Introduction to Fluid Mechanics
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Lecture – 37
Problems and Solutions

We continue with the example that we were discussing in the last lecture that there is a tank and through the bottom of the tank the water is being drained out, and the height of water in the tank is therefore, changing with time. Our objective is to find out how the height changes with time.

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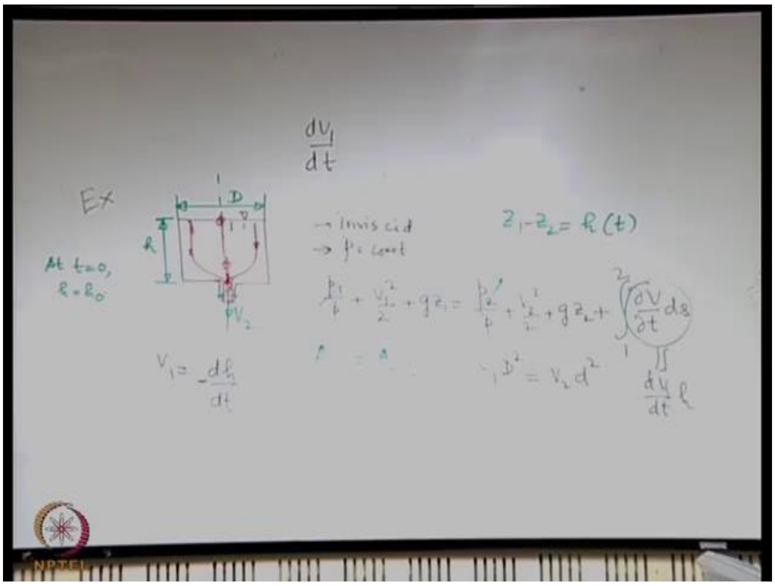

Now, we were discussing about what is the significance or impact of the unsteady term that is retained or that should not be retained or should be retained is our doubt in the Bernoulli's equation.

In fact, you can have a large number of streamlines their envelope may look like an imaginary pipe or a tube that is known as a stream tube. So, it is a collection of streamlines making an imaginary tube within which the fluid is flowing. So, if you consider such a tube you can always see that the extent of that tube that remains almost the same, till you come to the exist; where it is really accelerating because now the area available to it is so small that it has to get adjusted to itself.

$$\int_1^2 \frac{\partial V}{\partial t} ds \Rightarrow \frac{dV_1}{dt} h$$

$$\frac{V_2{}^2 - V_1^2}{2} - g(Z_1 - Z_2) = -\frac{dV_1}{dt} h$$ and $(Z_1\text{-}Z_2)=h$

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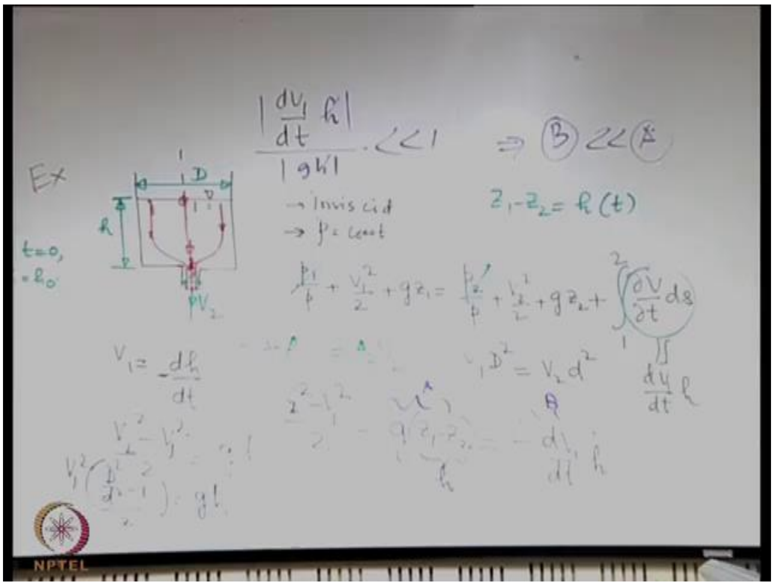

$$\frac{\left|\frac{dV_1}{dt} h\right|}{|gh|} << 1$$

$$\frac{V_2{}^2 - V_1^2}{2} = gh \Rightarrow \frac{V_1^2\left(\frac{D^2}{d^2} - 1\right)}{2} = gh$$

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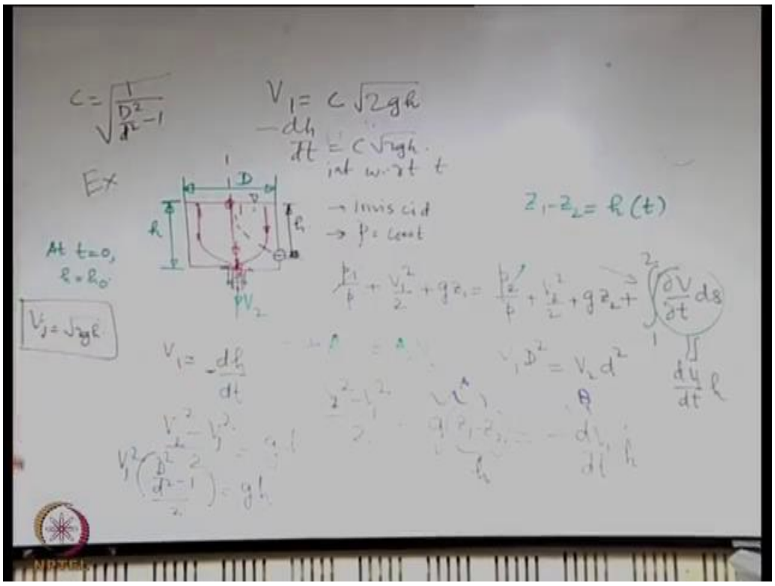

Let us say that now this height is h which is changing with time. So, there is no hole here but there is some hole here. There is a nozzle that is fitted and water is coming out, ok. So, when you are doing that the way in which most of you have done is like you have assumed the velocity $V_2 = \sqrt{2gh}$ this is known as Torricelli's formula.

The other approximations are that you are having a stream line like this with respect to which you have the points 1 and 2, and the unsteady term does not appear in that analysis and it is assumed to be an inviscid flow. The greatest deviation from reality is because of the assumption of the inviscid flow. So, that is one of the very important features that we have to keep in mind.

We will consider another example in the unsteady Bernoulli's equation in the use of the unsteady Bernoulli's equation that is given by the next problem.

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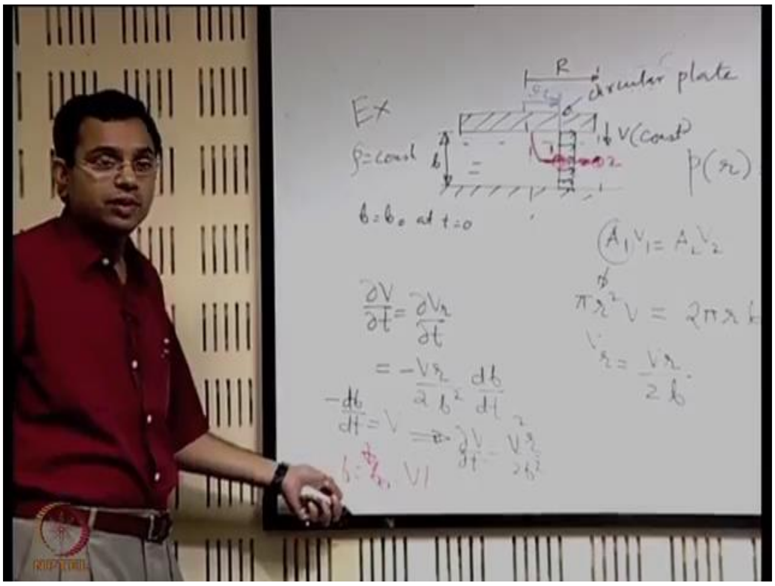

Let us say that you have two plates, these are circular plate. We have solved problems with rectangular plates just for a change let us consider that it is a circular plate. So, this plate is coming down with a uniform velocity V.

The radius of the plate is R and say we are considering a coordinate system the local coordinate as r. So, r is the local coordinate at a radius R. Height of water, $b=b_0$ at $t=t_0$.

The water will be squeezed out radially to make sure that the continuity is maintained. So, we are interested to find out how the pressure varies with r. So, if it is a invisicid flow the velocity variation over the cross-section is not there, so that velocity is uniform over each section but this uniform velocity is changing with radius.

$$A_1V_1 = A_2V_2 \Rightarrow \pi r^2 V = 2\pi rb.V_r$$
$$V_r = \frac{Vr}{2b}$$

So, let us take any two points located on the stream line and write the Bernoulli's equation between those two points located on this identified streamline. But because it is an unsteady flow we need to retain the unsteady term in the Bernoulli's equation.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2 + \int_1^2 \frac{\partial V}{\partial t} ds$$

P_2 is the atmospheric pressure .

$$\frac{P_1 - P_a}{\rho} = \frac{V^2}{4b^2}\left(R^2 - r^2\right) + \int_r^R \frac{V^2 r}{2b^2}\,dr$$

$$\int_r^R \frac{V^2 r}{2b^2}\,dr \to \frac{V^2}{4b^2}\left(R^2 - r^2\right)$$

$$\frac{\partial V}{\partial t} = \frac{\partial V_r}{\partial t} = -\frac{Vr}{2b^2}\frac{db}{dt}$$

$$-\frac{db}{dt} = V$$

$$\frac{\partial V}{\partial t} = \frac{V^2 r}{2b^2}$$

$$b = b_0 - Vt$$