Introduction to Fluid Mechanics

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Lecture – 44
Problems and Solutions

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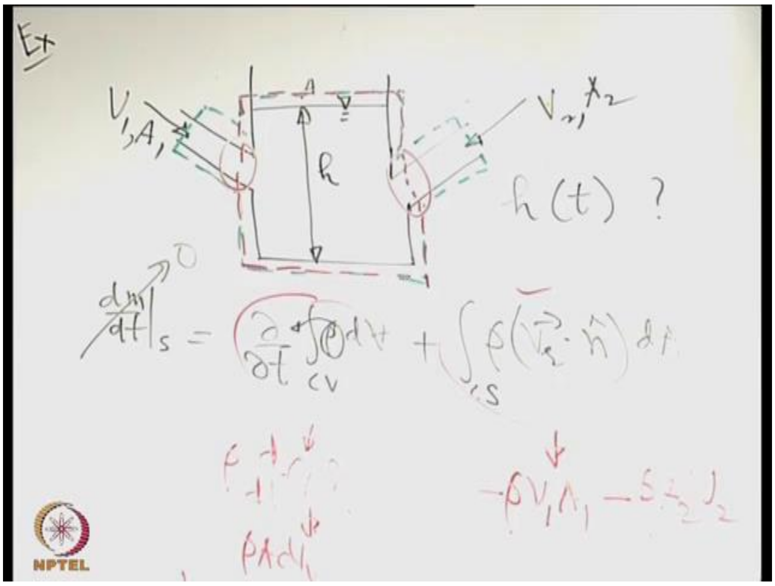

Let us see similar problem where you have a tank like this, and there is a free surface here. So, in the previous problem, we assumed that there is no change in level of the surface in the tank, but in reality that does not happen, so it is something which is a bit hypothetical. In reality it may happen that approximately the change in this level is 0, because this is such a large area that no matter whatever is entering and leaving, it is not changing the height. And these types of tanks are called as constant head tanks. So, they maintain constant head because with respect to the inflow and outflow the change in height of this is so small because of maybe this is a large reservoir. So, the area is so large that it does not change any level. But the more realistic version of the previous problem is that the level of the water will also change.

So, let us say that the height of the free surface from the bottom of the tank is h, just to simplify the situation we now go back to a case of a uniform velocity profile because we have already seen that if it is non-uniform that it is not a very difficult thing, we have to just integrate the velocity profile over the section. So, with that understanding let us say that you have a situation like this.

Let us say it is uniform, so you have a velocity V_1 here and let us say area of cross section A_1 you have a velocity V_2 area of cross section A_2. When we are talking about the previous problem, we could get rid of the situation of change of height of the tank, even in a real case by some approximation that the rate at which the water is entering is the same at which the water is leaving. So, it is not changing the height of the level of the tank, but when both are entering that is not the case.

So, here the only chance of the height of the tank or the level of the water in the tank not changing with time maybe only for the consideration that the area of cross section of the tank A is so large as compared to the others that the corresponding change in height is very small. Otherwise there will always be a change in height, smallness or largeness depends on the areas.

In the previous problem, the situation represents there is zero change in height. By making sure that the rate at which it enters is exactly the same at which it leaves. So, it appeared to be bit hypothetical, but it is not that hypothetical if it is really doing that. So, if you have a reservoir and water is entering and leaving at the same rate why should you change the level of the reservoir it will not, but here both are entering.

So, here our objective is to find out how the height is changing with time. Given these velocities, areas and the information that these velocities are uniform over the area. So, let us take a control volume. $\left.\frac{dm}{dt}\right|_{system} = \frac{\partial}{\partial t}\int_{CV} \rho dV + \int_{cs} \rho\left(\vec{V}_c.\hat{n}\right)dA$. So, the volume within the control volume is $A \times h$. A is the cross-sectional area of the tank, and h is the height.

$$\frac{\partial}{\partial t}\int_{CV} \rho dV \rightarrow \rho A\frac{dh}{dt}$$

$$\int_{cs} \rho\left(\vec{V}_c.\hat{n}\right)dA \rightarrow -\rho A_1V_1 - \rho A_2V_2 = 0$$

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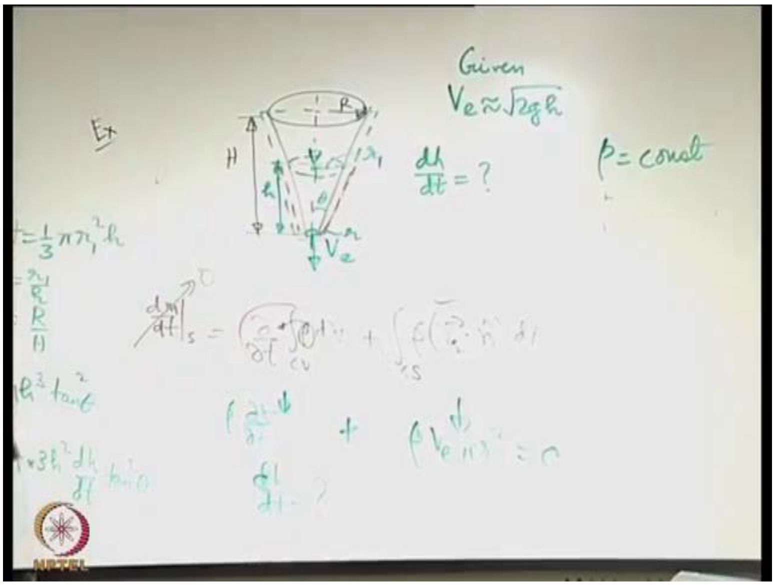

Let us say that you have conical tank just for a change of radius R, and height H. There is a small hole at the bottom of the cone through which water is coming out with a velocity V_e. And because of this leaving of the water, the height of the water in this conical tank is reducing, maybe initially it was the full height H, but because water is leaving at some instant of time say the height is like h. So, this h is changing with time, because water is continuously leaving. Let us say that it is leaving through the small hole with radius of r.

$$V_e = \sqrt{2gh}$$

Let us say that we identify a control volume like this, let us say with respect to that control volume we are writing this term. So, with respect to this control volume when we are writing the term, let us say that we neglect the density of the air which is there or at the top of the water in comparison to that of the water.

$$V = \frac{1}{3}\pi {r_1}^2 h$$

$$\tan\theta = \frac{r_1}{h} = \frac{R}{H}$$

$$\cancel{V} = \frac{1}{3}\pi h^3 \tan^2\theta$$

$$\frac{d\cancel{V}}{dt} = \frac{1}{3}\pi \times 3h^2 \frac{dh}{dt}\tan^2\theta$$

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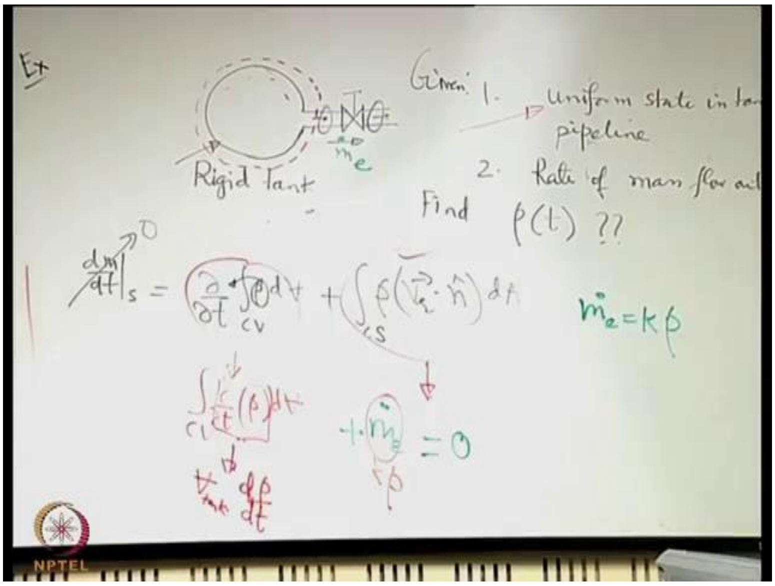

Let us say that we have a rigid tank, spherical tank, which contains air. And originally there was a valve located here, which was preventing the air inside to go outside. Now, the valve is opened. And once the valve is opened, the air will go out of this pipeline which is connected to the tank. It is given that

1) the state is uniform within the tank and pipeline that means the properties are the same at a given instant of time everywhere that we are considering.
2) It is given that the rate of mass flow out is proportional to the density at that instant .

Let us start with the choice of a control volume on which we want to have our analysis. So, the control volume let us say that we choose a control volume like this. If we choose a control volume like this, it does not matter whether the valve is opened or closed right, there will always be a flow across it.

So, if the density in the pipeline is a function of time, and if there is a possibility of like see variable density cases are very typical cases. So, for variable density cases, it is not so trivial to say that like the mass flow rate is the same in all cases, but here no matter how the density varies you can say that these two mass flow rates will be the same.

So, we are assuming that there is no accumulation it is a steady flow system. So, if it is a steady flow like whatever enters here the same leaves here then like it does not matter really whether your control surface goes through these or through these. So here it is a rigid tank in the previous class we try to solve a similar problem which is like a flexible balloon. So, there the volume could change with time that is maybe if it was a if it were a flexible balloon you could have expected that by taking away air out of it the balloon will try to shrink, but here it is a rigid tank. So, it cannot respond to that it can only respond to that change by having a change in density inside, but not its change of its own volume.

$$\frac{\partial}{\partial t}\int_{CV}\rho dV \rightarrow \cancel{V}_{TANK}\frac{d\rho}{dt}$$

$$\int_{cs}\rho\left(\vec{V}_c.\hat{n}\right)dA \rightarrow +\dot{m}_e = 0$$

$$\int_{\rho_1}^{\rho_2}\frac{dp}{\rho} = -\frac{k}{V_{TANK}}\int_{t_1}^{t_2}dt$$

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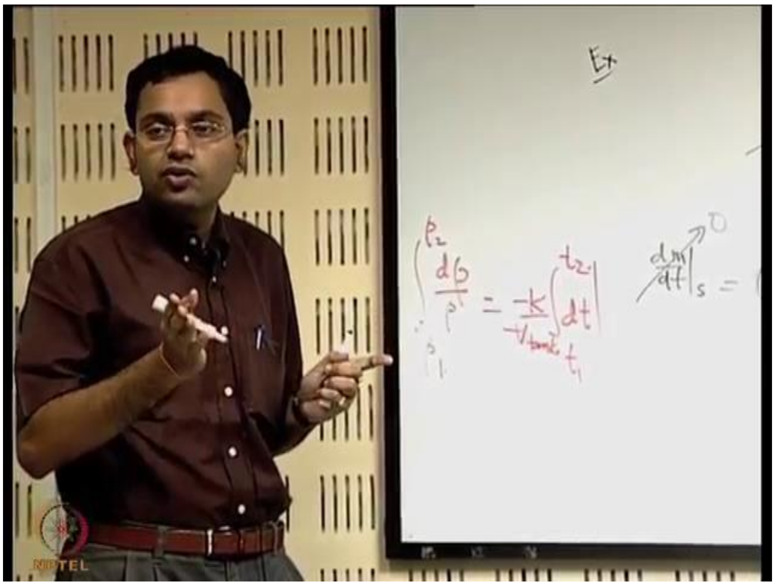

So, it is just a balance between the rate of outflow and inflow, and physically that represents a condition where rate of outflow is equal to rate of inflow. For that we have to keep two things in mind one is that what is the sense of the velocity, velocity component normal to the area is it going, is it opposite to the area vector or is it along the area vector. The second point is, is the velocity uniform over the cross section or is it non-uniform, accordingly we might need to integrate or not.