Introduction to Fluid Mechanics
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Lecture – 40
Application of Bernoulli's equation-Part – III

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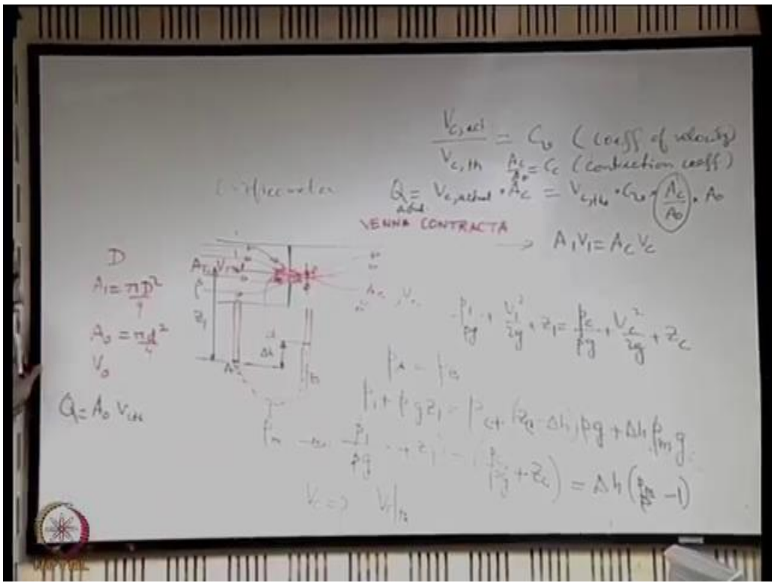

So, Orifice meter is another application of the Bernoulli's equation. So, Orifice meter is something like an the purpose is the same, that you have a pipe line you want to measure the flow rate through the pipe line. If it is a circular pipe, it may be a circular plate with a hole at the center which is called as Orifice. Here, if you consider the streamlines the streamlines were originally say parallel to each other. Not that they always have to be, but just as an example. Now, you know that because of this constriction the streamlines have to be forced to flow through these small section.

So, streamlines will converge like this. And then, when the streamlines pass through this. So, there is also a streamline at the center. So, after the streamlines pass, not that they become parallel; because of the inertial effects, the streamlines go on tending to converge. So, there it is not that after coming out of this they become parallel.

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Actually, we are interested about the pressure at a point which is at the centerline. But at the same section, there may be a difference in this pressures if the streamlines are curved but if the stream lines are parallel, they that will be not. So, the pressure read here and the pressure here will obviously, mean almost the same. Effect of streamline curvature will not change anything. Here also, if you want to utilize the same principle, you should come to a location where thus there is negligible streamline curvature and that is there only at the place when this has come to a minimum. So, if you consider a curve which has come to a minimum, that tangent is parallel to the axis.

So, at this location where the distance between the consecutive streamlines or the extreme streamlines the minimum, here almost streamlines are parallel to each other. So, there is negligible error because of neglecting the curvature of the streamlines at that location and this location where the distance between extreme streamlines is a minimum is known as a Venna Contracta and that is located somewhat away from the orifice. It is not exactly located at the orifice.

Let us say that area of cross section of this is A_c and the velocity of flow through this section entire section is uniform and is equal to V_c. Again, we are assuming uniform velocity profiles which is a deviation from the reality and with such a kind of abrupt change the deviation of from the reality is more severe.

Let us consider A_1 as the area of cross section, $A_1=\frac{\pi D^2}{4}$. Let us say, that d is the diameter of the orifice.

$A_o = \frac{\pi d^2}{4}$ where d is the diameter of this orifice and let us say that V_o is the velocity through the orifice. Let us say, we have a point 1 and a point c. Point c is located on the same streamline as that of one, but in the Venna Contracta section. So, we are writing the Bernoulli's equation between points 1 and c along the streamline.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_c}{\rho g}+\frac{V_c^2}{2g}+Z_2$$

So, let us say that you have the depth of the limb as marked in the figure and let us say that Δh is the difference in the reading of the two limbs of the manometer. So, utilizing the principle of manometry, you can write that if you have A and B as these two points.

$$P_A = P_B$$
$$P_1 + \rho g Z_1 = P_c + \left(Z_1 - \Delta h\right)\rho g + \Delta h \rho_m g$$
$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_c}{\rho g} + Z_c\right) = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\frac{V_{c,act}}{V_{c,th}} = C_v (Coeff.\ \text{of velocity})$$

$$Q_{actual} = V_{c,actual} \times A_c = V_{c,theo} \times C_v \times \frac{A_c}{A_o} \times A_o$$

$$\frac{A_c}{A_o} = C_c (contraction\ \text{coefficient})$$

$$Q_{act} = A_o V_{cth} C_c C_v$$
$$C_d = C_c C_v$$

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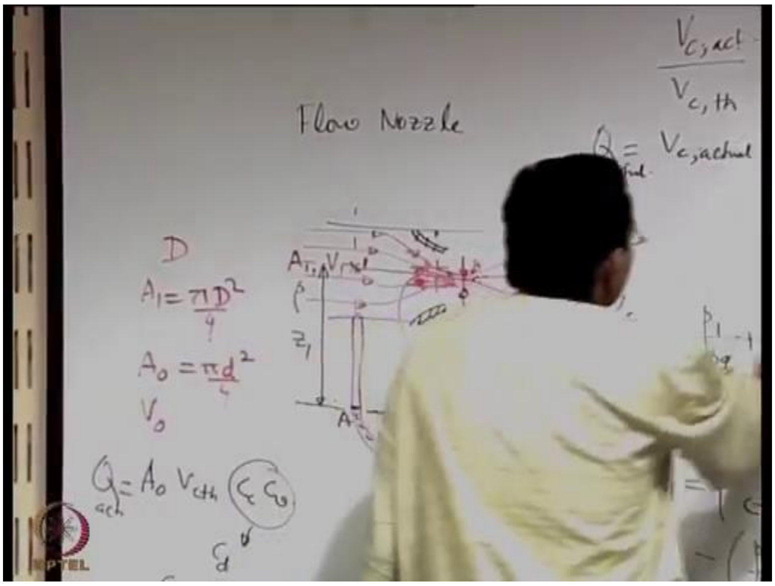

We will just try to go through the philosophy because flow nozzle is something in between the venturi meter and orifice meter. It is the cost is in between the accuracy is also in between. So, what it does is instead of putting a sharp orifice with an abrupt change puts a kind of a nozzle at the wall to have a more gradual change of cross section of the area. It does not make it as good as the venturi meter, but sort of compromise between the venturi meter and the orifice meter.

So, with this kind of flow through the orifice, let us consider a very simple example to illustrate it that in what other conditions these types of concepts of venna contracta also come into the picture. And one such example is something which you have a encountered many times that if you have a tank and if you have a hole through the tank, there is a water jet that goes out.

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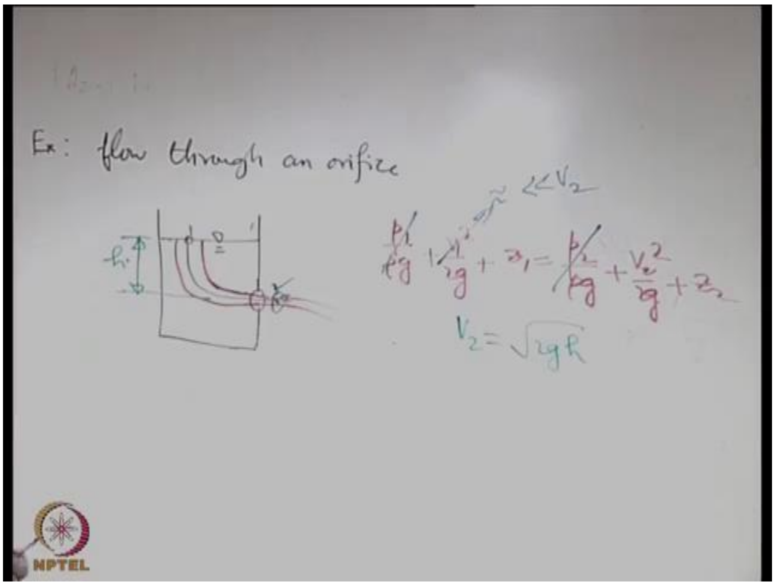

Let us try to draw a stream line. Say let us say, you are coming from the free surface the stream line gets bent or curved to accommodate this one. All the stream lines which are there they are getting bent or curved and just like the previous case, the stream lines come or converge to a location of minimum distance of separation between these two before they diverge and then maybe the water is falling like this.

So, the location where the extreme streamlines come to a minimum distance of separation is somewhere here which is the Venna Contracta here, but not at the orifice. So, this is the place what we are looking for and let us say we identify a stream line from going like this and we want to apply a Bernoulli's equation from in between the points 1 and 2. Assumptions to be taken are i) steady flow; ii) frictionless flow and iii) area of the thickness of the orifice is such that it is much less than the area of cross section of the main tank.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$V_2=\sqrt{2gh}$ (Torricelli's formula)

If there is a serious stream line curvature, then there is no guarantee that throughout 2 pressure is p atmospheric because of the stream line curvature, there will be a difference in pressure as you go across it. Only where it is a Venna Contracta, that is true because stream lines are parallel.

So, there is no normal gradient of pressure across the stream line. So, whenever you have called it that same as p atmosphere, you have to take this section 2 at Venna Contracta.

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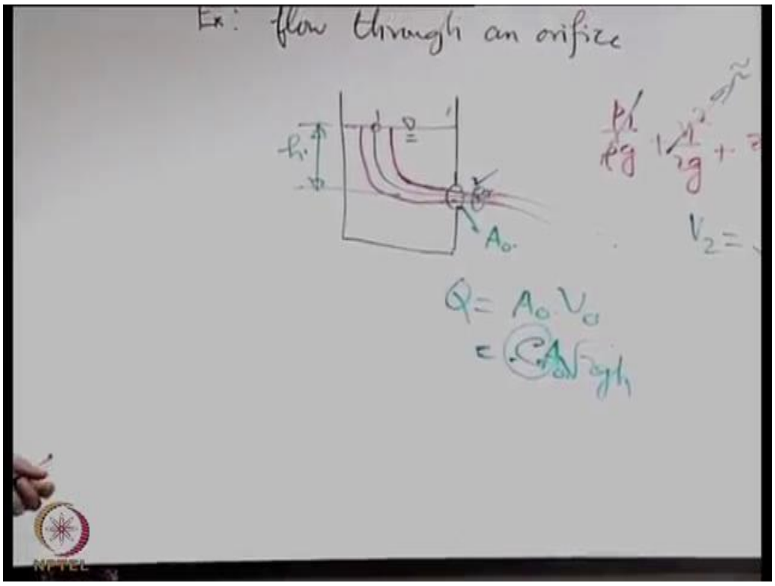

$$Q=A_oV_0=CA_o\sqrt{2gh}$$

Now finally, we will come in to one example where we show the use of an unsteady Bernoulli's equation for a practical device.

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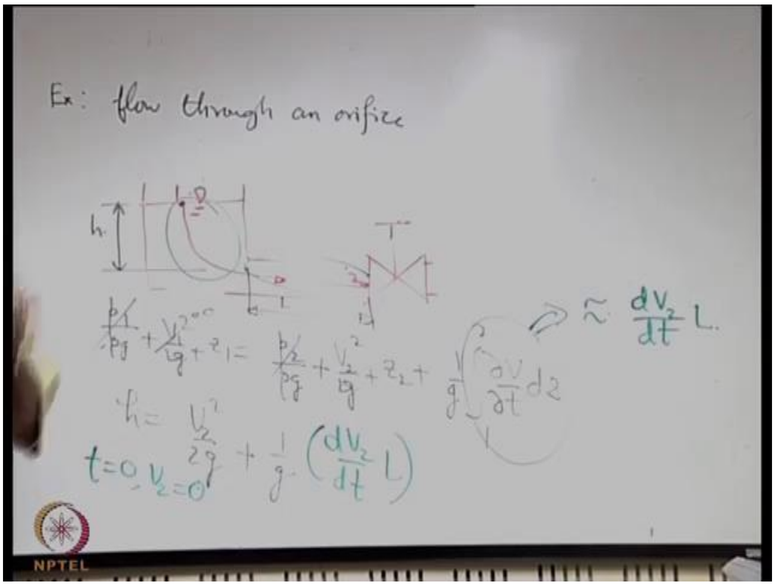

So, let us consider that we have similar arrangement like a tank with a pipeline. In this pipeline, there is a valve. This valve when it is fully closed, it does not allow the water to be discharged through this pipeline. Now, suddenly this valve is made open and water is allowed to flow. So, you have to find out that how the velocity changes with time assuming the flow velocities to be uniform over each section.

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^{\ 2}}{2g}+Z_2+\frac{1}{g}\int_1^2\frac{\partial V}{\partial t}\,dz$$

$$h=\frac{V_2^{\ 2}}{2g}+\frac{1}{g}$$

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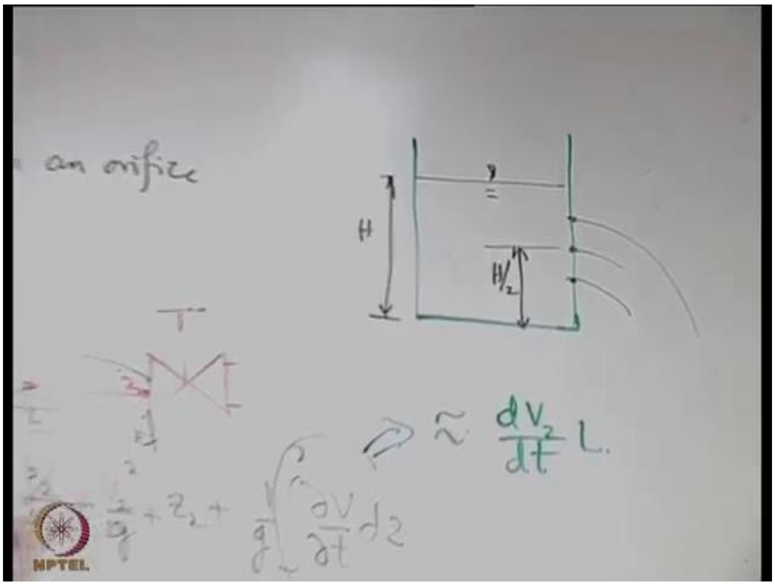

So, you have a tank like this. You have three holes in the tank. If this height is h, the central hole is at $\frac{h}{2}$ and the others are symmetrically located one at the top, one at the bottom. So, when water jets are ejected like this, which one will traverse the greatest distance?