Introduction to Fluid Mechanics
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Lecture – 42
Application of RTT: Conservation of mass

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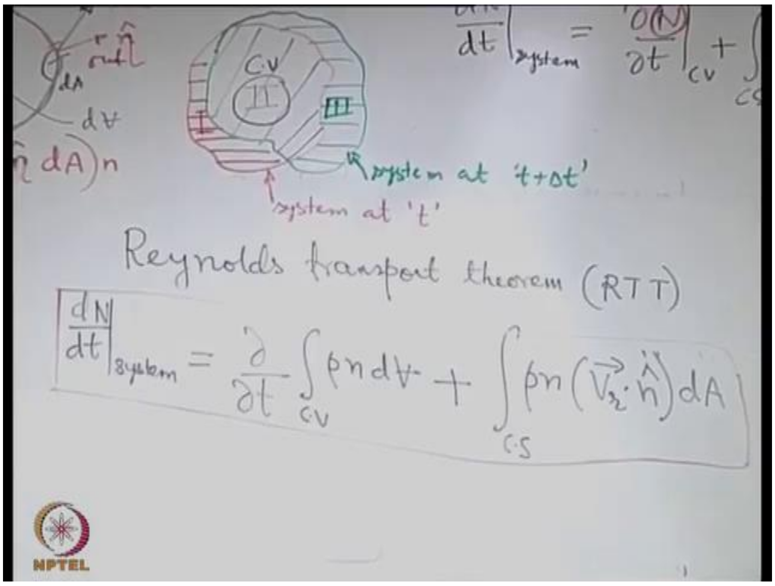

The good thing about this theorem is its generality. Till now we have never committed what is n. So we may try to apply it for different cases with n parameterized in different ways physically representing the different principles of conservation. As an example we will start with the conservation of mass.

So, in the fluid mechanics, we will be discussing about three conservation principles in this chapter conservation of mass, conservation of linear momentum and conservation of angular momentum. So, we will first start with the conservation of mass.

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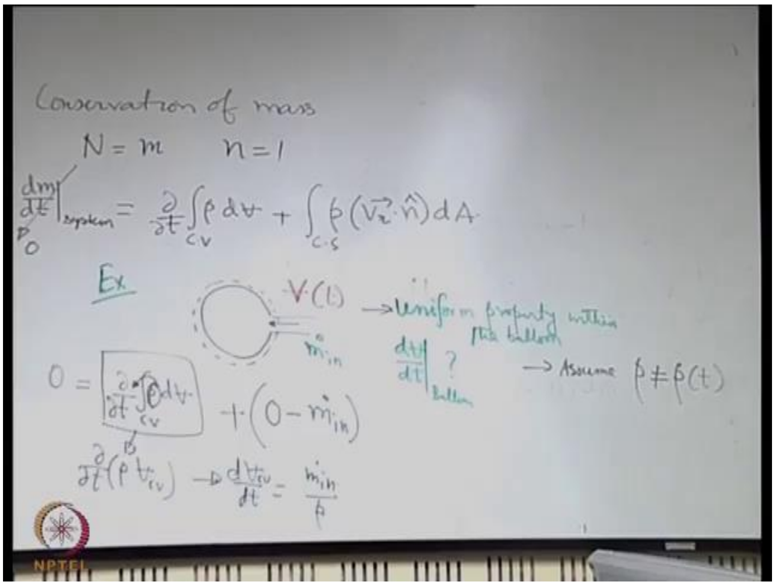

N = m ,   n = 1

$$\left.\frac{dm}{dt}\right|_{system} = \frac{\partial}{\partial t}\int_{CV} \rho d\cancel{V} + \int_{cs} \rho\left(\vec{V}_c.\hat{n}\right) dA$$

$$\left.\frac{dm}{dt}\right|_{system} = 0$$

Regarding the choice of the control volume, we have again remained very general. We have never committed that the control volume is stationary or fixed. So, that even if the control volume is moving it has no consequence, it can be still be applied. The other thing is we have not committed that the control volume is non-deforming.

Let us say that you have a balloon initially the balloon is very small, but you are pumping air in to the balloon. So, it is getting inflated. So, if you consider the control volume like this which is encompassing whatever air is there is within the volume, then that volume is changing with time. So, if you consider that air within the volume as the constitute air within the balloon as the constituent of the control volume and the balloon is being inflated then the volume of the control volume is a function of time.

So, here you have this V as a function of time, this is an example of a deformable control volume. So, if you have a deformable control volume, that is also not ruled out here because we have not committed here that V is not deformable. So, in the most general case V may be moving and V may be deformable. Moving and deformable are two different things when it is moving it need not be deformable. It is like it may be moving like a rigid body, and if it is deformable it might be locally stationary, but deforming.

And when it is moving and deformable, it is the most general case that is it might move as well as continuously deform. So, all those possibilities are there and let us try to see that in such a simple case how we make an analysis. Let us take an example that you have a balloon like this, there is a rate of mass flow of air into the balloon, which is being supplied by the pressurising mechanism. So, some air at a given rate of $\dot{m}_{in}$ is entering the balloon. The state within the balloon is such that you have a density, it is considered that the property within the balloon is uniform.

So, uniform property within the balloon is an assumption. Our objective is to find out what is the rate of change of volume of the balloon with respect to time of the balloon.

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho d\cancel{V} + \left(0 - \dot{m}_{in}\right)$$

As it is a flexible balloon, then whatever mass is coming in this balloon is getting adjusted to that. So, it might be possible that the density is changing, but changing only slightly. If it was rigid that assumption would have been a very bad assumption, but because it is flexible may be it has capability enough to adjust which depends on many parameters.

Assume $\rho \neq \rho(t)$

$$\frac{d\cancel{V}_{cv}}{dt} = \frac{\dot{m}_{in}}{\rho}$$

So, we have got a fair idea that it could be a deformable control volume and if it is a deformable control volume it is not a very trivial thing to deal with.

So, the first assumption may not be bad because if it is pumped well enough it is assumed to be a homogeneous distribution of the density. But this is perhaps not a very correct assumption it may be approximate. So, we will leave apart the deformable control volume and we will try to consider a special case of a non-deformable control volume. So, we will make two important assumptions first assumption is non deformable control volume to further simplify the equation.

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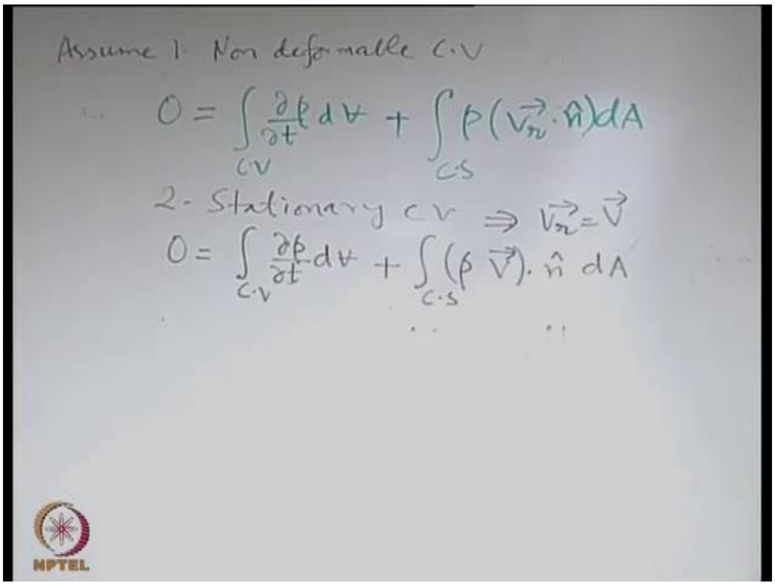

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} d\cancel{V} + \int_{CS} \rho\left(\vec{V}_r . \hat{n}\right) dA$$

Next we will make another assumption the assumption is that the control volume is stationary or fixed.

2. Stationary CV $\Rightarrow \vec{V}_r = \vec{V}$ -

$$0 = \int_{CV} \frac{\partial \rho}{\partial t} d\cancel{V} + \int_{CS} \left(\rho \vec{V}\right) . \hat{n} dA$$

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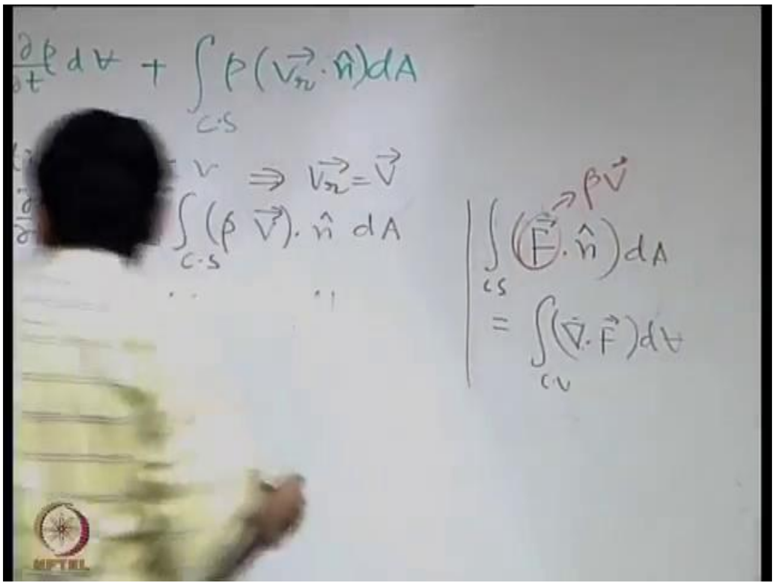

$$\int_{CS} \left(\vec{F}.\hat{n}\right) dA = \int_{CV} \left(\nabla.\vec{F}\right).d\forall$$

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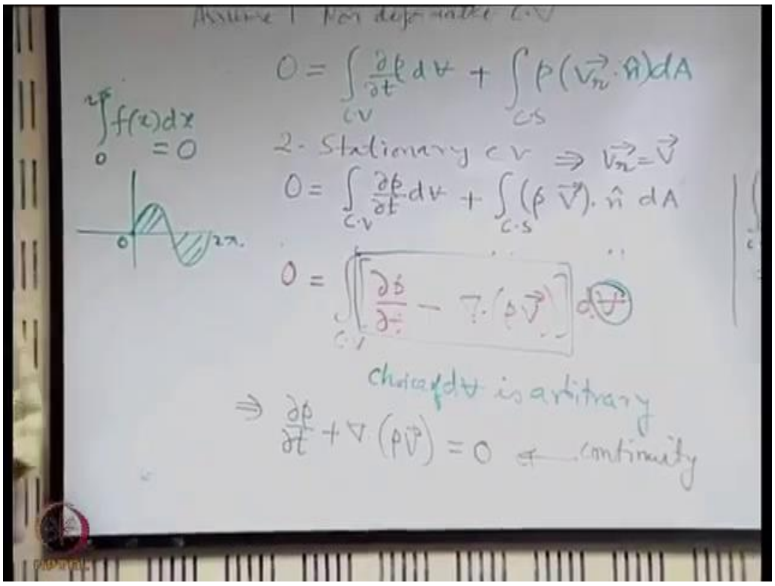

$$0 = \int_{CV} \left[\frac{\partial \rho}{\partial t} - \nabla.\left(\rho \vec{V}\right)\right] d\forall$$

Choice of $d\forall$ is arbitrary.

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

So, it is not for a general like interpretation for any case, but it is an interpretation based on the arbitrariness in choosing the control volume. So, we started with the Reynolds transport theorem integral form and we could show that we can come up with a differential form of the same conservation.

So, this is representing the mass conservation principle, but in a differential form. So, it is possible to convert one form to the other keeping the physical meaning intact. So, that is one of the important strings of the Reynolds transport theorem, that it is possible to derive almost all conservation equations in whatever form by starting with the most general integral form of the Reynolds transport theorem.