Introduction to Fluid Mechanics
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Lecture – 48
Problems and Solutions

Now, let us look into another problem.

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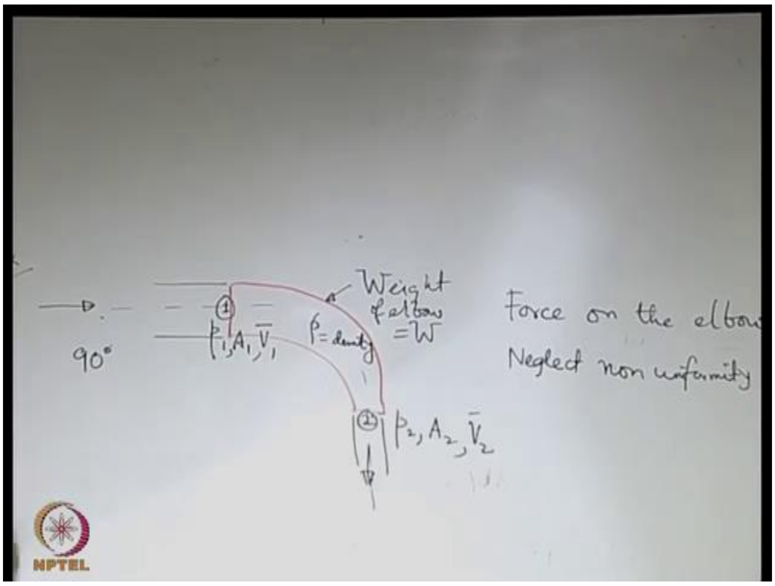

Let us say that you have points 1 and 2 or let us call these as sections 1 and 2. So, at the section one say there is a equivalent pressure p_1 which is given, area of cross section is given; at the section 2 you have p_2 and A_2 these are given.

Neglect non uniformity in velocity profile.

Weight of elbow=W

ρ is the density

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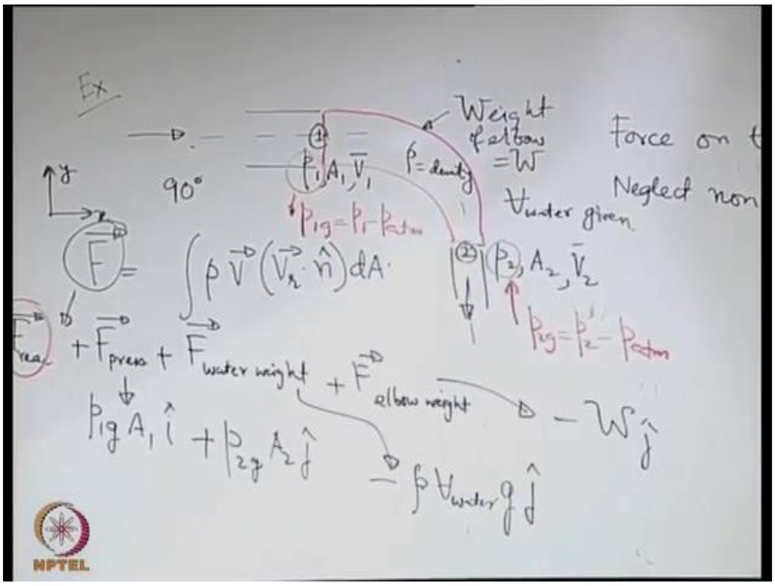

So, if stream lines are almost parallel to each other then the change in pressure is very very small or negligible. So, then we are assuming that streamlines here and are almost parallel to each other.

When non uniformity of velocity over each section is occurring then that means, it is always there until and unless it is a highly turbulent flow at the velocity profile due to high mixing is almost uniform. Otherwise if there is a velocity profile it gives an important understanding that yes viscous effects are important. And when viscous effects are important you cannot apply Bernoulli’s equation along a streamline between 1 and 2.

$$\vec{F} = \int \rho \vec{V} \left(\vec{V}_r . \hat{n}\right) dA$$

$$\vec{F}_{reac} + \vec{F}_{press} + \vec{F}_{water\ weight} + \vec{F}_{elbow\ \ weight}$$

$$P_{1g} = P_1 - P_{atm}$$
$$P_{2g} = P_2 - P_a$$

$$\vec{F}_{pressure} = P_{1g} A_1 \hat{i} + P_{2g} A_2 \hat{j}$$

$$\vec{F}_{water\ \ weight} = -\rho \cancel{V} g \hat{j}$$

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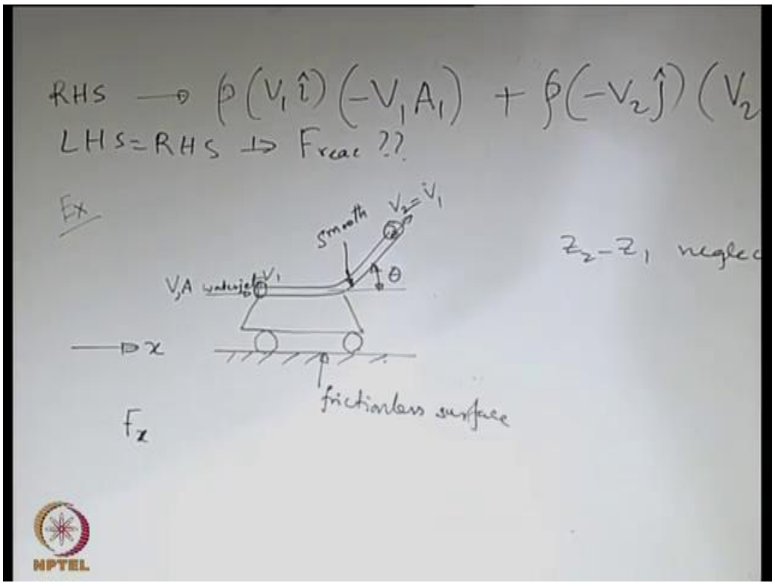

$$\text{RHS} \rightarrow \rho\left(V_1\hat{i}\right)\left(-V_1A_1\right)+\rho\left(-V_2\hat{j}\right)\left(V_2A_2\right)$$

$$LHS = RHS \rightarrow F_{reac}\,?$$

Let us say there is a cart like this and a water jet is striking on the cart and it is changing its direction; let us say this angle is θ. Let us say that the velocity of the water jet is V and the corresponding area over which the jet is moving here is A cross section area and let us assume that this is smooth.

Let us assume that this is a frictionless surface and maybe assume that the cart is having a particular weight.

Let us assume it is an inviscid flow.

If the velocity here is V_1, the velocity $V_2 = V_1$ provided that the difference in height between 1 and 2 is neglected.

In general if there is a friction here V_2 will be somewhat less than V_1, but because of the frictionless nature V_2 is V_1 and then you can apply the continuity equation then A_2 also must be same as A_1.

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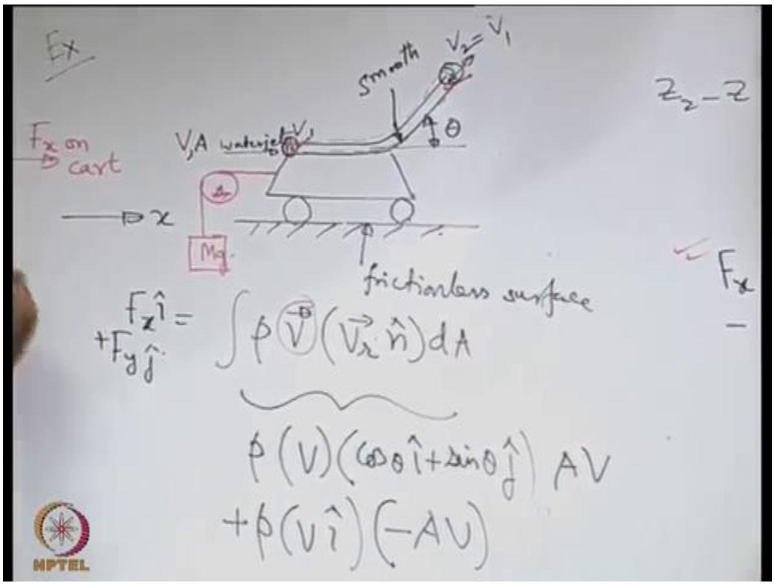

$$F_x\hat{i}+F_y\hat{j}=\int\rho\vec{V}\left(\vec{V}_r\hat{n}\right)dA$$

$$\rho(V)\left(\cos\theta\hat{i}+\sin\theta\hat{j}\right)AV+\rho\left(V\hat{i}\right)(-AV)$$

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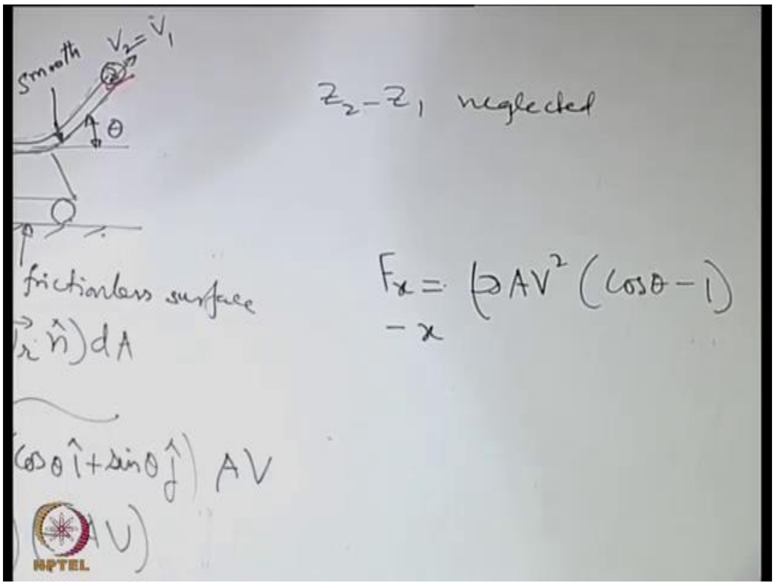

$$F_x=\rho AV^2\left(\cos\theta-1\right)$$

The force is along negative direction that is -x direction.

Let us say we have a pulley like this. And let us say there is a weight Mg which is there this pulley is hinge supported like this

It depends on what is this weight and that you can design exactly because you know what is the exact magnitude of the force.

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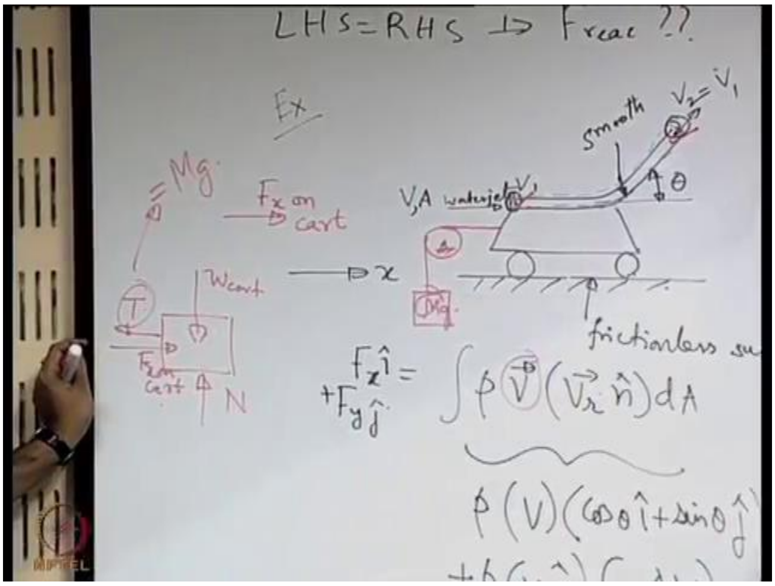

So, if you draw the free body diagram of the cart there will be a tension in the string and you force effects exerted by the water on cart.

When it is not in equilibrium with these forces it might have a velocity that velocity itself might change with time. So, it might have a situation when the reference frame which may be attached to the cart itself is moving and moving with arbitrary velocity or arbitrary acceleration.

When we say an accelerating reference frame we mean accelerating frame reference frame in all respects; that means, it could be linearly accelerating, it might have an angular velocity because of which it has its original acceleration. So, we have to next go for an analysis for accelerating reference frames.

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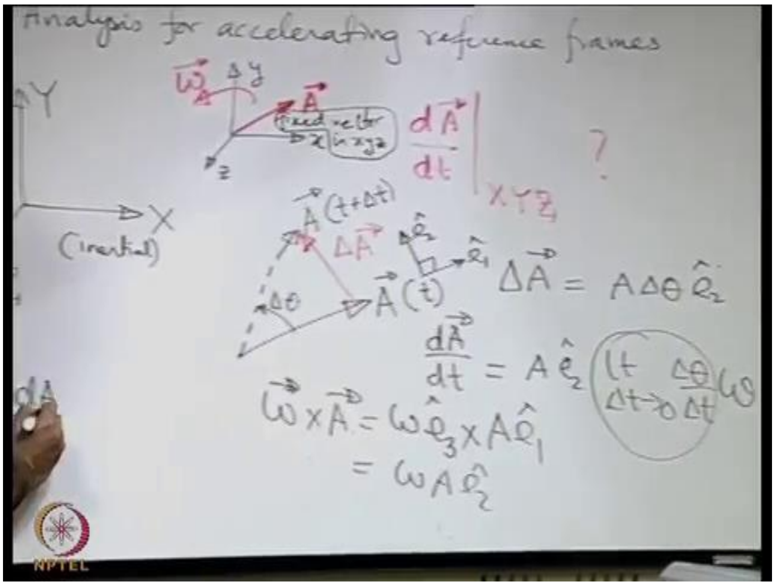

So, we will use certain nomenclature; we will consider an axis X, Y, Z for an inertial reference frame. And x y z reference frame as an arbitrary it may be; it may be inertial may be non inertial, but is a moving reference frame.

If it is moving with an acceleration then it is non inertial, if it is moving with a uniform velocity it is still inertia.

Let us say there is a vector A, which is there in a reference frame that is rotating with an angular velocity ω.

$$\Delta\vec{A} = A\Delta\theta$$

$$\frac{d\vec{A}}{dt} = A\widehat{e}_2 \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\omega$$

$$\vec{\omega}\times\vec{A} = w\widehat{e}_3 \times A\widehat{e}_1 = \omega A\widehat{e}_2$$

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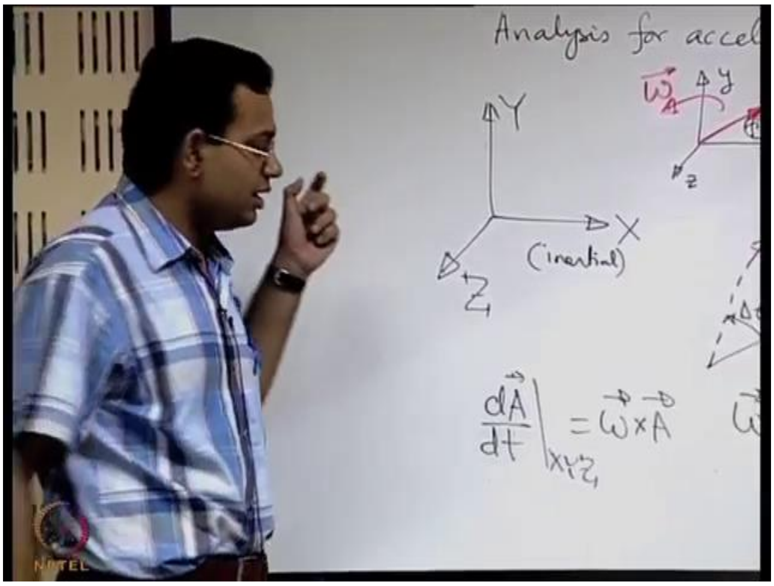

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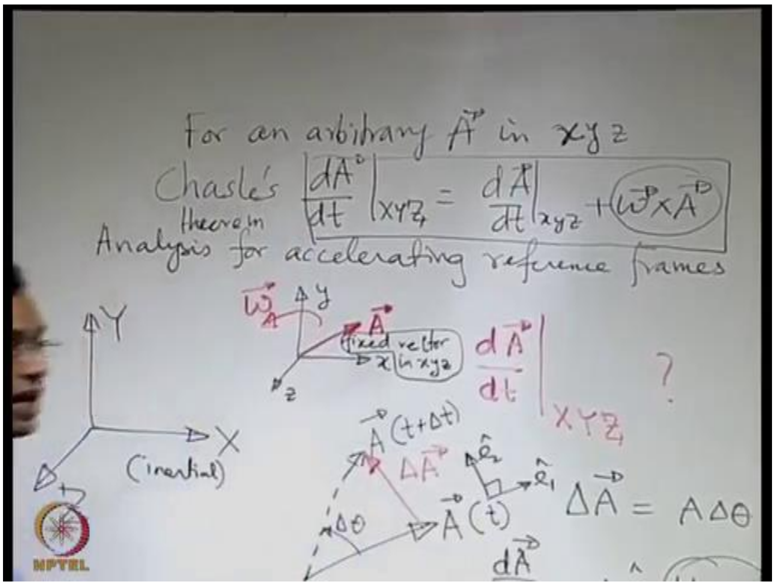

For an arbitrary $\vec{A}$ in xyz

$$\left.\frac{d\vec{A}}{dt}\right|_{XYZ} = \left.\frac{d\vec{A}}{dt}\right|_{xyz} + \vec{\omega} \times \vec{A}$$

This is known as Chasle's theorem.