Introduction to Fluid Mechanics

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Lecture – 39
Application of Bernoulli's equation – Part – II

In our previous lecture, we were discussing about the venturimeter and we will continue with that as an example.

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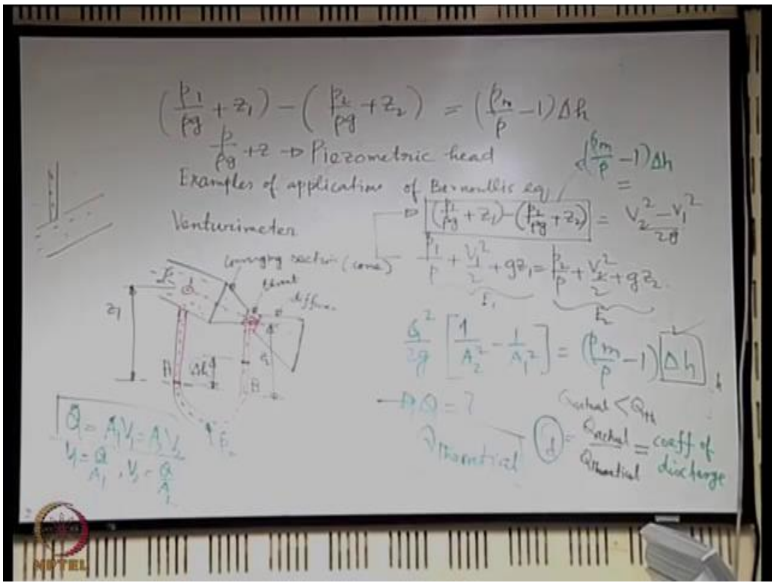

So, if we recall what was the purpose of the venturimeter? The purpose was to measure the flow rate, volume flow rate through a pipe. Now, for that we utilize this type of an arrangement where you have a converging section which is sometimes also called as a converging cone, then you have a throat where the area of cross section of the entire arrangement is a minimum and then a diverging section.

And the purpose is obvious that this part has to fit with the pipe. So, if we have reduced the cross section area somehow you have to increase it, so that it again matches with the pipe. So, this black colored portion is like a fitting which is fitted with the pipe to measure the flow which is occurring through it.

The question is why you have such a converging section; we have seen that it gives rise to an accelerated flow. So, it gives rise to a high velocity and therefore a change in pressure.

So, the difference in piezometric head is reflected by the difference in the heights of the liquid columns in the two limbs. And that Δh therefore will be an indicator of the rate of flow which we found out by utilizing the Bernoulli's equation.

The question is if it a very reliable way of finding out the flow rate. The answer is straightforward in some sense that it is not that reliable. The reason is that we have utilized an ideal type of equation, this all the assumptions which are there in built with the Bernoulli's equation in steady form are inbuilt with this. And therefore, all the idealities which are also inbuilt with this form of equation, those are assumed to hold; at the same time we understand that in practice such idealities do not hold true.

It is never a frictionless flow. Because of the viscous effects it is a representative of the total energy at section 1 and this is a representative of the total energy when we say energy here we only mean the mechanical form of energy at section 2.

E_1 is more, because you expect that there may be a loss of energy. And the loss of energy will be because of the travel of the fluid from the initial point to the final point. So, when traveling from 1 to 2, it will have a loss, so that means, E_1 will be more, so that when you come to the section-2, it is E_2 and some losses. The losses which have taken place between 1 and 2.

If E_2 is greater than E_1, the flow is taking place from 2 to 1. So, it is basically taking place from a high head to a low head; it cannot be the other way. Now, here because of this loss, what will happen see this eventually will boil down to a large, larger drop in the piezometric head. So, if it is flowing from 1 to 2, the piezometric head which is which is coming into the picture, the piezometric head drop that is the difference between the two terms present there in the bracket, this you expect to be more.

Because you expect a more severe drop in pressure because of the overcoming the frictional resistance effects, so that means, whatever Δh you read here it is not the ideal Δh see, in this formula what Δh you put to get the value of the ideal q, this Δh is what you experimentally observe. We have to keep one thing in mind, what is the basic principle of measurements that we use in experiments that we have an expression in which we have certain measurable quantities, certain easily measurable quantities and we express some more some more difficult to measure quantities in terms of the easily measurable quantity.

So, here Δh is something which we measure easily, flow rate you do not measure directly, but you use this formula to write flow rate express flow rate in terms of the measured Δh. So, here may be to get Q ideal, it would have been better if you could put Δh ideal, but that you cannot do because Δh is what you are reading from the practical thing. So, it is giving the Δh which has got manifested because of by considering all practicalities. So, this Δh also considers the practicality that there is some loss of energy to overcome the fluid friction effects; that means, this Δh is higher than what Δh would have been if it were a frictionless flow.

Now, incidentally engineers are such classes of people who are happy to get the final result disregarding maybe some mistakes which have already been done. And then to adjust that mistake let us say that some adjustment factor is put let us say we call our new coefficient $C_d = \frac{Q_{actual}}{Q_{theoretical}}$.

The coefficient is known as coefficient of discharge. So, this takes into account that we have realized that if it is an ideal case altogether, this would have been equal to 1. So, deviation of these from one represents the extent of non-ideality in the flow. And not only non ideality in the flow in general, but more specifically how that non-ideality has got manifested in the prediction of flow rate. So, that means, what is the total influence of this friction in terms of the Δh and what is the influence of the inaccuracy in the velocity distribution that has already got inbuilt in the corresponding expression for energy.

when we will be discussing about turbulent flows, we will see that turbulence is a kind of situation which will create almost a uniform velocity profile over the section, so that we will in some way take care of some of our acts of ignorance in writing or describing the correct velocity profile. But even then let us try to see that at which section, the error will be more severe at section-1 or section-2. To do that let us say that we consider two streamlines which are very close to each other.

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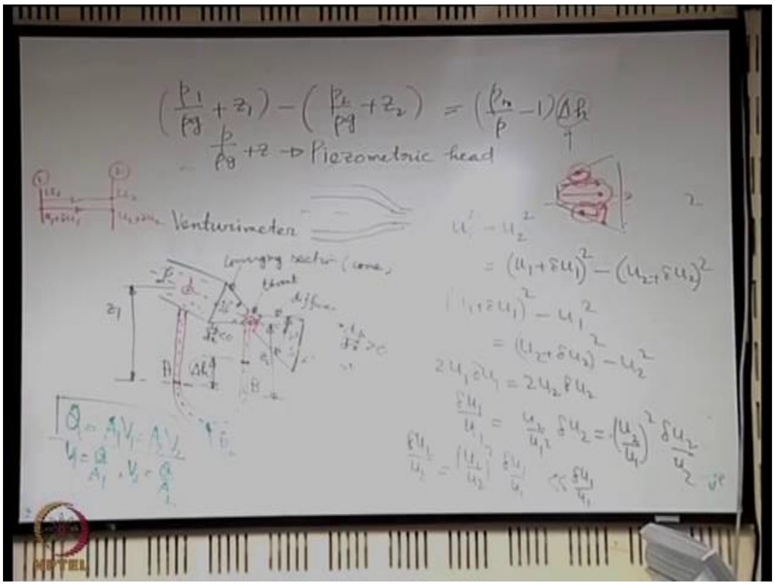

Say you have a stream line like this and another stream line which is very close to each other. Both stream lines are connecting the sections contained by 1 and 2. So, here 1 and 2 are points, but let us say that these are sections which contain the points 1 and 2. So, we have one stream line and we have another stream line. Now, these two stream lines are very close. So, close that let us say that here the velocity is u_1, here the velocity is u_2. Here the velocity is $u_1 + \delta u_1$; and here it is $u_2 + \delta u_2$, where Δis the small change in comparison to the other value.

Now, if this stream lines are very close to each other, what will happen, there will be negligible difference in pressure between these two streamlines. There is a difference in pressure between that stream lines because of the curvature effects of the stream lines, but stream if they are very close that effect is negligible. So, if these two are very close, there is negligible difference in the pressure head between these two stream lines at 1 and at 2. And if they are very close, there is negligible difference in the height also that is the z-coordinate.

So, what we can say is that the difference in kinetic energy heads between the two points, it remains same for the stream line above and the stream line below, because the other terms they do not change.

$$u_1^2 - u_2^2 = (u_1 + \delta u_1)^2 - (u_2 + \delta u_2)^2$$

$$(u_1 + \delta u_1)^2 - u_1^2 = (u_2 + \delta u_2)^2 - u_2^2$$

$$2u_1\delta u_1 = 2u_2\delta u_2$$

$$\frac{\delta u_1}{u_1} = \frac{u_2}{u_1^2}\delta u_2 = \left(\frac{u_2}{u_1}\right)^2 \frac{\delta u_2}{u_2}$$

$$\frac{\delta u_2}{u_2} = \left(\frac{u_2}{u_1}\right)^2 \frac{\delta u_1}{u_1}$$

If it is approximately uniform at section-1, it will be even better at section-2, because the non-uniformity is much less. What, this represents a non-uniformity, when you go to a different streamline along the same section, you expect the velocity to be different and that difference give rise to a non-uniformity.

So, when you have this non uniformity, but again see this is an estimation, because for estimating the non uniformity we have again utilize the ideal equation which is like the Bernoulli's equation. But what we have considered that even for a non-ideal case this is not very very invalid because whatever is the frictional effect that also has got cancelled out when you have subtracted the two equations.

Assuming that the frictional effects are also same as the fluid flows from 1 to 2 along the two stream lines above and below. So, even if frictional effects are considered and these the Bernoulli's equations for the two or the modified Bernoulli's equations considering the frictional effects, they are canceled or they are subtracted one from the other that effect will be cancelled. So, this is not a bad estimation.

So, this estimation shows that if the velocity is such that you are going towards a cross section of reduce size if at the bigger cross section the velocity was more or less uniform, the smaller cross section it is expected to even be even more uniform. The reason is quite clear that if there were stream lines like these, stream lines will more converge to each other, because they are now confined to be there within a very small space as compared to how they were earlier. So, if the streamlines were quite a large distance apart, so if the streamlines were like this now when there are stream lines are confined, so what will happen all the stream lines will try to converge.

So, by having a section two like this which is like a convergent section, it is not bad. It sort of eliminates one non-ideality. The other non ideality because of negligible friction that may be reduced to some extent by minimizing the length travelled between 1 and 2, because the frictional resistance will be related to how much length the fluid has traveled against the viscous effects.

Now, there are different issues like you cannot make it as large angle as you like there are issues of manufacturing the device and so on. So, it is not that whatever angle you want and you propose, one has to also fabricate it and put it in practice. One particular aspect on which one may not make a compromise this is like by putting by locating this section 1 where you are having this manometer limb. It should be preferably somewhat away from the place where the reduction has started, so that this disturbance is not influencing the velocity at this point significantly and that is why it is kept a little bit away from this one.

So, the experimental objective is the Δh is if the Δh is more it is better because that is your reading; if it is very small your error in resolution will be affecting your results significantly. But if the readability of this is good then the error corresponding error is less and that is why you are trying to trying desperately to reduce the cross section area, so that there is a change in the kinetic energy head very severely which is manifested in terms of this Δh. Now, after this section has come, and then what you have to do then you have to revert back to the pipe diameter again. So, you have a diffuser which is like a diverging section.

There are two conflicting requirements engineering is such an area where when you want to design something, there are two aspects that you have to keep in mind. One is it should satisfy the fundamental scientific requirements, so that the device is based on a thorough scientific principle; the other important thing is that it must optimally satisfy the performance requirements.

So, in a relatively short length, this device will merge with the pipe. So, the loss due to frictional resistances will be less.

$$\frac{dP}{dx} > 0$$

So, at those locations, what will happen is the flow may not be capable enough of being dragged with the main or the core flow because it is slowed down so severely that it just creates a local rotation, but it does not contribute to the main flow, so that type of thing is called as a flow separation .

So, local vortices are created close to the wall how do these vortices contribute they contribute in a sort of negative way. See, these vortices by virtue of the rotation have some energy, but that energy is not contributed to the main flow. The main flow is like this which is moving. Now, here this energy which is there because of the rotation of these a vortices because of flow separation that does not contribute.

So, effectively as if some energy is taken away from the main flow to sustain the rotation of these vortices. So, effectively there is a kind of loss of energy of the main flow and that loss has been created because of this flow separation. And this flow separation effect is stronger more is this angle of diffuser. The reason is more is this, more severe will be the adverse pressure gradient, because more severe will be the pressure increase over a given length the length becomes smaller.

So, this is a conflict with the requirement of the frictional resistance. So, we have seen that if you increase the length of this one or maybe reduce this angle, then this effect will be less. So, the adverse pressure gradient effect will be less if you make this angle quite small, so that this length is large, but if this length is large the direct frictional resistance will be more. So, these two are two conflicting parameters in the design that is where you have to come to an optimal design where you cannot keep this angle may be as large as this. And the common optimization is that this is typically like 5^0 , 6^0 much less than the angle of the converging section.

So, this is something we have to understand very carefully that why in the practical design the diffuser angle is much much less than the converging section angle. When you have the converging section, you do not have such a case of flow separation. So, only the frictional resistance is because of the length is the only important resistance, because flow separation will be there when the flow is decelerated, but in the converging section the flow is accelerated. So, it does not suffer from a resistance because of adverse pressure gradient in fact the pressure gradient here is favorable, which makes it move in a much more convenient manner.

So, the design aspects are quite clear that why you should have different angles for converging and diverging sections and what are the parameters we should decide the range of these angles. So, and keeping these things in mind one may if one designs this device quite well by minimizing the losses, then the coefficient of discharge which is the ratio between the actual flow rate and the ideal flow rate, it is actually very close to 1, 0.98, 0.97 like that. So, somehow the device is very cleverly designed, some of the non-idealities are taken care of in some way not that it becomes ideal, but our ignorance about non ideality does not get manifested so much.

The reason is that one is you are using a like a continuously converging section in this way and the diffusing section is also say properly well designed. Now, this venturimeter is therefore, a very common device which may be used in a pipeline to measure the flow rate. At the same time, this is not a very inexpensive device, it is not very highly expensive, but at the same time for very routine applications one might look for some cheaper device which are broadly following the similar principles.