Introduction to Fluid Mechanics
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Lecture – 45
Application of RTT: Conservation of linear momentum

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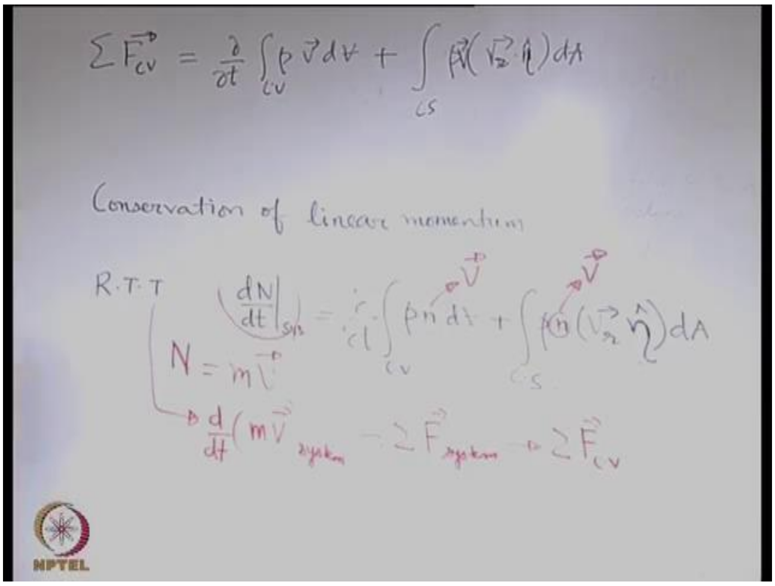

In this lecture, we will look into the integral form of the Conservation of linear momentum. To do that we will start with the Reynolds Transport Theorem general expression which is valid for any conservation.

$$N = m\vec{V}$$

$$\left.\frac{dN}{dt}\right|_{system} = \frac{d}{dt}\left(m\vec{v}\right)_{system} = \sum \vec{F}_{system} \rightarrow \sum \vec{F}_{CV}$$

n is N/unit mass.

It is not necessary to have a stationary reference frame. We may have moving reference frame not only that a reference frame moving arbitrarily with arbitrary rotation as well. But here we are considering inertial and a special case of inertial which is stationary.

$$\sum \vec{F}_{CV} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall + \int_{CS} \rho \vec{V}\left(\vec{V}_z \hat{\eta}\right) dA$$

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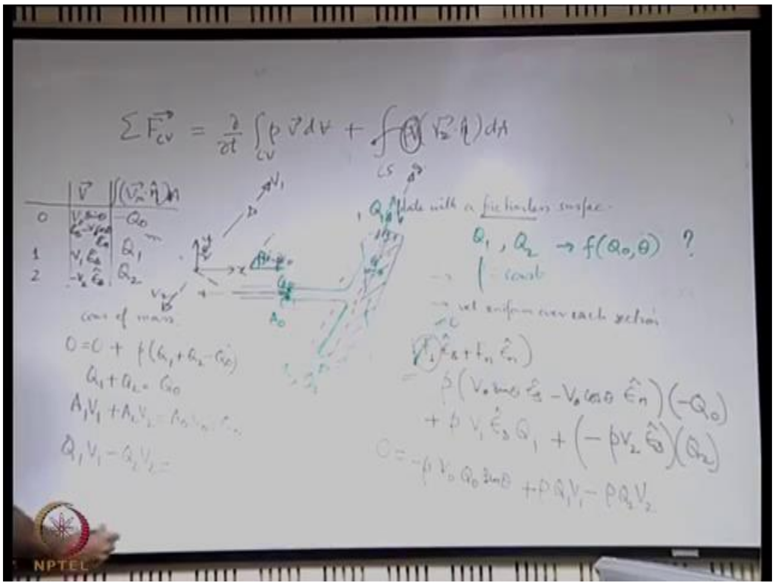

Let us say that you have a water jet which comes out of a tube or maybe a small pipe. And say there is a plate, which is frictionless and oriented inclined to the jet. So, this is a plate with a frictionless surface.

We have to find out, if the flow rate volume flow rate here is Q_0 and here the volume flow rate is Q_1 and if the volume flow rate is Q_2 and let us say that, the angle made by the plate with the vertical is theta which is the angle between the plate and the vertical. When we say vertical let us neglect the change in height between various points so, just to simplify the situation.

The change in potential energy between various points is insignificant as compared to the other forms of energy, like the kinetic energy. Now let us say that this area of cross section of this extreme streamline. So, these are like stream tubes these are called. So, if you see that this is not a pipe basically. So, this is a free water jet. Exposed to the atmosphere, but the water jet takes a form like a tube its outer periphery takes the form of a tube and outer ones are the extreme streamline. So, the figure that is drawn here the lines represent the envelope of all the streamlines.

So, this is like this called a stream tube which engulfs all the streamlines, the same is there for the exit ones. $Q_1, Q_2 \to f\left(Q_o, \theta\right)$.

Let us say that this is the boundary or the surface of the control volume that we are looking for.

Now when this control volume is drawn we are interested to find out a write and expression first for the mass balance, conservation of mass. Assume that the density of the fluid is a constant. Let us say that the areas of flow here are like here it is A_2 and here it is A_1. These are the parameters which we do not know because these are not within our direct control. What is in our direct control may be this area, because this depends on the ejected jet from the tube or the pipe whatever nozzle, but when it comes on the plate and moves this depends on the many other things.

Consv. Of mass

$$0 = 0 + \rho\left(Q_1 + Q_2 - Q_0\right)$$

$$Q_1 + Q_2 = Q_0$$

$$A_1V_1 + A_2V_2 = A_0V_0$$

See one very important thing is it is considered to be a frictionless surface. That means, there is no shear force between the fluid and the plate in that direction tangential to the plate. So, if you write a linear momentum conservation, with a component say along x, x is like say tangential to the plate or maybe let us call it s to indicate that it is a tangential to the plate. And maybe a direction n which is like normal to the plate, but we are bothered about for the tangential to the plate. The situation is that there should not be any force on the water which is there in the control volume because it is a frictionless thing.

$$F \rightarrow \left(F_s\hat{e}_s + F_n\hat{e}_n\right)$$

So, we have a section say 0, at which we are interested to find out what is V and what is the normal vector. The normal vector will be insignificant we will see because eventually we are dealing with the flow rates.

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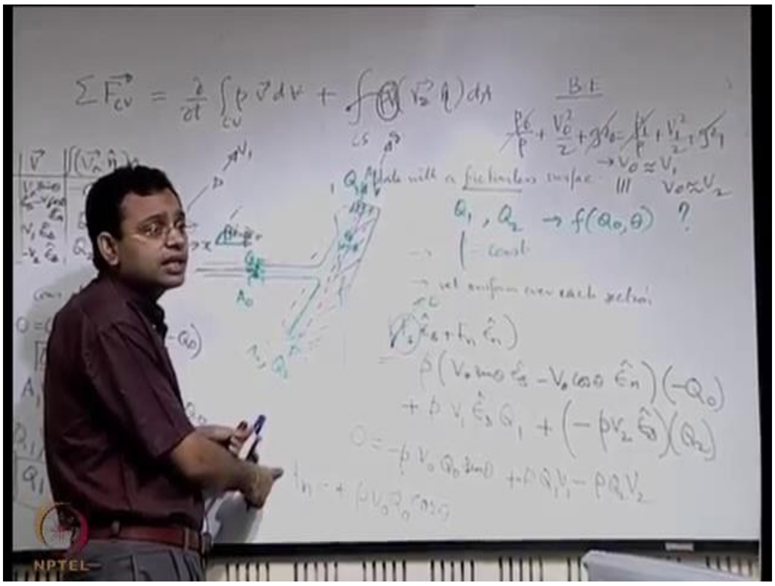

So, the velocity vector for this one like, let us say that we have our original axis like this is x and this is y. And the s axis is like this which makes an angle, θ with y. May be you can also choose a n axis which is normal to that.

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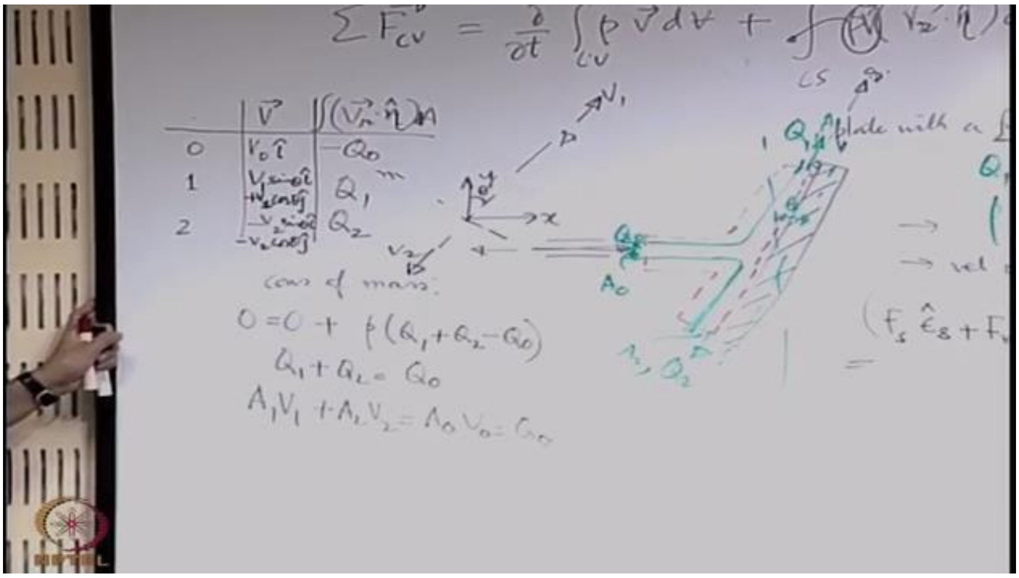

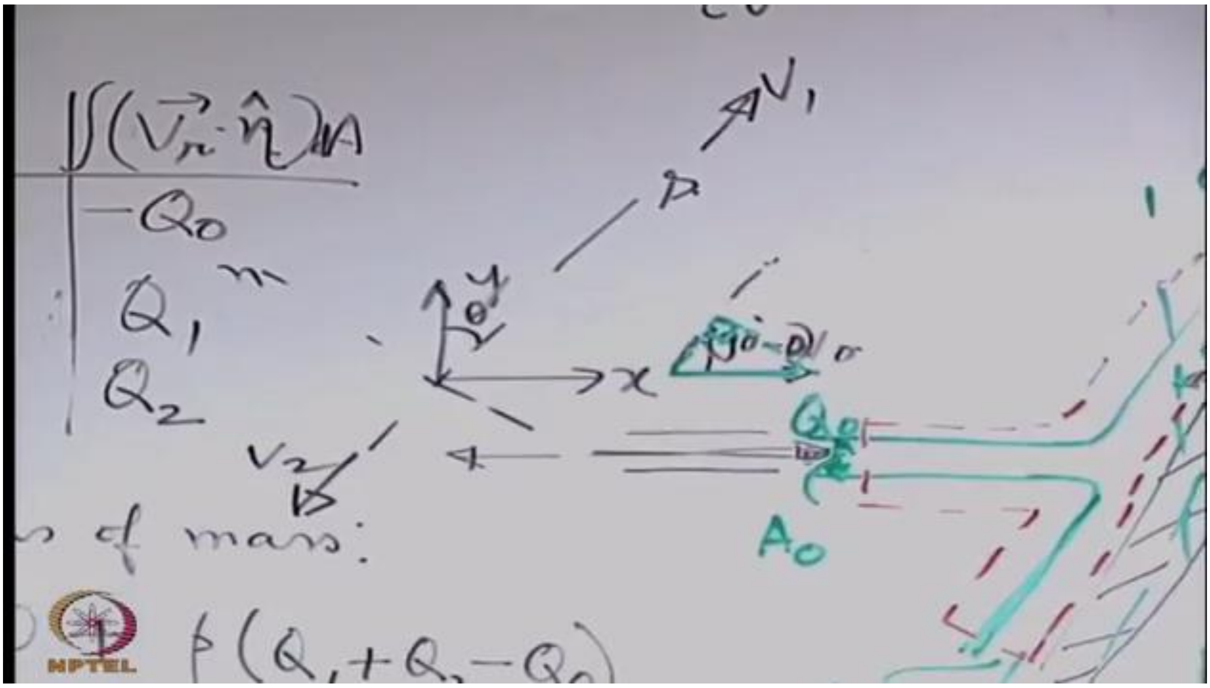

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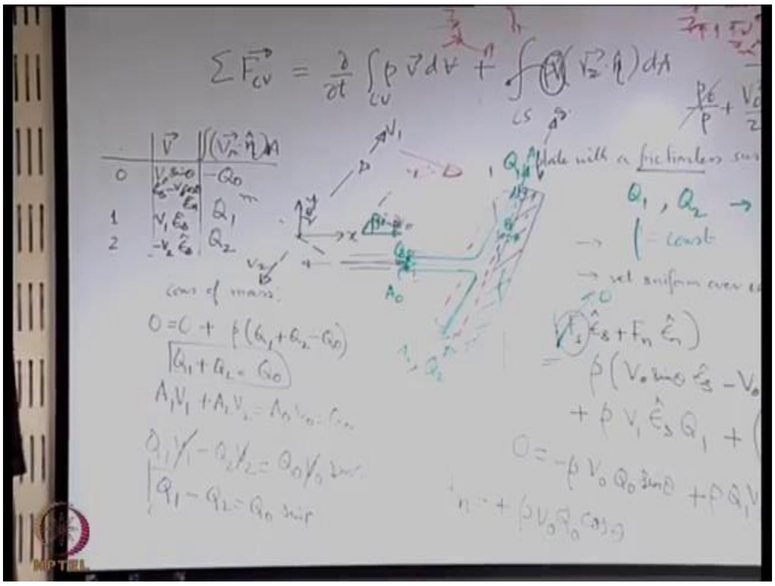

$$\left(F_s\widehat{e}_s + F_n\widehat{e}_n\right) = \rho\left(V_0 \sin\theta\widehat{\varepsilon}_s - V_0 \cos\theta\widehat{\varepsilon}_n\right)\left(-Q_0\right) + \rho V_1\widehat{\varepsilon}_s Q_1 + \left(-\rho V_2\widehat{\varepsilon}_s\right)\left(Q_2\right)$$

$$0 = -\rho V_0 Q_0 \sin\theta + \rho Q_1 V_1 - \rho Q_2 V_2$$

$$Q_1 V_1 - Q_2 V_2 = Q_0 V_0 \sin\theta$$

Let us consider a stream line that goes from the section 0 to the section 1. Because it is a uniform velocity like if you take a point 0 here and a point 1 here these are representatives of the velocities as good as at section entire section 0 and entire section 1. Because it is a uniform velocity at each section that we have assumed. Now if we apply the Bernoulli's Equation between any two points on the stream line.

$$\frac{P_0}{\rho}+\frac{V_0^{\,2}}{2}+gz_0=\frac{P_1}{\rho}+\frac{V_1^2}{2}+gZ_1$$

$\frac{P_0}{\rho}$ and $\frac{P_1}{\rho}$ are equal and because the height difference is not appreciable as we assumed earlier. So, Z_0 and Z_1 are approximately the same, or their difference is negligible.

$$Q_1-Q_2=Q_0\sin\theta$$

$$F_n=+\rho V_0 Q_0\cos\theta$$

So, if you have drawn a free body diagram of the control volume, if you would have done. Then what would have been the forces acting on the on that control volume that would have shown in the free body diagram.

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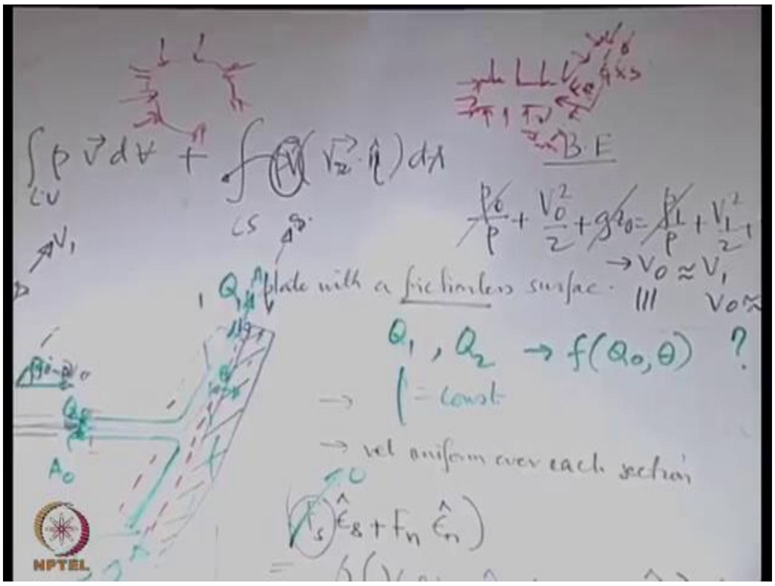

Let us say that this could be your control volume. You could have a force sorry, in the tangential direction F_s and normal direction F_n. Let us assume that the same thing is occurring almost in a horizontal plane.

So, body force we have considered the weight just not shown in this plane because it is not significantly there in this plane. Surface force, pressure distribution is there. So, there is a surface force, from all sides, normal to the areas. We have not considered, but magically we are expecting that it is giving us the correct information on forces. And this magical thing is possible because if you have a closed contour, over which are you have a uniform pressure distribution then irrespective of the shape of the contour the resultant force is 0.

Because of a very special nature that pressure local is always normal to the direction of the surface. So, no matter how bad the shape of the contour of the surface is. So, long as it is closed and if you have a uniform pressure throughout then the resultant force due to that will be 0.

Because it is a uniform pressure, and that is why you will see that what is important for us is not what is the atmospheric pressure; but the deviation from the atmospheric pressure that only can give rise to a local force. So, always when we consider forces here due to pressure we consider the gauge pressures, at different sections because the deviation from atmospheric pressure is what is interesting for us. Atmospheric pressure if it was the sole feature, it would have existed throughout equally and it would have given rise to a 0 net force. So, in terms of giving rise to a net force any deviation from atmospheric pressure is important.

F_n is force exerted by plate on the water because it is force exerted on the control volume. And what is there in the control volume is not plate, but water. So, this is force exerted by the plate on the water present in the control volume. By Newton's third law, you will be having the force exerted by the water on the plate as minus of these. So, the force exerted by water on the plate will be opposite to the positive direction.

So, force exerted by water on the plate will be in this direction. This is you see this is in the positive n direction that we have considered. So, opposite to that is the force exerted by water on the plate. And that is quite obvious like logically if you see a jet stripes on the plate like this it should have a force in this direction not the opposite direction.

So, this there is a resultant force which is exerted on the plate in its normal direction. So, this force might be good enough to make the plate move, and the plate should be adequately supported to prevent such movement. In general if that jet is falling on the plate, the plate will also move because of this normal force.