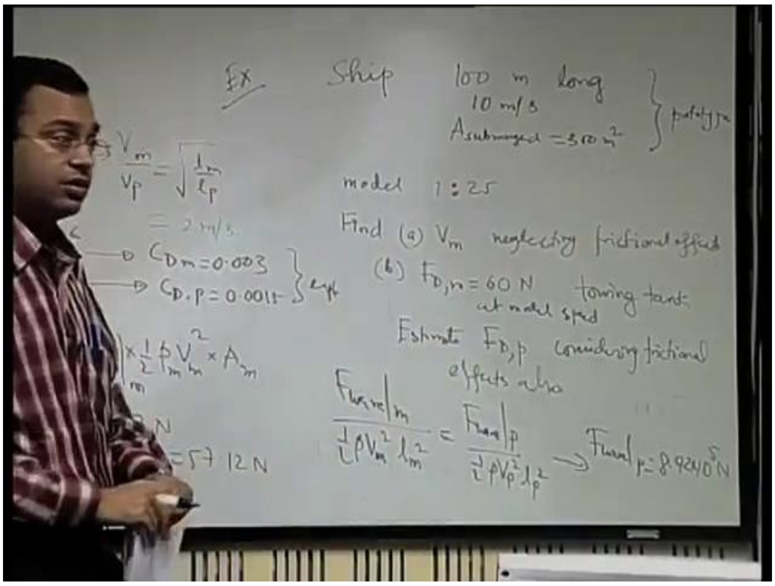

Estimate the total drag force on the prototype considering fictional effects also. There are some other data given for the problem.

One of the important resistance is the frictional resistance. The other important resistance is called as wave making resistance, so because of formation of the water waves and there it is sort of a gravity dependent phenomena.

Considering the wave making resistance the important forces are inertia force and gravity force. So, then the similarity will be determined by the Froude number which is the ratio of the inertia force and gravity force.

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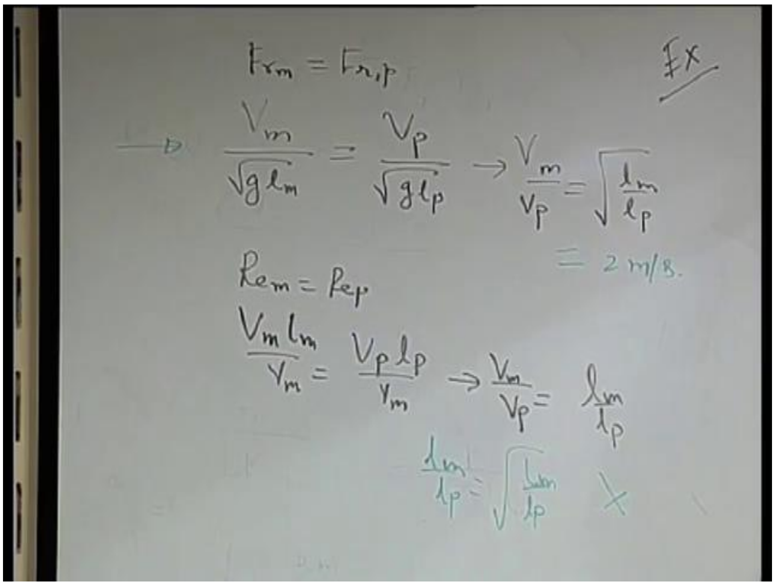

$$Fr_m = Fr_p$$

$$\frac{V_m}{\sqrt{gl_m}} = \frac{V_p}{\sqrt{gl_p}} \rightarrow \frac{V_m}{V_p} = \sqrt{\frac{l_m}{l_p}} = 2m/s$$

$$\mathrm{Re}_m = \mathrm{Re}_p$$

$$\frac{V_m l_m}{\nu_m} = \frac{V_p l_p}{\nu_p} \rightarrow \frac{V_m}{V_p} = \frac{l_m}{l_p}$$

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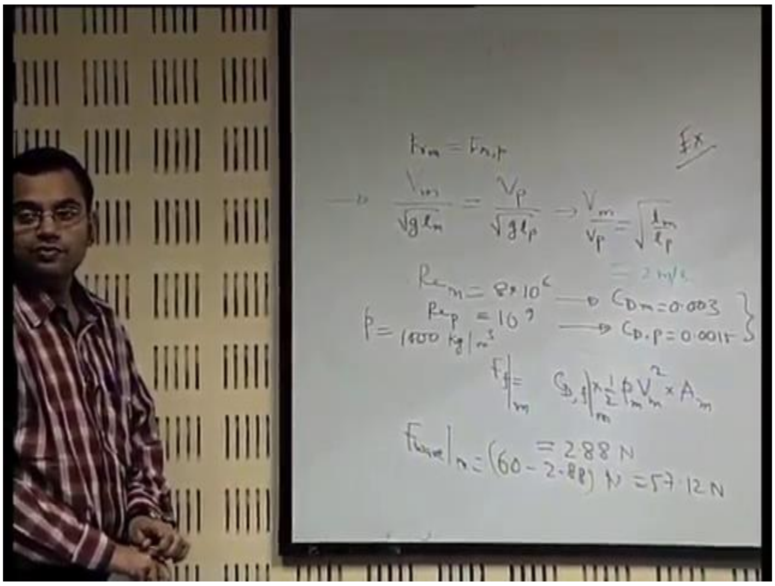

$$\mathrm{Re}_m = 8\times10^6 \rightarrow C_{D,M} = 0.003$$

$$\mathrm{Re}_p = 10^9 \rightarrow C_{D,P} = 0.0015$$

$$\rho = 1000 Kg / m^3$$

$$F_f = C_{D,f} \times \frac{1}{2} \rho V^2 \times A$$

$$F_f\Big|_m = C_{D,f}\Big|_m \times \frac{1}{2} \rho_m V_m^{\ 2} \times A_m = 2.88N$$

$$F_{wave}\big|_m = \left(60 - 2.88\right) N = 57.12N$$

$$\frac{F_{wave}\big|_m}{\frac{1}{2}\rho V_m^{\ 2} l_m^{\ 2}} = \frac{F_{wave}\big|_p}{\frac{1}{2}\rho V_p^{\ 2} l_p^{\ 2}} \rightarrow F_{wave}\big|_p = 8.92 \times 10^5 N$$

$$F_f\Big|_p = C_{D,p} \times \frac{1}{2} \rho V_p^{\ 2} \times A_p = 0.225 \times 10^5 N$$

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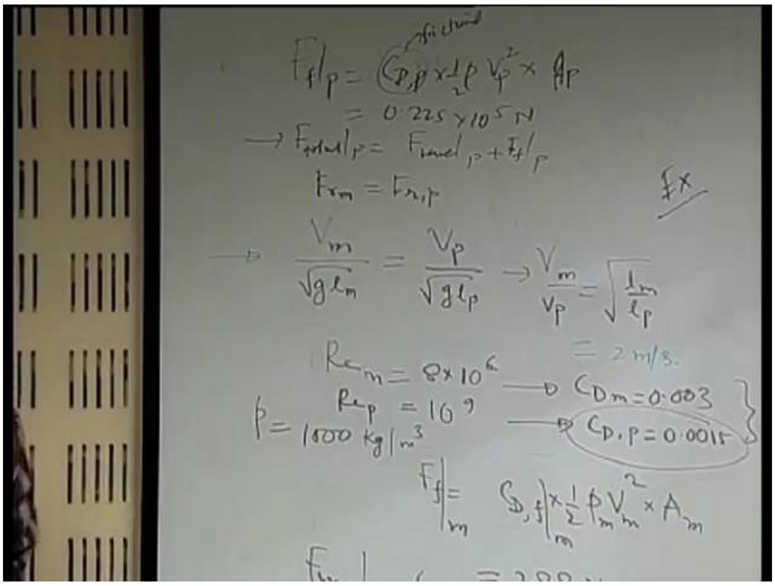

$$F_{total}\big|_p = F_{wave}\big|_p + F_f\Big|_p$$

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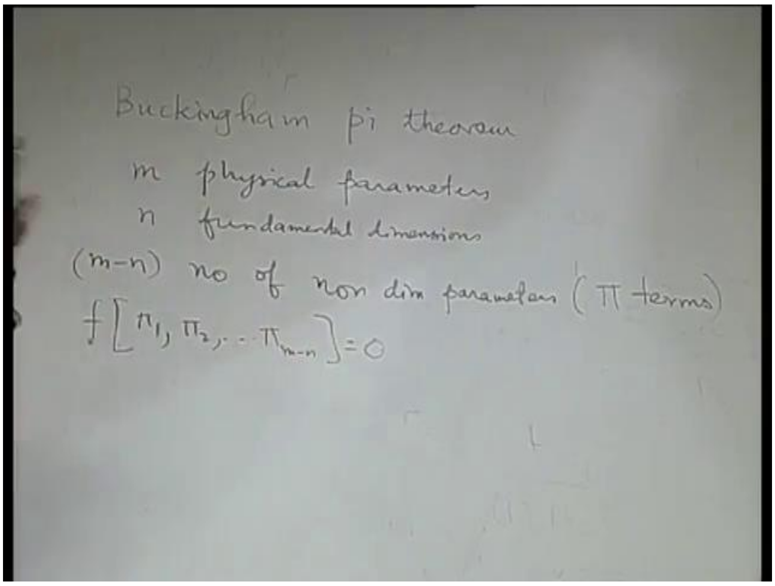

Buckingham pi theorem

m physical parameters

n fundamental dimensions

(m-n) no. of non. Dimensional parameters $\left(\pi \quad terms\right)$

$f\left[\pi_1,\pi_2,...,\pi_{m-n}\right]=0$

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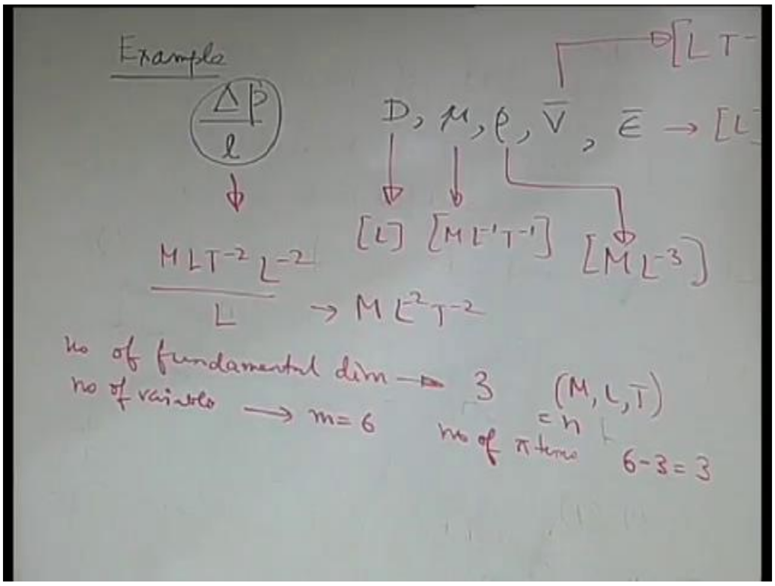

Example

$$\frac{\Delta p}{l}$$

$$D, \mu, \rho, \bar{V}, \bar{\varepsilon}$$

$$D \rightarrow [L]$$
$$\mu \rightarrow \left[ML^{-1}T^{-1}\right]$$
$$\rho \rightarrow \left[ML^{-3}\right]$$
$$\bar{V} \rightarrow \left[LT^{-1}\right]$$
$$\bar{\varepsilon} \rightarrow [L]$$

$$\frac{\Delta P}{L} = \frac{MLT^{-2}L^{-2}}{L} \rightarrow ML^{-2}T^{-2}$$

No. of fundamental dimensions $\rightarrow 3(M, L, T)$

No. of variables $\rightarrow m = 6$

No. of π terms 6-3=3

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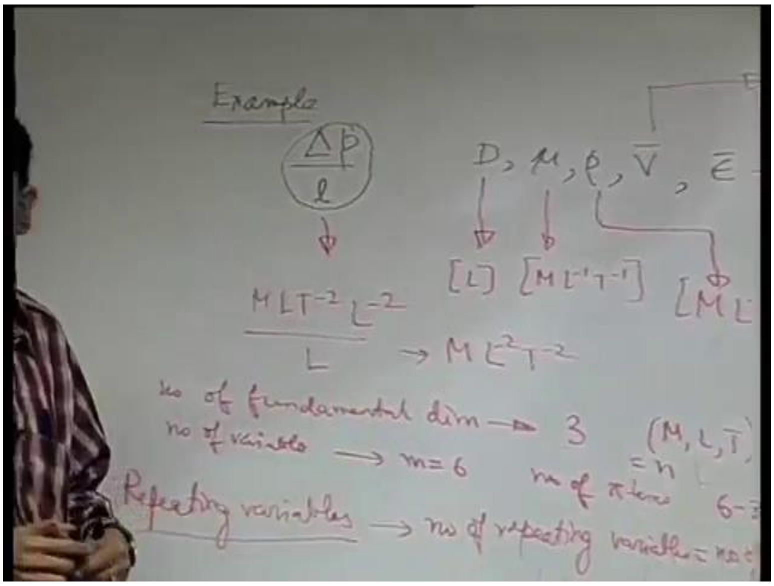

Repeating variables $\rightarrow$ *no.* of repeating variables=no. of fundamental dimensions=3

First of all out of the 3 repeating variable one choose none of those should be the dependent variable that is the first thing, none of those should be dimensionless and none of those should be having same dimension and may be the most important thing is collectively there should content all the dimensions.

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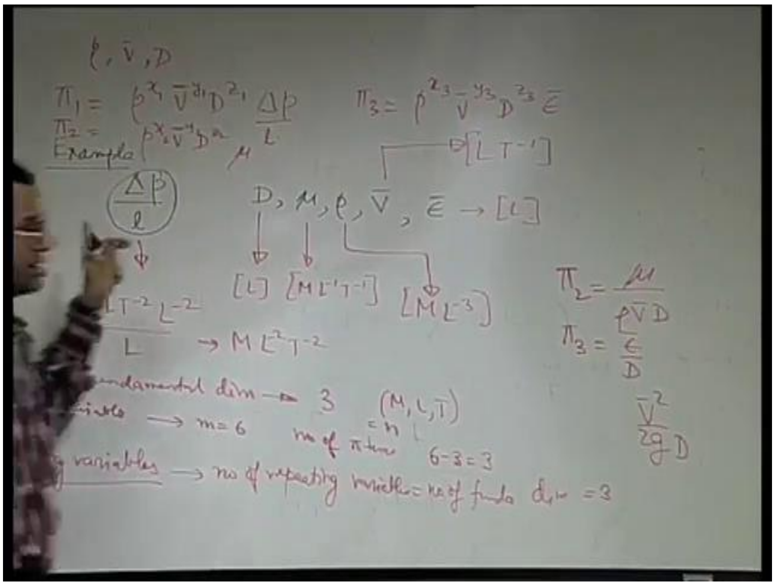

$$\pi_1 = \rho^{x_1} \bar{V}^{y_1} D^{z_1} \frac{\Delta p}{L}$$

$$\pi_2 = \rho^{x_2} \bar{V}^{y_2} D^{z_2} \mu$$

$$\pi_3 = \rho^{x_3} \bar{V}^{y_3} D^{z_3} \bar{\varepsilon}$$

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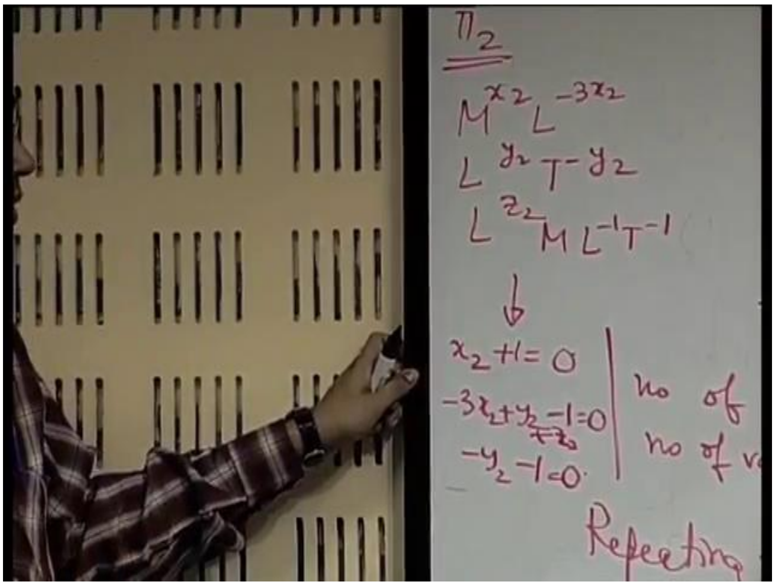

π_2

$M^{x_2} L^{-3x_2}$

$L^{y_2} T^{-y_2}$

$L^{Z_2} M L^{-1} T^{-1}$

$x_2 + 1 = 0$

$-3x_2 + y_2 - 1 = 0$

$-y_2 - 1 = 0$

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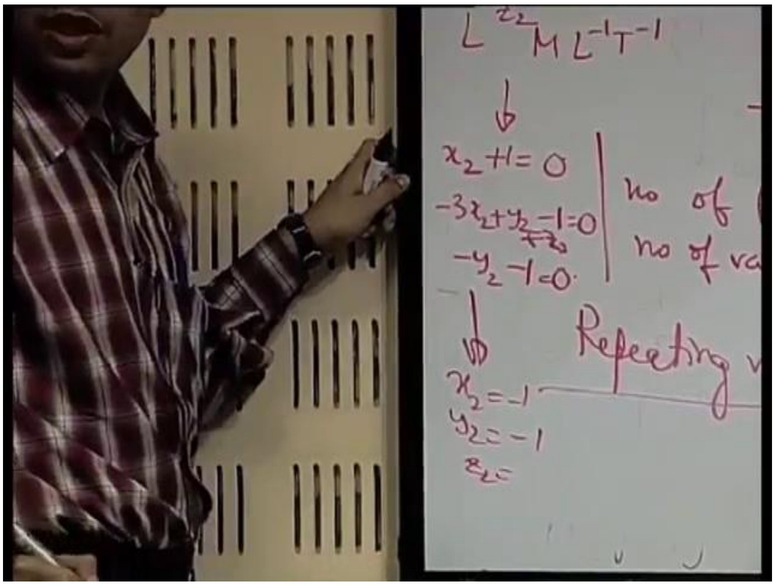

$x_2 = -1, y_2 = -1, z_2 = -1$

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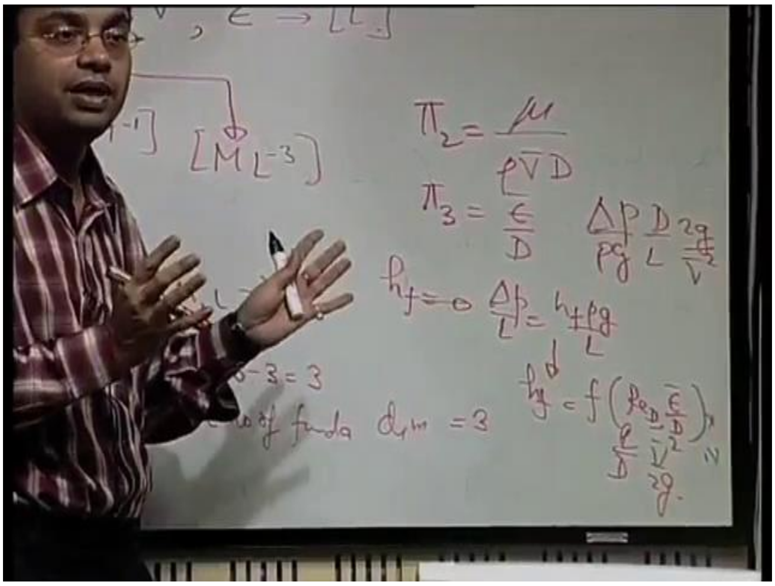

$$\pi_2 = \frac{\mu}{\rho \bar{V} D}$$

$$\pi_3 = \frac{\bar{\varepsilon}}{D}$$

$$\pi_1 = \frac{\bar{V}^2}{2gD}$$

$$h_f \rightarrow \frac{\Delta p}{L} = \frac{h_f \rho g}{L}$$

$$h_f = f\left(\mathrm{Re}_D, \frac{\bar{\varepsilon}}{D}\right) \times \frac{l}{D} \frac{\bar{V}^2}{2g} \rightarrow \frac{\Delta p}{\rho g} \frac{D}{L} \frac{2g}{\bar{V}^2}$$