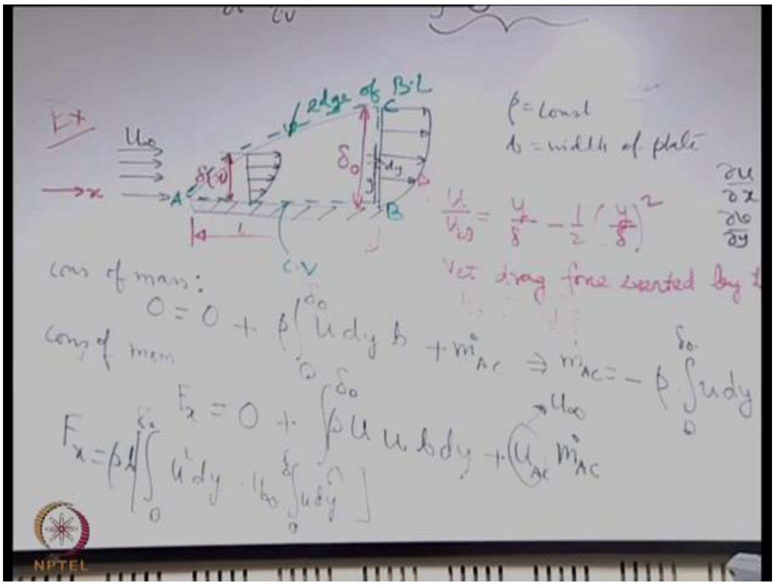
Introduction to Fluid Mechanics

Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 46
Problems and Solutions

(Refer Slide Time: 00:46)



Let us try to solve another problem. So, in the next problem, we will revisit the case of a boundary layer on a flat plate that we discussed quite some time back when we were discussing about viscosity. So, what is the situation that you have a flat plate fluid is coming from far stream with a uniform velocity, say u_∞. This fluid is falling on the plate and because of its viscous interactions the effect of viscosity within the plate so the within the fluid.

The first thing is the plate tries to slow down the fluid which is in contact with that and that effect is propagated to the outer fluid through the viscosity of the fluid. So, there is a thin layer close to the wall where this effect of the plate is failed and a velocity gradient is created. So, outside that layer the fluid does not actually feel the effect of the plate and that thin layer is known as the boundary layer.

So, let us say that at the edge of the plate, you are given the velocity profile. Let us say that locally the thickness of the boundary layer is given by δ which is a function of x, $\delta(x)$ The value of this δ is the boundary layer thickness at this location where the x is l.

The velocity profile is given, $\frac{u}{u_\infty} = \frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2$

Net drag force exerted by the plate on the fluid. This is a physically interesting parameter because of the viscous effects there is a drag force that is there that is the plate tries to slow down the fluid. And our objective is to find out what is that net force acting over this length l.

So, to find out the force we may understand that like we might require a linear momentum conservation, because the force is involved. And when we have our linear momentum conservation, the mass part of that should also be conserved. So, we should also be consistent with a mass conservation. So, it depends like we should choose a control volume. Let us first choose a control volume which is intuitive and try to solve the problem. And then see we will try to investigate whether that in intuitive choice of the control volume is good or bad.

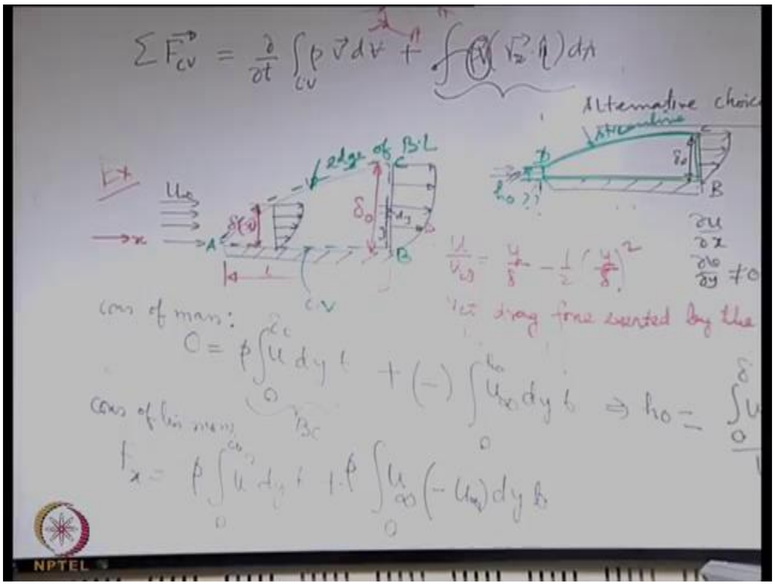
So, let us say that we choose a so this black line what is there, it is the edge of the boundary layer. What is the edge of the boundary layer, we know that within this layer only the velocity profile is there; beyond this the velocity is uniform equal to u infinity almost. So, if you want to take a control volume let us say we choose a control volume like this, just engulfing the edge of the boundary layer control volume.

Consv. Of mass: $0 = 0 + \rho \int_0^{\delta_0} udy.h + \dot{m}_{AC} \Rightarrow \dot{m}_{AC} = -\rho \int_0^{\delta_0} udy.h$

Consv. Of momentum: $F_x = 0 + \int_0^{\delta_0} \rho u.u.b.dy + u_{AC}\dot{m}_{AC}$

Now, let us just look into that what could be an alternative choice of the control volume, maybe to solve the problem a bit more elegantly, not that it is too dirty, but one could even like do it in a bit more elegant manner. One interesting thing you observe from here that there is only x-component of velocity here, and y-component of velocity in the free stream is 0, but there is some flow across the surface AC. Now, could we choose a surface ac or surface of similar type such that there is no flow across it?

(Refer Slide Time: 14:33)



So, let us try to draw separate sketch with an alternative choice of the control volume, so alternative choice of control volume. So, for alternative choice of the control volume, and let us say that again we have the velocity profile and everything you have this AB. We want to choose a line from here C such that there is no flow across that line; we have seen there is a flow across the edge of the boundary layer.

$$F_x = \rho b \int_0^{\delta_0} u^2 dy - u_\infty \int_0^{\delta_0} u dy$$

Alternative choice of Control Volume

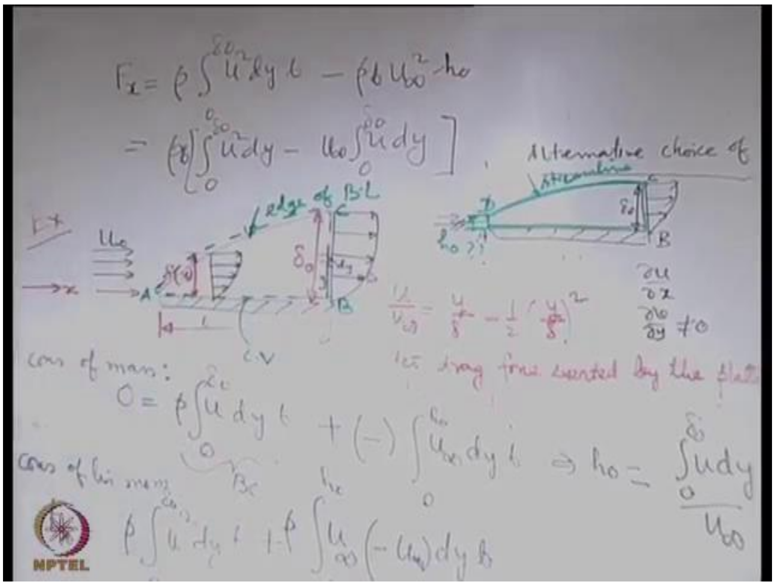
Consv. Of mass: $0 = \rho \int_0^{\delta_0} u dy.b + (-) \int_0^{h_0} u_\infty dy.b \Rightarrow h_0 = \dfrac{\int_0^{\delta} u dy}{u_\infty}$

Consv. Of linear momentum: $F_x = \rho \int_0^{\delta_0} u^2 dy + \rho \int_0^{h_0} u_\infty (-u_\infty) dy.b$

So, what should be that line that we choose here, so that there is no flow across it? Yes, through what type of line you have no flow across, through a streamline. So, if you choose a stream line which is passing through the point C, this is a stream line; then we know that there is no flow across it. But it gives rise to our new unknown because we do not know that where that stream line is intersecting with this one let us say h_0.

So, now, let us make a new control volume say ABCD, where we have tried to get rid of a problem by considering the stream line when there is no flow across it, but we have a new flow boundary AD. So, with for that new control volume let us write the conservation of mass.

(Refer Slide Time: 19:27)



$$F_x = \rho \int_0^{\delta_0} u^2 dy.b - \rho b u_\infty^2.h_0 = \rho b\left[\int_0^{\delta_0} u^2 dy - u_\infty \int_0^{\delta_0} udy\right]$$

This is the force exerted by the plate on the control volume.

So, two different choices of the control volumes are giving back the same expression and to be the alternative choice of control volume is not bad, because it gives a better visualization of what is happening, because this gives a direct visualization that something is entering here and something is leaving, and these two are not participating. For the case of the edge of the boundary layer, it is physically not that intuitive mathematically it is not that difficult at straightforward, but the better physical picture is being provided by this control volume.

But both are fine and the remaining part is easy, you may substitute u as a function of y and integrate, because u as a function of y is given to find out the expression.

This is the force exerted by the plate on the control volume. So, here u is multiplied with a number which is greater than u itself. So, this term must be less than the second term.

So, intuitively this should come as negative and that is what is obvious, because it is a drag force. So, it tries to slow down the motion of the fluid. So, this is something which is a physical understanding that is important. Once you solve a problem, you get a sign out of the problem, and you should develop an intuition of what that sign implies. Let us stop here today, and we will continue again in the next class.