Introduction to Fluid Mechanics
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Lecture – 57
Pipe Flow – Part-III

We have till now discussed about the head loss, but how is the head loss related to the energy of the fluid. When you are talking about inviscid flows we were discussing about the Bernoulli's equation and we found out later that the Bernoulli's equation sort of gives the mechanical energy balance for a system for flowing fluid.

Now, therefore, here we are seeing that even we might be tempted in using the Bernoulli's equation, but because of certain losses that may not directly be applicable there might be certain errors. So, these losses must have some relationship with the energy consideration in the pipe flow. So, let us look into a bit more details of the energy considerations in pipe flow.

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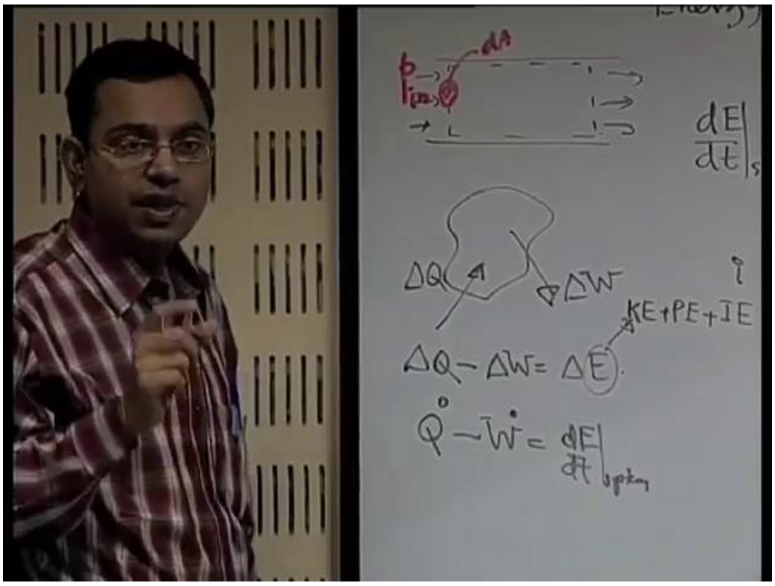

The objective will be to figure out that how the head losses are related to the total energy balance. Let us say that you have a pipe of whatever sections a circular pipe as an example and we are looking for the fluid in the pipe and trying to write an expression for the energy balance. So, for any conservation equation we may start with the Reynolds transport theorem. So, let us start with the Reynolds transport theorem where E is the total energy of the system.

$$\left.\frac{dE}{dt}\right|_{system} = \left.\frac{\partial E}{\partial t}\right|_{CV} + \int_{CS} \rho e\left(\vec{V}_r.\hat{n}\right)dA$$ where E is the total energy and e is the energy per unit mass.

$$\Delta Q - \Delta W = \Delta E$$

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right|_{system}$$

$$\int_{CS} \rho e\left(\vec{V}_r.\hat{n}\right)dA \rightarrow \int_{out} \rho eudA - \int_{in} \rho eudA$$

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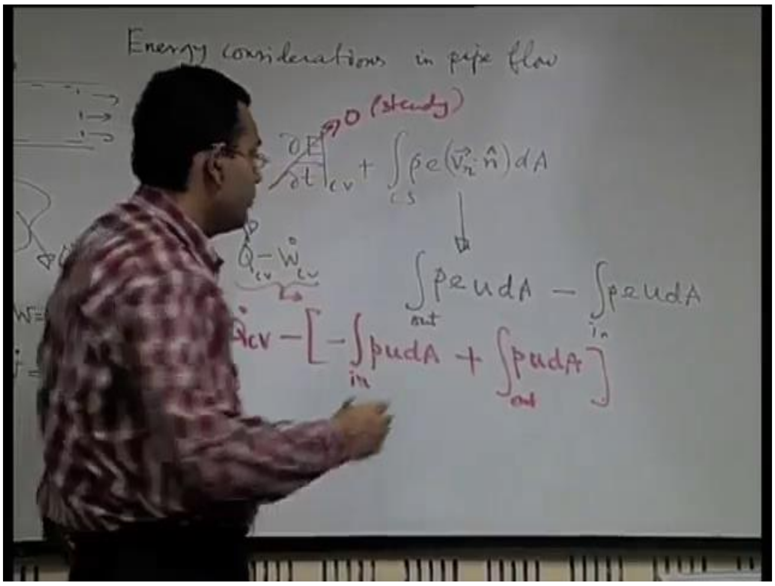

$$\dot{Q}_{CV} - \left[-\int_{in} \rho udA + \int_{out} \rho udA \right]$$

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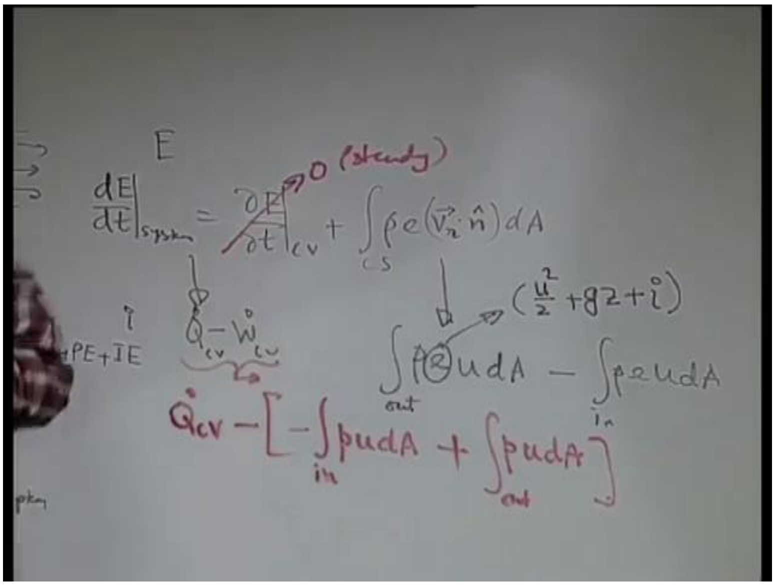

$$e \rightarrow \left(\frac{u^2}{2} + gz + i \right)$$

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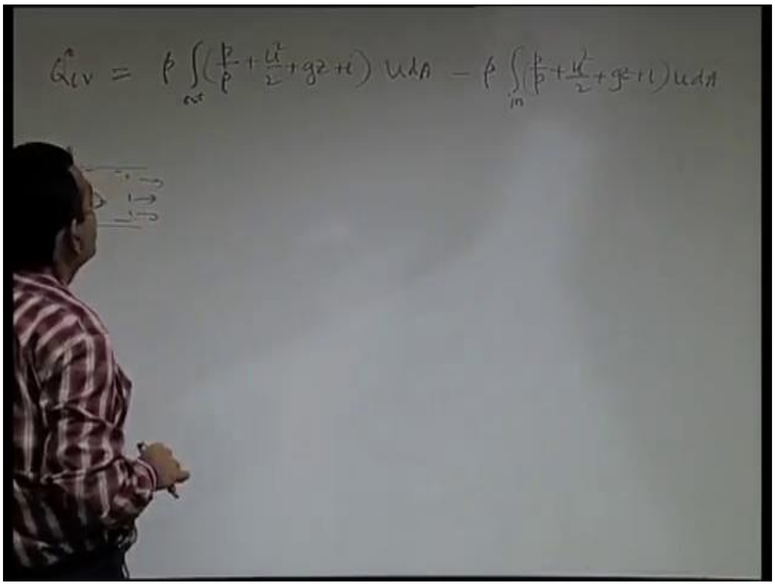

$$\dot{Q}_{CV} = \rho \int_{out} \left(\frac{P}{\rho} + \frac{u^2}{2} + gz + i \right) udA - \rho \int_{in} \left(\frac{P}{\rho} + \frac{u^2}{2} + gz + i \right) udA$$

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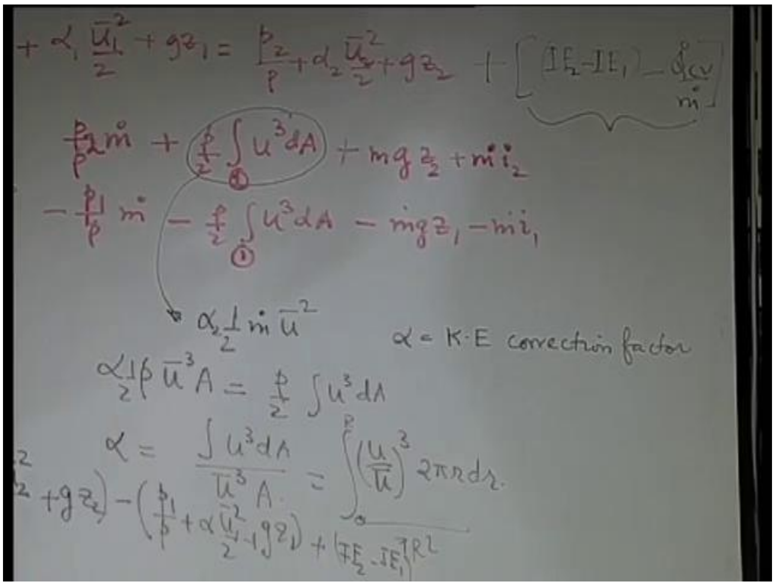

$$\dot{Q}_{CV} = \frac{P_2}{\rho}\dot{m} + \frac{\rho}{2}\int_2 u^3 dA + \dot{m}gz_2 + \dot{m}i_2 - \frac{P_1}{\rho}\dot{m} - \frac{\rho}{2}\int_1 u^3 dA - \dot{m}gz_1 - \dot{m}i_1$$

$$\frac{\rho}{2}\int_1 u^3 dA \rightarrow \alpha\frac{1}{2}\dot{m}.\bar{u}^2$$

$$\alpha = K.E.\ correction\ \ factor$$

$$\alpha\frac{1}{2}\rho\bar{u}^3 A = \frac{\rho}{2}\int u^3 dA$$

$$\alpha = \frac{\int u^3 dA}{\bar{u}^3 A} = \frac{\int_0^R \left(\frac{u}{\bar{u}}\right)^3 2\pi r dr}{\pi R^2}$$

$$\frac{\dot{Q}_{CV}}{\dot{m}} = \left(\frac{P_2}{\rho} + \frac{\alpha\bar{U}_2^2}{2} + gZ_2\right) - \left(\frac{P_1}{\rho} + \frac{\alpha\bar{U}_1^{\ 2}}{2} + gZ_1\right) + \left(IE_2 - IE_1\right)$$

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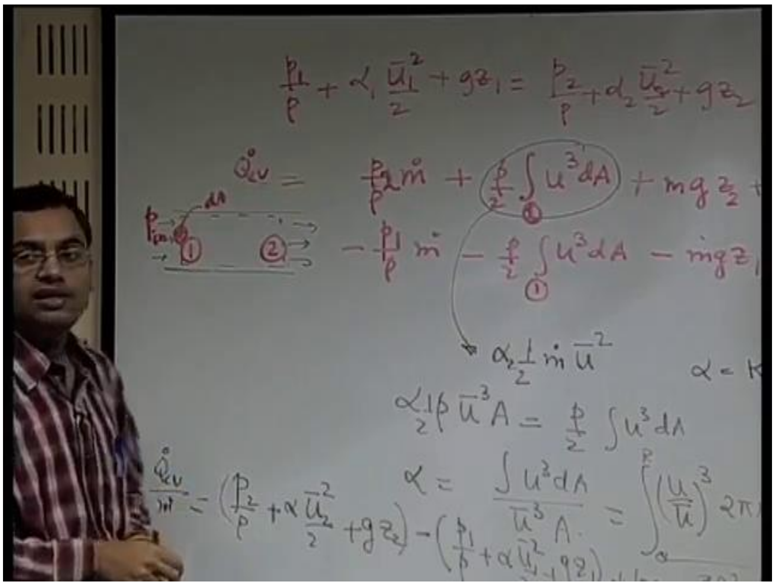

So, because of viscous action whatever work is necessary to overcome that that is eventually manifested in the form of an increased temperature this is known as viscous dissipation.

Only, whatever is the work internally that is manifested in the form of this internal energy change, but not where as sort of a useful work by the displacement at the wall; obviously, because it is a no slip boundary condition.

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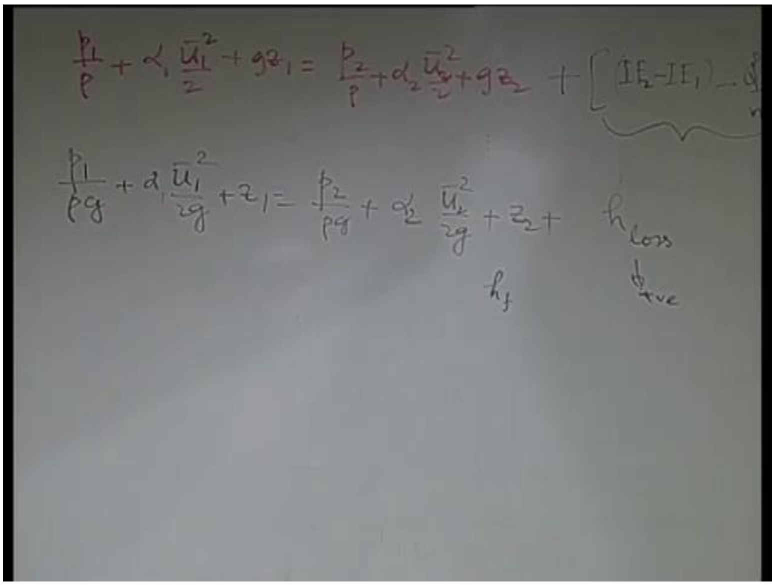

So, from this analysis what we understand is that we can cast the energy consideration between two sections 1 and 2 as

$$\frac{P_1}{\rho g}+\alpha_1\frac{\overline{u}_1^{\,2}}{2g}+Z_1=\frac{P_2}{\rho g}+\alpha_2\frac{\overline{u}_2^{\,2}}{2g}+Z_2+h_{loss}$$

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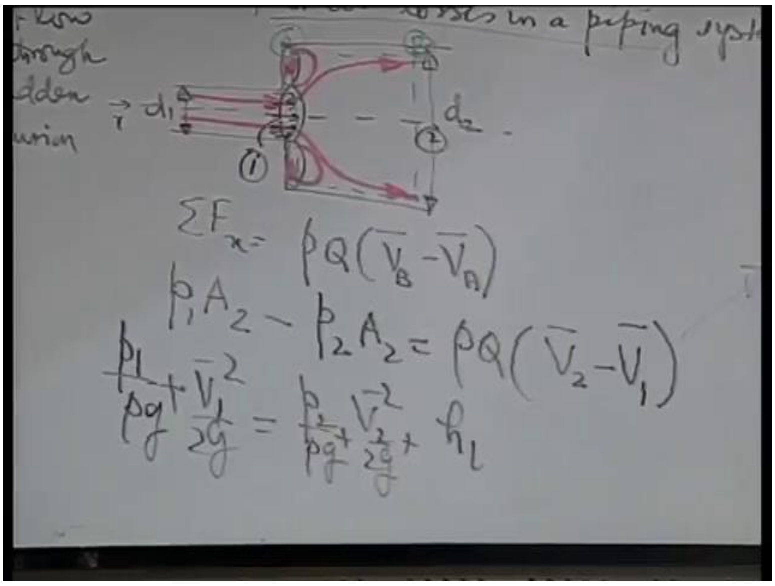

For example, there is a small pipe which is getting changed to a larger diameter. Now, there is a loss. So, considering the stream lines(shown by red lines in diagram), the stream lines because of the sudden change in cross section will be having their curvature in this way. So, locally there will be eddies formed in this way. These eddies do not participate in contributing to the energy of the main flow.

So, whatever energy is there associated with the rotation of these eddies that is the loss so far as the main flow transmittal is concerned. So, this is also a loss. If the valve is partially closed and partially open the fluid may flow. So, obviously, there may be other forms of resistances and those losses because of the other forms of resistances are known as minor losses in a piping system.

Minor losses in a piping system

Ex 1) Flow through sudden expansion

Let us say that the diameters of the smaller and the larger pipes are d_1 and d_2.

We take a control volume and try to apply the Reynolds transport theorem momentum conservation. So, let us say that we take this section as section A and take this section as section B.

$$\sum F_x = \rho Q\left(\bar{V}_B - \bar{V}_A\right)$$

$$P_1 A_2 - P_2 A_2 = \rho Q\left(\bar{V}_2 - \bar{V}_1\right)$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^{\,2}}{2g}$$

$$h_l = \frac{P_1 - P_2}{\rho g} + \frac{\bar{V}_1^2 - \bar{V}_2^{\,2}}{2g}$$

$$P_1 - P_2 = \bar{V}_2\left(\bar{V}_2 - \bar{V}_1\right), \qquad \because Q = A_2\bar{V}_2 = A_1\bar{V}_1$$

$$h_l = \frac{\bar{V}_2\left(\bar{V}_2 - \bar{V}_1\right)}{\rho g} + \frac{\bar{V}_1^2 - \bar{V}_2^2}{2g} = \frac{\left(\bar{V}_1 - \bar{V}_2\right)^2}{2g}$$

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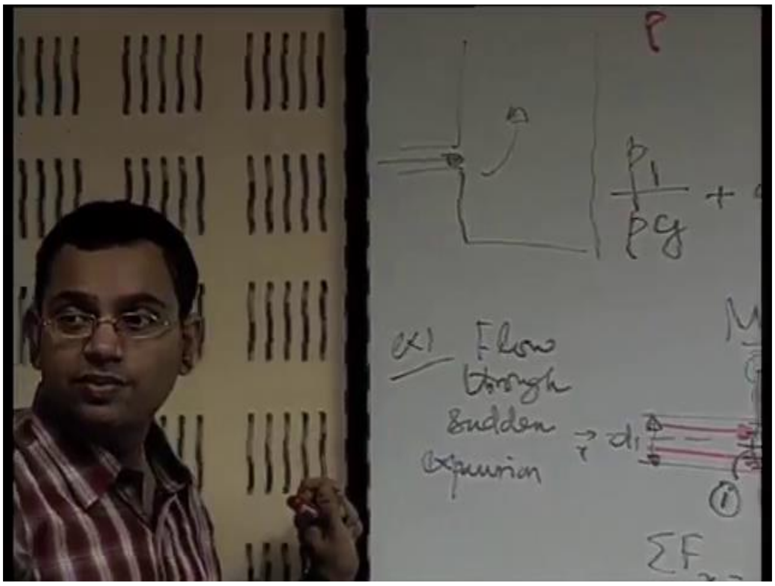

One special case is that let us say this is a small pipe entering into a very large reservoir. So, what is happening is fluid is exiting from the pipe to a reservoir.

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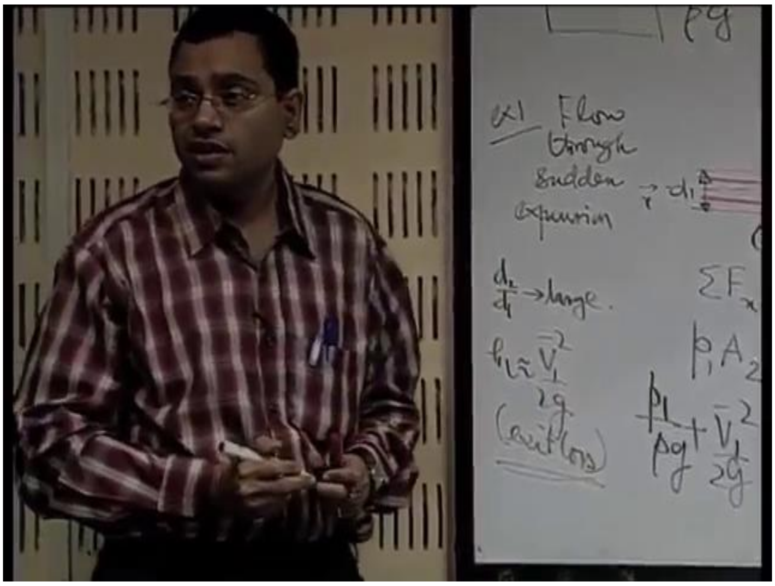

$$\frac{d_2}{d_1} \rightarrow \text{large}$$

$$h_l = \frac{\bar{V}_1^2}{2g} \text{ (exit loss)}$$