Introduction to Fluid Mechanics
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Lecture – 38
Application of Bernoulli's equation–Part-I

Now, the next topic that we are going to discuss in the context of these Bernoulli's equation is the use of such equations. See the Bernoulli's equation has been one of the very popular equations in fluid mechanics, not just because of it is simplicity, but because of it is applicability in an approximate sense in terms of quantifying the nature of or the principle of working of many engineering devices and we will look into such examples of Applications of Bernoulli's equations.

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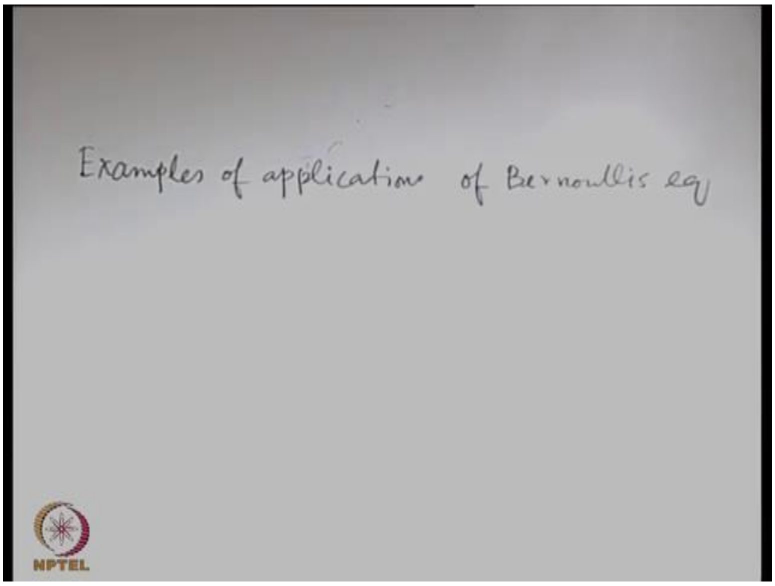

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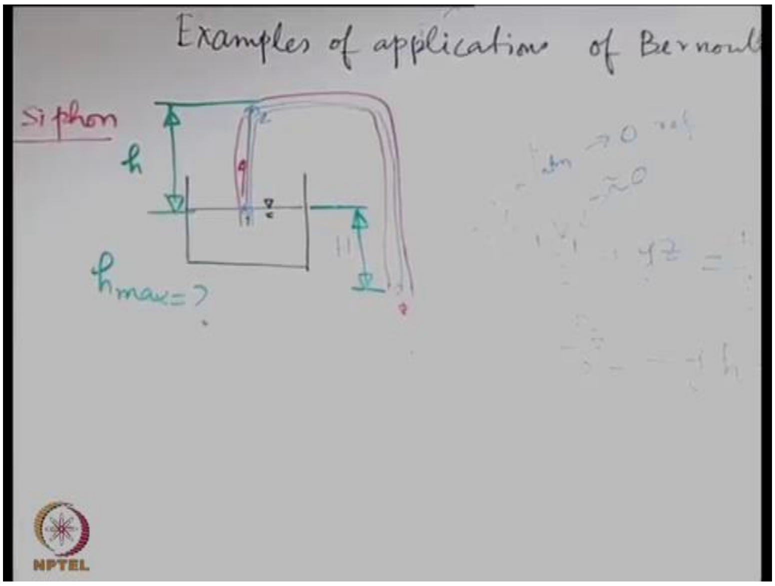

So, if you have say water in a tank like this and you are having a bend tube which is a sort of sucking water and ejecting water to a different place from the tank. So, this is called as a Siphon. The apparent amazing feature of the siphon is out of nothing, it is pulling the water in the upward direction. That is the apparent amazing feature. But if you look into it a bit carefully, it is not at all any amazing feature because eventually when it is discharged, it is discharged at a level below. So, the actual head difference which is working on it is this one which is a favorable one because effectively it is coming from this elevation to this elevation and this net elevation difference is actually giving it a velocity.

So, with that velocity, the water is being sucked. So, the fact that it is going up is nothing very special because eventually it comes down and it gets ejected from a height which is less than or below the level of the tank. But the good thing is that while doing it, it can traverse a vertically upward distance. Question is how much distance it can vertically traverse? So, what should be this say if we call this as h, then what is this h max? This is given by a practical consideration. Let us try to identify a stream line which connects the points say the stream lines will be bend like this.

But let us just consider a stream line which is confined between that points one and two which are almost like a located on a vertical line.

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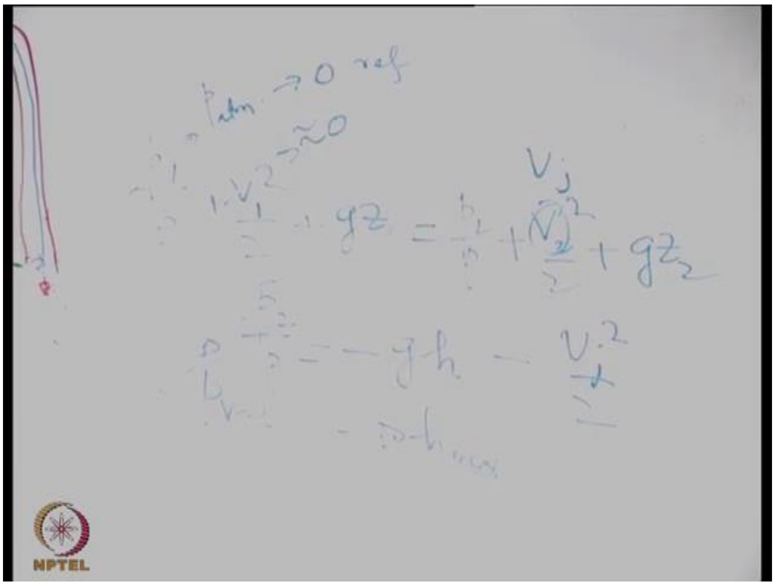

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gZ_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gZ_2$$

The area here is so large that the velocity with respect to which this level is changing is very small as compared to the velocity here; V_2 is same as the velocity at which the jet is ejected here if the area of cross section remains the same. So, V_1 is small because A_1 is large as compared to the area available at 2. Then, V_2=V_j that is the velocity at which the jet is coming out if the cross section is same. $V_j=\sqrt{2gH}$

$$\frac{P_2}{\rho}=-gh-\frac{V_j^2}{2}$$

So, if it is below atmospheric, it may come to a state when it comes to the vapor pressure local vapor pressure. So, when the pressure falls below the local vapor pressure, then what happens, then vapor bubbles are found. So, when the vapor bubbles are found, it is it is nothing very special that vapor bubbles are found. But what is special is that when these vapor bubbles are transported or moved to a different place where the pressure is again higher, they will collapse again to form a liquid. And once they collapse, what happens basically, then they were occupying a large volume. But when they collapse again to be converted to liquid, again there is a volume change.

So, it creates an unsteadiness in the flow and it can create a lot of vibration and noise and that is not so good for the flow and that type of phenomenon is known as cavitation. We will see in details what is cavitation when we will be discussing about the fluid machinery which will be our last chapter in this particular course. So, we will not go into the details of like what is cavitation at this tail, but we have to keep in mind that it is better if we keep the pressure at to below the local vapor pressure that is below the vapor pressure which should be there at that corresponding temperature so that vapor is not formed. So that means, we are keeping a restriction that P_2 must be less than the vapor pressure at that local temperature of the fluid.

So, the siphon in principle may be designed to be very like tall in height in terms of this bent tube. But in practice, one should not make it too tall because if you make it too tall, it is possible that the pressure is so low that vapors are found and that can create other disadvantages in terms of operation of the device.

The next application when we will consider, we will keep in mind that now whatever applications we are going to study our objective will be to have the Bernoulli's equation utilized in devices through which we are interested to measure the velocity or the flow rate in a say pipeline.

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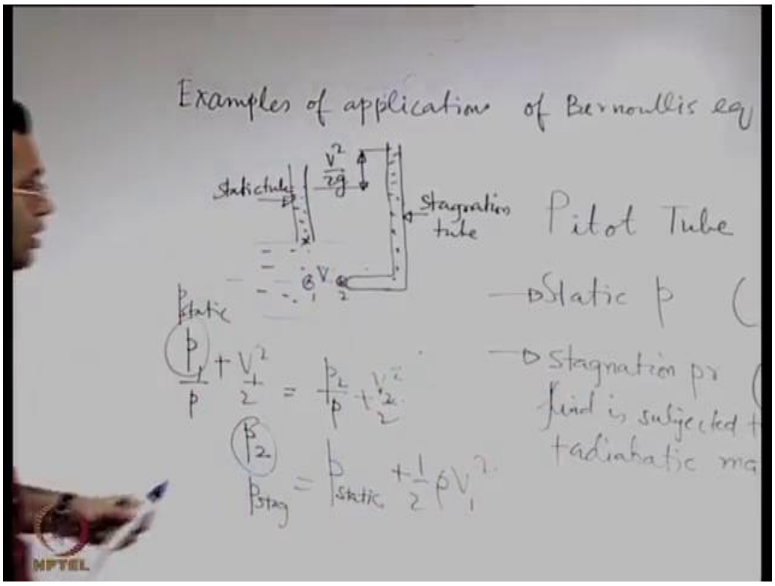

So, let us take an example. Let us say that you have a pipe like this, a horizontal pipe. The water will rise and it will come to a height. The height with respect to which the water rises will be an indicator of the local pressure at that location. Pressure at where see we are interested about the center line.

At the same axial location, we are probing at a point which is different from the center line and we know that it is very much possible that the pressure at the centre line should be different in general from pressure at this. But here if we consider that the stream lines are parallel to each other, then you do not have that effect of the stream line curvature in terms of the pressure gradient.

So, whatever is the pressure here, should be the same as the pressure here. So, then this is an indicator of the local pressure. Now, say we are in we are interested to have an indication of the velocity. We can have another tube where we make a penetration in the wall but before that we have the tube directly confronting with the flow. So, this tube and this tube is different. This is not directly interfering with the flow but this is directly interfering with the flow. When it is directly interfering with the flow, it is bringing the flow to a standstill or a dead stop.

So, it is creating like a creating like a stagnation point where the flow comes to a dead stop it, it cannot go further. So, whatever water was coming here, it comes to a dead stop. It will enter, it will rise through the tube and the question is will the rise will be greater than this one or less than this one? See, this rise was the function of the pressure, now what. So, the entire energy which was there in the flow, if we assume that assumptions of the Bernoulli's equation those are valid now we have made the kinetic energy to 0.

So, the entire energy, now contribution of pressure term plus the kinetic energy term will be successful to make it go further up because that where mean that energy go you have made the fluid to a dead stop, you are assuming that it is a frictionless flow. It will obviously, make the fluid rise to a greater height and the difference between these two heights is if these points are very close to each other the pressures are almost the same, the difference between these two heights is $\frac{V^2}{2g}$.

So, from this principle, V is the velocity of flow at this point. So, from this principle, it is possible to make an estimation of the velocity and if you know the estimation of the velocity and if we assume it to be uniform, then you can also have an estimate of the flow rate. Now, if it is not uniform, you can keep it at different radial locations and you can even find out how velocity varies radially because this tube you can put at different radial locations. So, this is put at r = 0 at the center line but you can also keep it away from the center line.

So, at different radius if you put, it will give you a picture of velocity at different radius. So, it is possible even to get a velocity profile if this is quite accurate. Of course, there are many doubts about the accuracy of such a simple arrangement, but it gives us a conceptual understanding. So, the device which is based on this conceptual understanding is known as a Pitot tube. And it is a very simple device and the working principle of this device is based on two important definitions which we will tell now.

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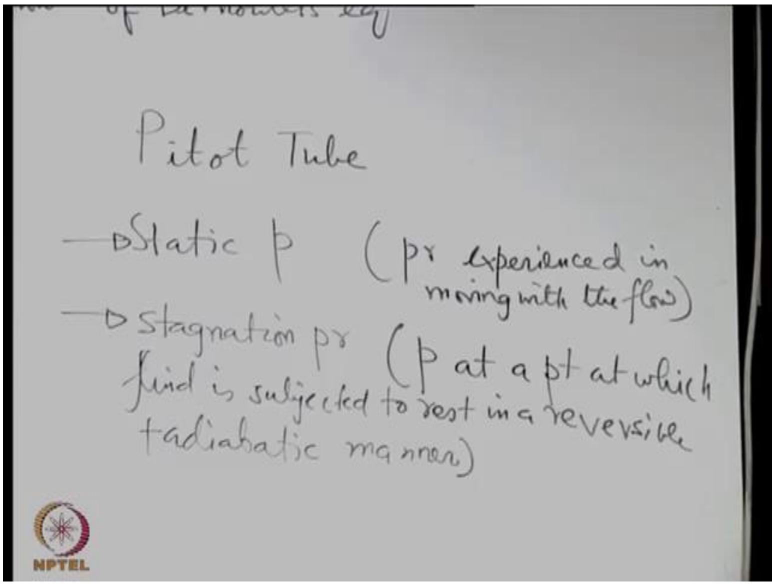

One is known as static pressure. Static pressure is the pressure which is there because of the intermolecular collisions. So that means, if one is moving with the flow, then what is the pressure felt because of just moving with the flow is the static pressure. So, this is the pressure experienced in moving with the flow. So, this is the result of the intermolecular collisions and this is the pressure that we fundamentally define. Now, we are also going to define something called a stagnation pressure.

Stagnation pressure is the pressure that is there at a point if the fluid is subjected to 0 velocity at that point in a reversible and adiabatic manner. So, pressure at a point at which fluid is subjected to rest in a reversible and adiabatic.

So, that means, when the fluid is subjected to rest at a point, you have to make sure that it is subjected to rest in a frictionless manner. So, whatever is the pressure that this tube is getting is the stagnation pressure. So, this is also known as a Stagnation tube because it is reading gives an indication of the stagnation pressure and this is known as a static tube. So, you can if you want to write the Bernoulli’s equation between two points, 1 and 2 which are located in such a closed manner.

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}=\frac{P_2}{\rho}+\frac{V_2^2}{2}$$

V_2 is 0 because it is a stagnation point. So, the definition of the stagnation point is velocity is 0.

$P_2 = P_{static}+\frac{1}{2}\rho V_1^2$. P_2 is $P_{stag.}$

Stagnation pressure is a sort of property of the flow if you know the velocity of flow.

So, the definition of the stagnation pressure is to be kept in mind stagnation pressure is not just pressure at a stagnation point. The stagnation point is a point where you have 0 velocity, but it does not mean that pressure at that point is a stagnation point. Pressure at that point will be a stagnation pressure only if the flow is subjected to rest in a frictionless manner. Because the stagnation pressure is defined in that way, it is not just sufficient, it is necessary that you must have the velocity to be 0 at that point so that the pressure measured the stagnation pressure. But at the same time, it is not velocity subjected to 0 in any way.

But it is subjected to 0 in a frictionless way. The second important thing is since these two points are very close to each other and you can just say stagnation point a stagnation pressure at a point just as a property which is depend which is dependent on the local velocity. So, stagnation pressure need not always be measured through a stagnation point.

Now, the next we will discuss one or two important flow measuring devices. And the first device that we will discuss is known as a Venturi meter.

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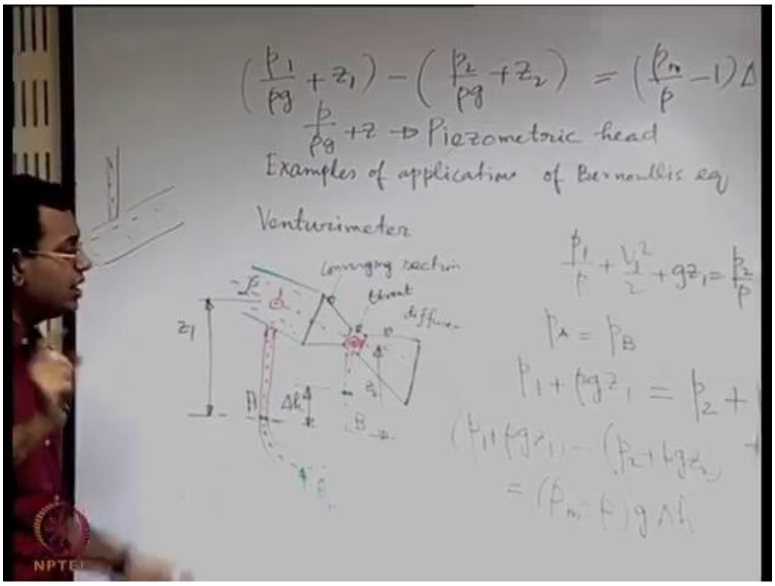

So, what is the venturi meter? Say you have a pipeline and you are interested to measure flow through a pipeline. Say you have a pipeline like this, you want to measure what is the rate of flow through the pipe.

So, how will you do it? There are many ways in which it can be done it. One of the ways is by utilizing a device called as Venturi meter. So, what is done? A part of the pipe is like replaced with a device. So, you have an accelerating section by having a converging cone. Then, you have a section of uniform zone of uniform cross section and then, you again come back to the pipe dimension. So, this is known as diffuser. This is known as a throat and this is the converging section. The objective is by this way you are reducing the cross sectional area.

So, to maintain the continuity in a steady state, you are what you are doing. So, if you consider now the points, let us say that you consider points 1 and 2. The point 1 was having the velocity as same as that of the velocity of flow in the pipe. Now, at the point 2, the velocity will be more or less. It will be more because the area of cross section has reduced. Now, let us say that you make a tapping of a manometer; that means, let us say that you consider a hole in the pipeline and a hole here and connecting that with a manometer.

So, when you are connecting that with a manometer see, we have not taken the point 1 at the inlet of the converging section. But at some location, which is sufficiently away from that because here the stream line curvature effect will tend to become more and more dominant. So, you want to take it away from such a place where the streamlines are almost parallel to each other. So, pressure at this point and maybe pressure at this point should not be very different because of the streamline curvature effect. So, we are having a manometer in which we have a fluid. Let us write the equation, the Bernoulli's equation along a streamline between the points connecting the points 1 and 2.

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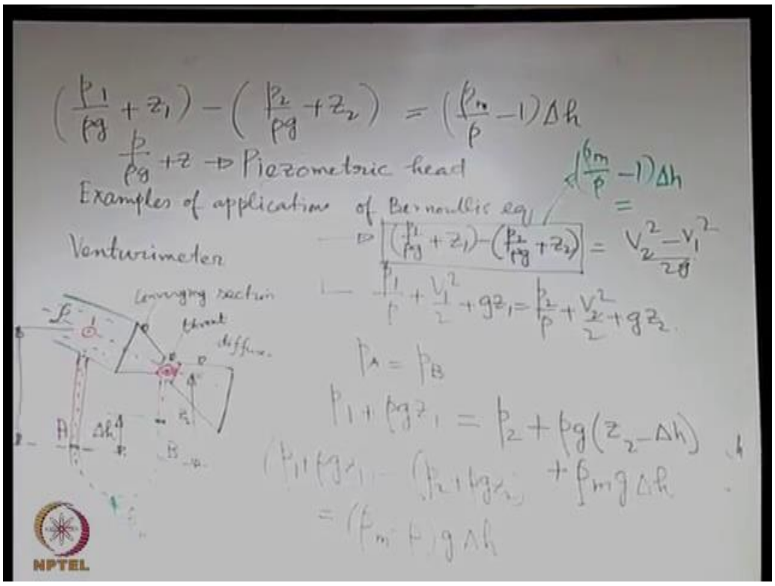

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gZ_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gZ_2$$

So, let us write let us say that this is the difference in height that we measure say that is equal to Δh.

$$P_A = P_B$$
$$P_1+\rho g Z_1 = P_2+\rho g(Z_2-\Delta h)+\rho_m g\Delta h$$

$$(P_1+\rho g Z_1)-(P_2+\rho g Z_2)=\left(\rho_m-\rho\right)g\Delta h$$

$$\left(\frac{P_1}{\rho g}+Z_1\right)-\left(\frac{P_2}{\rho g}+Z_2\right)=\left(\frac{\rho_m}{\rho}-1\right)\Delta h$$

If you say have a pipe and if you puncture the pipe or if you penetrate the pipe say and if you have a tube through which the water goes up, it is just like that static tube that we considered in this in the previous example. Then, the elevation that it assumes here is the elevation because of it is vertical location plus because of the pressure and static pressure at that point and this tube is commonly known as a piezometer tube. So, that is why the name piezometric head. So, in the manometer in this kind of an example, we do not measure the pressure difference, but we measure piezometric pressure or piezometric head difference.

$$\left(\frac{P_1}{\rho g}+Z_1\right)-\left(\frac{P_2}{\rho g}+Z_2\right)=\frac{V_2^{\,2}-V_1^2}{2g}$$

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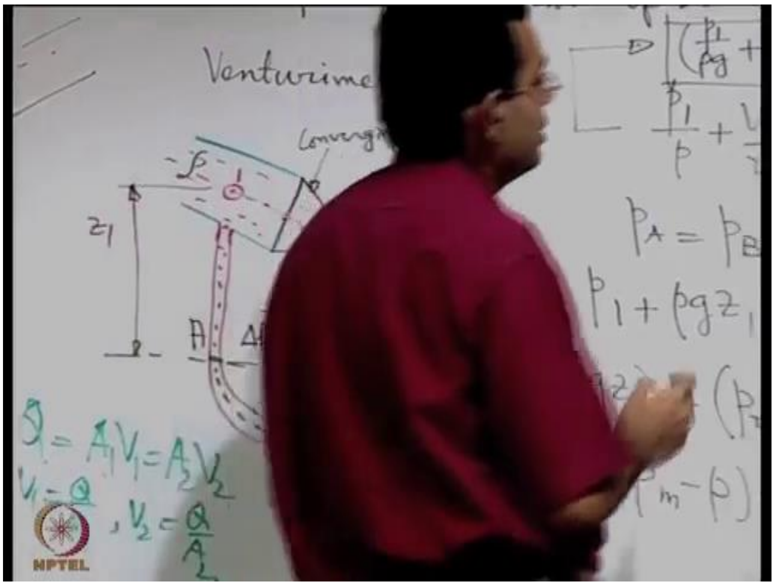

$$\left(\frac{P_1}{\rho g}+Z_1\right)-\left(\frac{P_2}{\rho g}+Z_2\right)=\left(\frac{\rho_m}{\rho}-1\right)\Delta h$$

$$Q=A_1V_1=A_2V_2$$

$$\frac{Q^2}{2g}\left[\frac{1}{A_2^{\,2}}-\frac{1}{A_1^{\,2}}\right]=\left(\frac{\rho_m}{\rho}-1\right)\Delta h$$

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