Introduction to Fluid Mechanics
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Lecture –59
Princie of Similarity and Dynamical Analysis-Part-I

In this lecture, we will discuss on the topic of Principles of Similarity and Dimensional Analysis. Let us say that we are trying to have an idea of the how to design an aircraft. And we know that if you want to design an aircraft, it is important to have a clear idea of the lift and the drag forces as some of the fundamental entities to design it for the real system. At the same time, we know that it is somewhat prohibitive to have many, many experiments on aircrafts of real sizes. So, if you have aircraft of real size, and if you want to test it, it is not only very expensive, but also there are many other drawbacks associated with such experiments.

So, one might be interested to have a reduced model that is a model of aircraft of maybe a reduced size and then test it in a wind tunnel. So, in a wind tunnel say keep the aircraft model and have a control flow of air relative velocity between the aircraft and the air. And then from that if the pressure distribution is measured, other parameters say the velocity distribution can also be measured.

So, the first question that we would like to answer is that given a study on the basis of a model of a different size how or whether you can extrapolate the results of those experiments to predict what is going to happen in reality in the real situation. So, the in the real situation, whatever is the entity that is being used that is considered to be a prototype. And the model is a scaled version of the prototype, in this example the model is smaller than the prototype. It is not always true that models have to be smaller than the prototype.

Sometimes the prototype itself where maybe inconveniently small and you might want to have a model a bit larger than that. Question is whether that scaling will affect the results or not. And if the scaling does not affect your prediction, then the next question comes that how can you utilize the results from the experiments with the model to predict, what is going to be for the prototype that is the first question.

The second question is let us say that you are doing experiments with a model, you may have many parameters which are influencing the results of your analysis or the results of your experiment. Now, how could you reduce the number of parameters to a fewer, but more effective ones may be more effective non-dimensional parameters. Non-dimensional parameters are important, because sometimes you may parameterize the result as a sole function of certain non-dimensional parameters. So, as an example, if you have say flow through a pipe, you may have many experiments with different lengths different diameters or may be different densities of fluid different viscosities of fluid.

But if the result is parameterized in terms of Reynolds number, keeping the combinations of these such that the Reynolds number is unaltered, the physical behaviour is unaltered. That means we in some in such a case may reduce the parameters from say four parameters to an equivalent single non-dimensional parameter. So, the big exercise or the big understanding is how can we make the parameterization of the experiments in terms of the reduced number of parameters using certain non-dimensional parameters. So, these are the important questions that we would like to answer through the study of principles of similarity and dimensional analysis.

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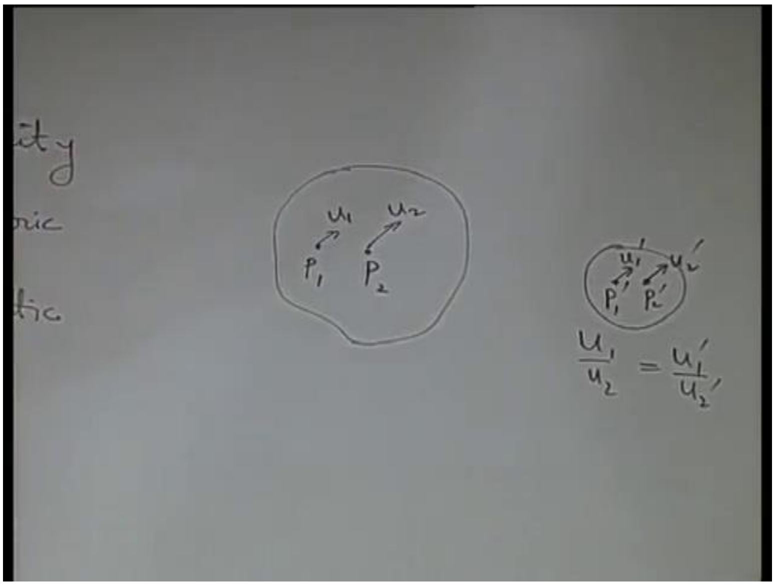

Similarity

(1) Geometric
(2) Kinetic

$$\frac{u_1}{u_2} = \frac{u_1'}{u_2'}$$

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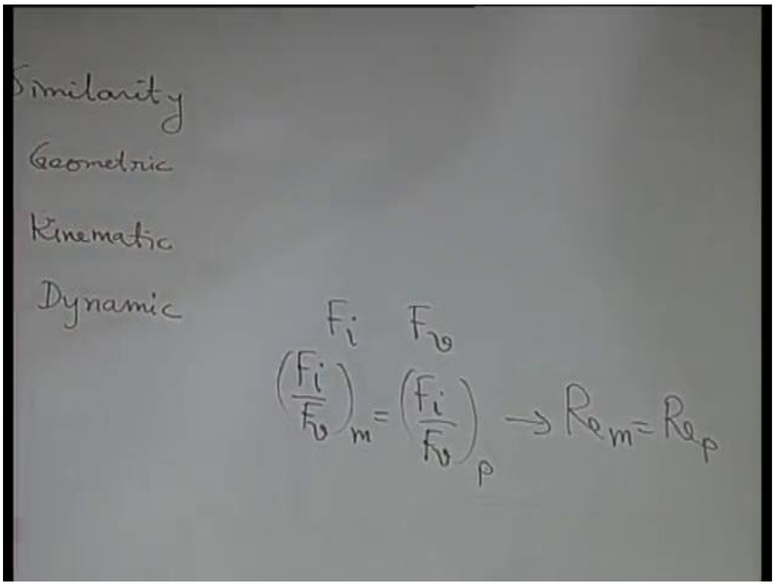

(3) Dynamic

The third important concept regard to the similarity is the dynamic similarity. Dynamic similarity is the similarity in forces that means if there are two types of dominating forces, which have certain ratio in the model, the same ratio should be preserved in the prototype. That means let us say that you have inertia force and viscous force. So, if you have these two as the important forces and competing forces, then you must have the ratio of the inertia force and viscous force in the model same as the ratio of the inertia force by viscous force in the prototype or equivalently in this case that means Reynolds number in the model same as Reynolds number in the prototype.

$$\left(\frac{F_i}{F_v}\right)_m = \left(\frac{F_i}{F_v}\right)_\rho \rightarrow \text{Re}_m = \text{Re}_\rho$$

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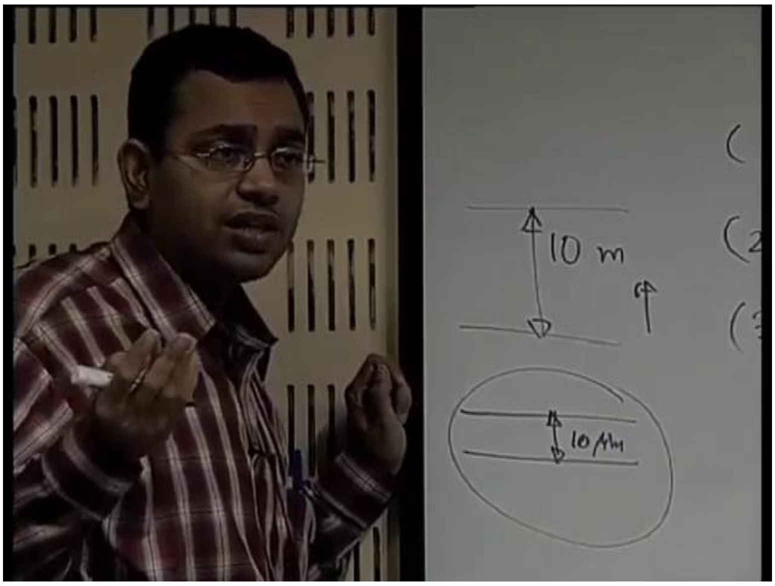

So, instead of going through the ratios of these certain forces or the non-dimensional numbers, you have to first be ascertain whether this non-dimensional number is physically relevant for the physics, which is occurring over that scale or not. So, it is recommended not to change from a scale to another scale for predicting the relative behaviour between the model and the prototype, in such a way that the physics of the problem changes all together and that is one of the very important tasks for an experimental designer that one should not design an experiment, which changes the physics altogether from what happens for the prototype.

Now, when you have these forces the ratio of these forces, let us just look into certain examples where we consider the ratios of different forces. And those will give certain non-dimensional numbers such as Reynolds number.

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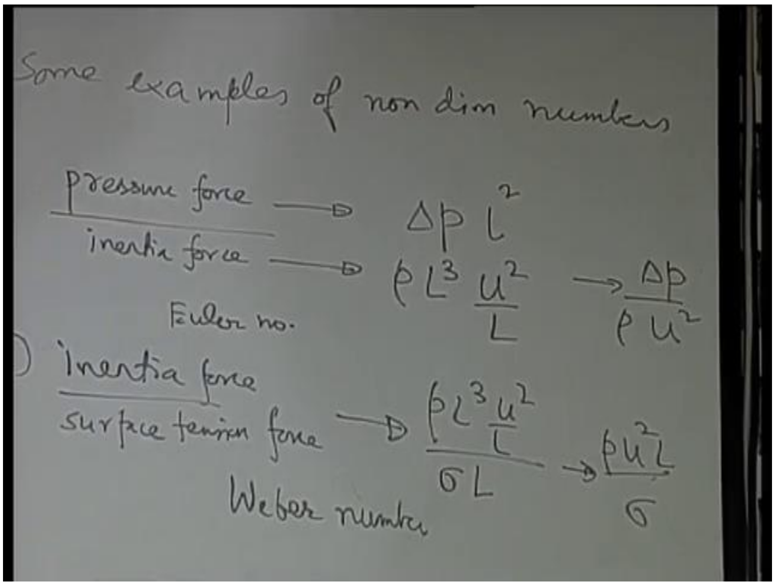

Let us say that we want to find out a non-dimensional number. So, let us say that we consider the ratio of the pressure force by inertia force as an example.

$$\text{Pressure}\ \ force \rightarrow \Delta PL^{2}$$

$$Inertia\ \ force \rightarrow \rho L^{3}\frac{u^{2}}{L}$$

$$Euler\ \ No. \rightarrow \frac{\text{Pressure}\ \ force}{Inertia\ \ force} \rightarrow \frac{\Delta P}{\rho u^{2}}$$

$$Inertia\ \ force \rightarrow \rho L^{3}\frac{u^{2}}{L}$$

$$Surface\ tension \rightarrow \sigma L$$

$$Weber\ \ number \rightarrow \frac{Inertia\ \ force}{Surface\ tension} \rightarrow \frac{\rho uL^{2}}{\sigma}$$

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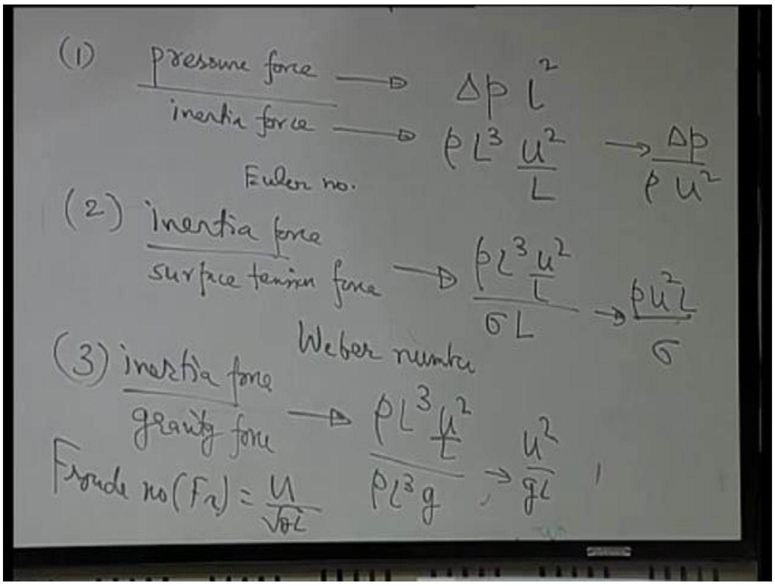

$$Inertia\ \ force \rightarrow \rho L^3 \frac{u^2}{L}$$
$$Gravity\ \ force \rightarrow \rho L^3 g$$

$$\frac{Inertia\ \ force}{Gravity\ \ force} \rightarrow \frac{\rho L^3 \frac{u^2}{L}}{\rho L^3 g} \rightarrow \frac{u^2}{gL}$$

$$Froude\ \ No.\left(Fr\right) = \frac{u}{\sqrt{gL}}$$

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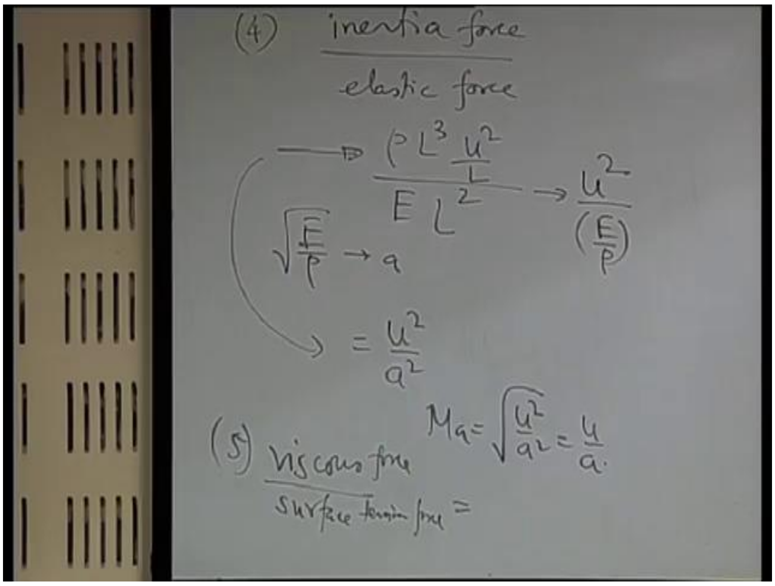

$$\frac{Inertia\ \ force}{Elastic\ \ force} \rightarrow \frac{\rho L^3 \frac{u^2}{L}}{EL^2} \rightarrow \frac{u^2}{\left(\frac{E}{\rho}\right)} = \frac{u^2}{a^2}$$

$$\sqrt{\frac{E}{\rho}} \rightarrow a$$

$$Ma = \sqrt{\frac{u^2}{a^2}} = \frac{u}{a}$$

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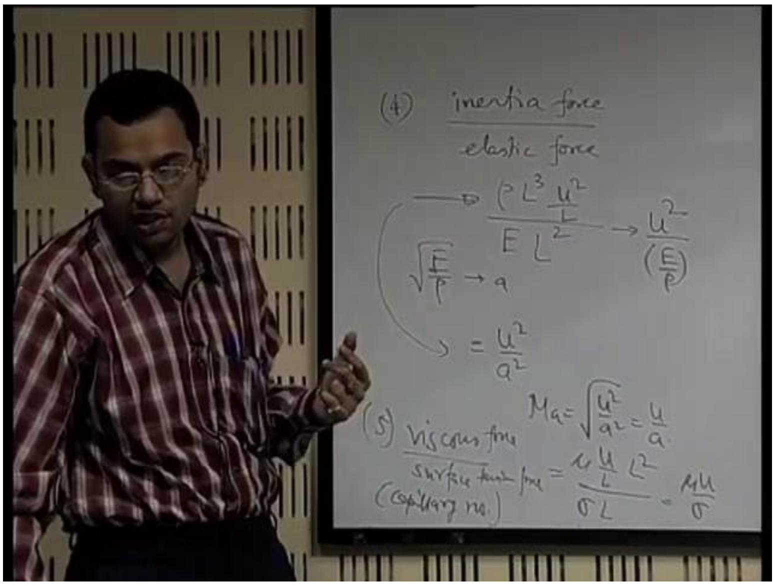

$$Capillary\ \ No. = \frac{Viscous\ \ force}{Surface\ Tension\ \ force} = \frac{\mu \frac{u}{L} L^2}{\sigma L} = \frac{\mu u}{\sigma}$$

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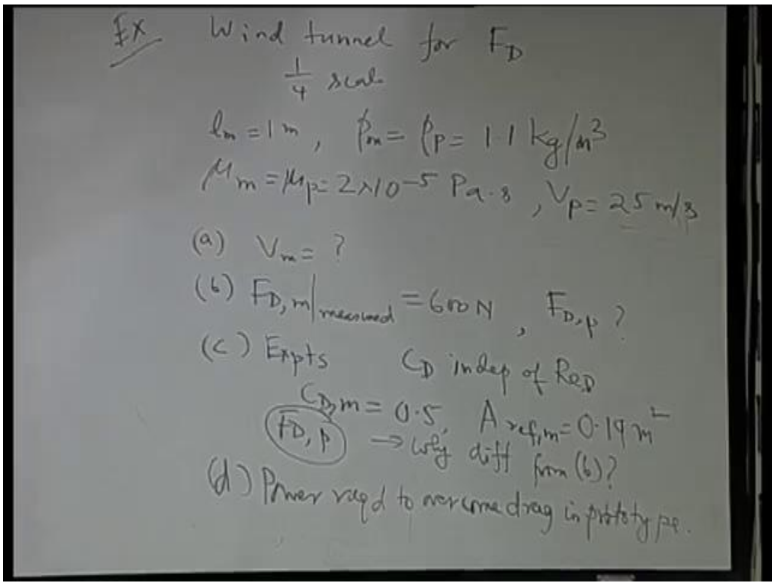

Wind tunnel for $\mathrm{F_D} \rightarrow \frac{1}{4}$ *scale*

$$l_m = 1m, \rho_m = \rho_P = 1.1 Kg / m^3$$
$$\mu_m = \mu_p = 2 \times 10^{-5} Pa.s, v_p = 25 m / s$$

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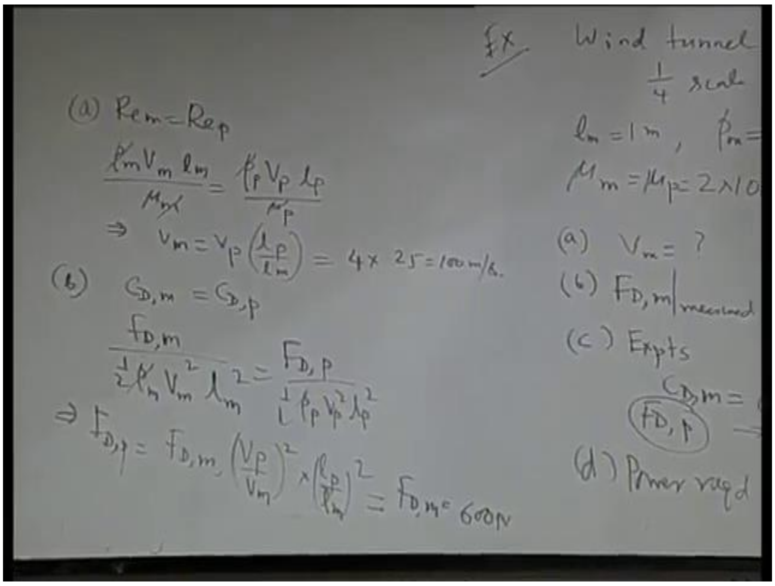

$$(a) \operatorname{Re}_m = \operatorname{Re}_p$$
$$\frac{\rho_m V_m l_m}{\mu_m} = \frac{\rho_p V_p l_p}{\mu_p}$$

$$V_m = V_p\left(\frac{l_p}{l_m}\right) = 4\times 25 = 100 m/s$$

$$(b) C_{D,m} = C_{D,p}$$

$$\frac{F_{D,m}}{\frac{1}{2}\rho_m V_m{}^2 l_m{}^2} = \frac{F_{D,p}}{\frac{1}{2}\rho_p V_p{}^2 l_p{}^2}$$

$$F_{D,p} = F_{D,m}\left(\frac{V_p}{V_m}\right)^2 \times \left(\frac{l_P}{l_m}\right)^2 = F_{D,M} = 600N$$

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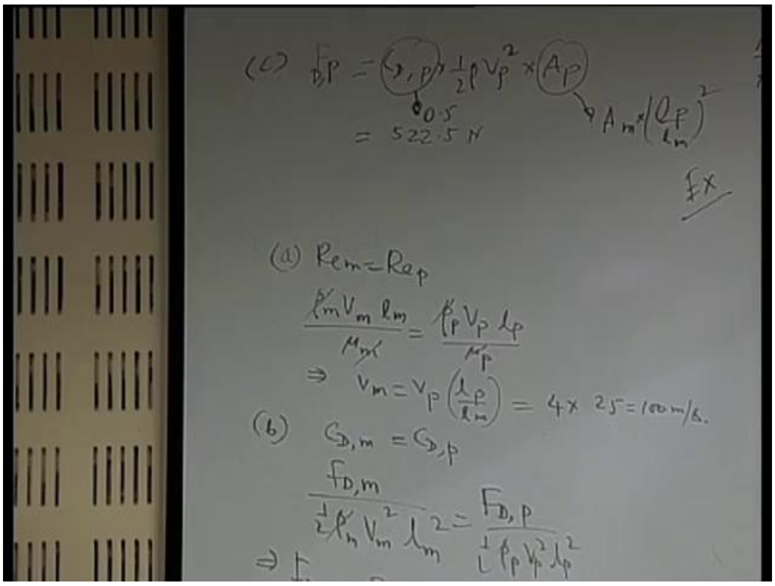

$$F_P = C_{D,p}\times\frac{1}{2}\rho V_p{}^2\times A_p$$

=522.5N

$$\frac{A_p}{A_m} = \frac{l_p{}^2}{l_m{}^2}$$

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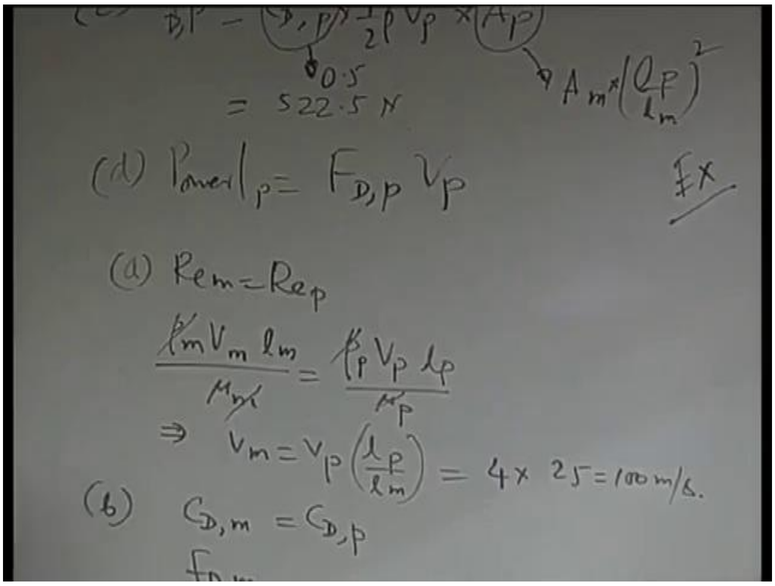

$(d)\, Power = F_{D,p} V_p = 13062.5 W$