Introduction to Fluid Mechanics
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Lecture –50
Problems and Solutions

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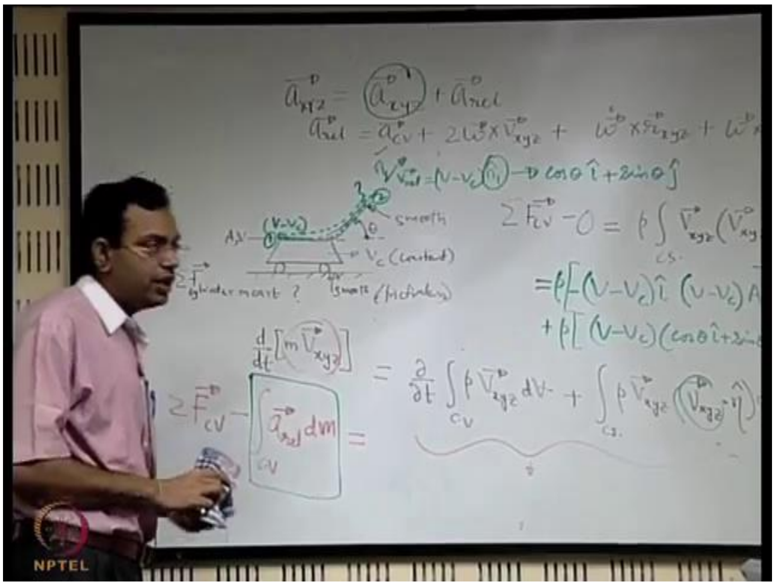

On a frictionless ground and there is a water jet which comes on it and which leaves the cart, the angle change is θ relative to the horizontal. And the surface over which it is changing its direction the water that is considered to be a smooth surface.

We have seen that because of the impingement of the water jet on the system and because of its change in direction, there will be a force and because of the force the cart will have a tendency of moving. So, we saw that one way of like restraining it from moving is like maybe making a pulley mass arrangement. So, that it is restraining from its motion, but in general that will not be the case. So, we will consider a first case when it is moving, but moving with a uniform velocity; just as a simple case. We will then move on to more and more complicated cases when it may move with an acceleration.

So, first let us consider that it is moving with a uniform velocity. So, this cart is moving with a velocity V_c towards the right. The water jet comes out with a from an area A with a velocity V and it gets deflected like this. And we are interested to find out what is the resultant force exerted by the water on the cart. So, let us try to write it relative to the moving reference frame.

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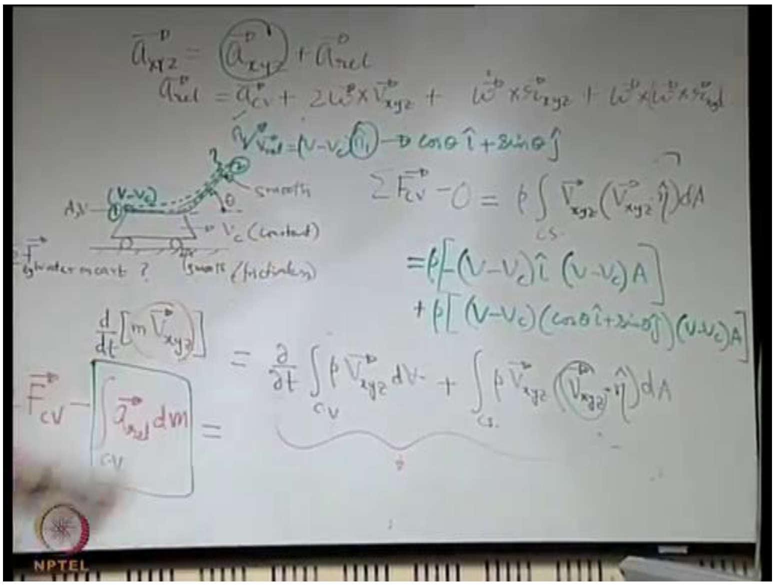

So, xyz is moving with a uniform velocity V_C.

$$\sum \vec{F}_{CV} - 0 = \rho \int_{CS} \vec{V}_{xyz} \left(\vec{V}_{xyz} . \hat{\eta} \right) dA$$

$$= \rho \left[-\left(V - V_c\right) \hat{i} \left(V - V_c\right) A \right] + \rho \left[\left(V - V_c\right) \left(\cos\theta \hat{i} + \sin\theta \hat{j} \right) \right]$$

Many times it is commonly understood that Bernoulli's equation you know is one of the easiest things in fluid mechanics. To me it is one of the toughest things in fluid mechanics; it is very easy formula at the end, but its restrictions are not well understood in many times. So, the question is can you directly use it keeping in view that all the assumptions that we have learnt otherwise are valid except we are now in a dilemma. Why we are now in a dilemma, because this is itself moving.

See although this is moving we are saved with one important thing this is moving with a uniform velocity. So, this is still an inertial frame. So, we have seen that in the Bernoulli's equation all the quantities are relative like pressure is relative; you may express it relative to something. Potential energy is relative you may express it relative to some reference or datum the height. Similarly, kinetic energy is also relative provided you are still using an inertial reference frame.

Relative velocity here is V -V_c and that relative velocity will be preserved because, the kinetic energy is preserved even in a reference frame which is moving, but non-accelerating.

$$\vec{V}_{rel} = \left(V - V_c\right)\hat{n}_i \rightarrow \cos\theta\hat{i} + \sin\theta\hat{j}$$

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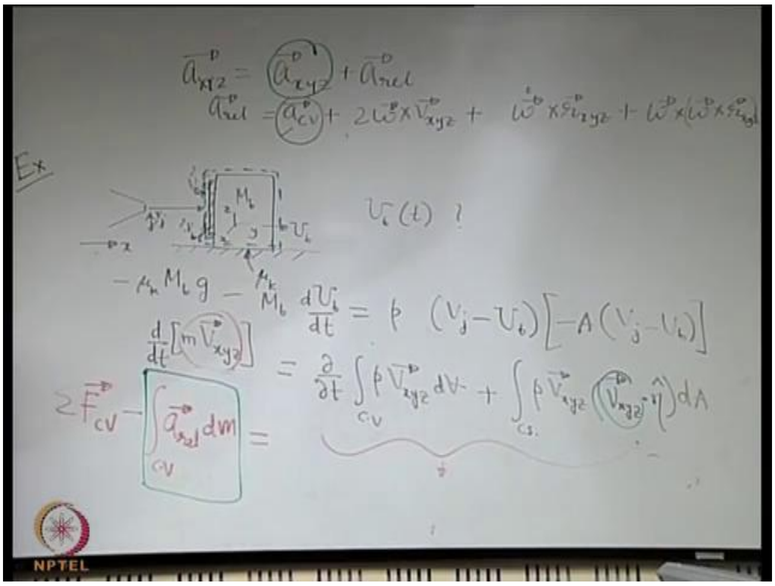

Let us say that there is a block which is moving on a surface, from a nozzle a water jet comes strikes this one and gets deflected in the two sides. The area of the velocity jets area of the jet that comes out is given, the velocity of the jet that comes out V_j that is also given $A_{j,}$ V_j area of the jet and velocity of the jet. The coefficient of kinetic friction between this surface and the block that is given. And our objective is to find out an expression of if let us say U is the velocity of this block or let us give it a different name maybe U_b for the U block, then how this U block is changing with time, $U_b\left(t\right)$?

The mass of the block is given let us say M_b is the mass of the block. Let us try to look into this problem with the framework of an arbitrarily moving reference frame. Now, this is also a simple form of arbitrarily moving reference frame because, this cannot have angular motion.

In general it should be like that because, before the jet is striking on it is stationary when the jet has jet is striking on it will try to make it move. So, its motion from 0 velocity will come to a non-zero velocity over a finite time. So, there will be a finite acceleration. So, it is expected in reality to be a accelerating reference frame.

Assuming that other assumptions other restrictions of the Bernoulli's equation are somehow satisfied; answer is no because it is now an accelerating reference frame. So, kinetic energy in an accelerating reference frame cannot be straight for in a straightforward way brought out from the Bernoulli's equation as preserved. Now it is not moving with a uniform velocity so, we do not know actually what these are even in a relative sense. But, the good thing is that since these are vertical these will not have any consequence on the horizontal momentum transfer.

So, whatever mass of water is just like flowing in contact with the in contact with this plate that water mass it is so, small as compared to this Mb that for weight or mass calculation aspect we will not consider the water weight. That is a very that is an assumption that we have to keep in mind, otherwise whenever we write the total weight it should be the weight of the water in the control volume plus the weight of the weight of the block which is there ok. But, we are neglecting that weight of the water in the control volume which is a very valid assumption because it is a very small amount of water which is there that is very practical.

So, then the normal reaction will be capital Mb.g because, of the equilibrium along the y. So, the friction force will be $-\mu_k M_b g$.

And, we could have been in trouble if there was lot of water in the control volume because, then for water we cannot have such a rigid body type of assumption. Then at each and every point the acceleration would be different, but we are neglecting the mass of water present in the control volume for writing these expressions. The unsteady term is fundamentally not 0, but we can approximate it by 0 because, again if you see the volume of the water that is present in the control volume is small. So, it is not because that V_{xyz} is small, but because of the volume of water which is there in the control volume is small then its time rate of change with respect to time is also very small.

So, these are certain important things. See we drop this term for solving the problem in exam fine. When you drop the term and solve the problem in the exam, if you solve it correctly we have no way to deny you from the credit. But, your conceptual understanding is not satisfied there, you must be convinced that why this term is dropped despite the fact it is an unsteady flow.

$$-\mu_k M_b g - M_b \frac{dU_b}{dt} = \rho\left(V_j - U_b\right)\left[-A\left(V_j - U_b\right)\right]$$

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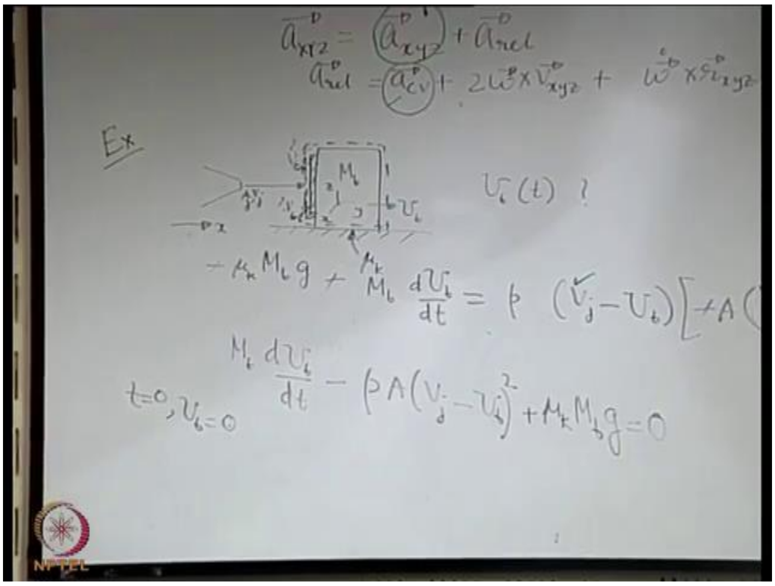

$$M_b \frac{dU_b}{dt} - \rho A\left(V_j - U_b\right)^2 + \mu_k M_b g = 0$$

It is a situation that occurs when the driving force on the block is just sufficient to overcome the kinetic friction. And then it is in equilibrium, but it takes some time for the driving force to be in equilibrium with the kinetic friction; till the time the velocity will be changing with time. And that how it changes with time is governed by this equation.

Now, when we have considered this problem one important thing that our when we have considered this theory as such; one important assumption that we have made is that the fluid in the control volume is of constant mass. But it might so, happen that the mass itself is a variable within the control volume. So, what we will do if the control volume is accelerating plus the mass within the control volume is a variable, it is not a constant. So, we will see that in the next class. In the next class we will take up the cases of variable mass in the control volume.

Thank you.