Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 35
Euler's equation in streamline coordinates

We are now going to discuss the Euler's equation of motion in streamline coordinate system.

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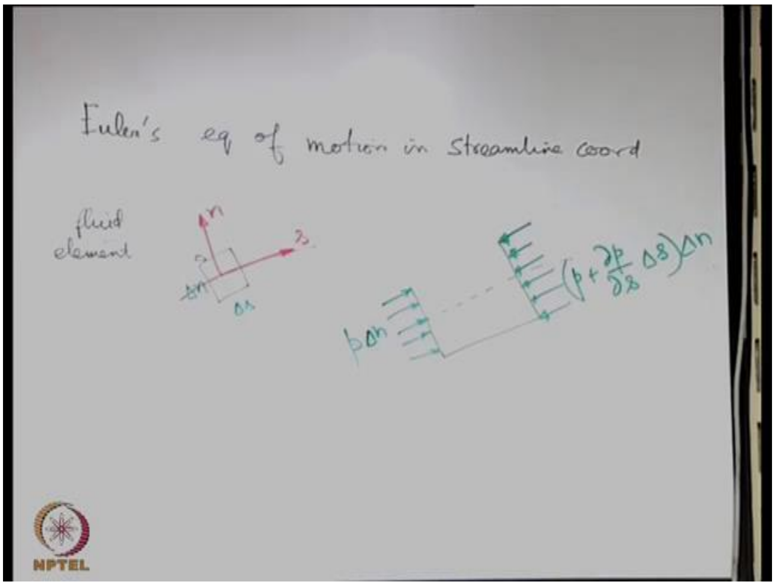

So, let us consider a streamline coordinate system as we were discussing in the previous class. Euler's equation of motion in stream wise coordinates or streamline coordinates. So, we consider that there is a streamline and we consider a coordinate system such that you have a coordinate system s & n, where s is given by the stream wise coordinate.

Fundamentally it is not exactly the tangential coordinate, but it is oriented just along with the streamline but effectively it is just like a tangential coordinate. The tangential coordinate is a coordinate along direction which is given by a slope and this is along a direction given by the curve. So, it moves it is a coordinate system that is aligned with the streamline itself. But locally it is as good as like a tangential coordinate and n is the normal coordinate.

So, in this coordinate system let us say that we have a small element of fluid like this. So, this is a fluid element, but this fluid element has a specialty. We have now considered this fluid element to be sort of coaxial with the streamline at a given location. Let us try to identify all the forces which are acting on the fluid element we separately draw it for clarity.

So, let us say that we have a fluid element like this and let us consider that the centerline of the fluid element is such that it is representing the streamline locally. Now we are going to identify all the forces which are acting on this fluid element. Again, there are forces which may be resolved in the coordinate directions s and n. we will first write the equation of motion along s.

So, we will only identify the forces which have components along s. So, we consider the left face; where you have a pressure distribution. If p is the pressure let us identify the element by it is dimensions along n and s say Δs and Δn. So, $P\Delta n.1$ is the width perpendicular to the plane of the figure. Then there will be a pressure distribution here $\left(P+\frac{\partial P}{\partial s}\Delta s\right)\Delta n$.

The most fundamental assumption that we are making that it is an inviscid flow.

The weight of the fluid element should have a component along s. So, let us identify that let us say this is the weight of the fluid element. And let us say that it makes an angle θ with the direction of s. The weight of the fluid element is $\rho\Delta s\Delta ng$

One important thing is although this is a curvilinear coordinate system remember this is not a rectangular coordinate system is a curvilinear coordinate system. But when you take small elements this almost behaves like a rectangle although it is a curvilinear coordinate system. That is one of the advantages of taking a small element.

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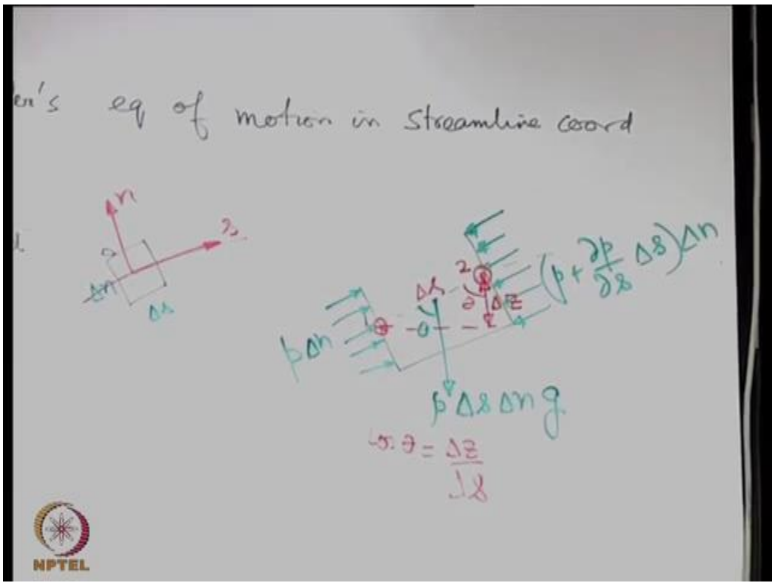

Now, we can write this component of this force as you can see from this figure. The component of the force along the direction of s as related to cosine of the angle θ. And that may be described in terms of the difference in vertical elevation between the points 1 and 2 which are located to say at the centers of the two faces the left face and the right face.

Let us say that these points are located at a vertical elevation of Δz. So, Δz is the difference in height between the points 1 and 2 which are the centers of the faces. $\cos\theta = \frac{\Delta z}{\Delta s}$.

Now we can write the Newton’s second law of motion for the fluid element because we have identified all the forces which are having some components along the s direction.

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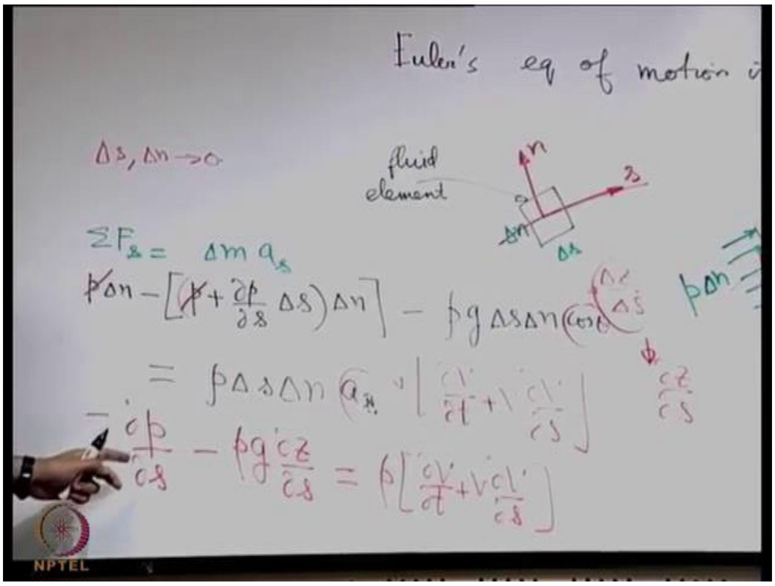

So, resultant force along s $\sum F_s = \Delta m a_x$

$$p\Delta n - \left[\left(p + \frac{\partial p}{\partial s}\Delta s \right)\Delta n \right] - \rho g \Delta s \Delta n \cos\theta$$
$$= \rho \Delta s \Delta n$$

$$a_s = \left[\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \right]$$

. So, this is like a one dimensional case of course, the velocity may be function of time. But otherwise in terms of spatial variation it is only a function of s because flow is always tangential to the streamline that is how the streamline is defined.

$$-\frac{\partial P}{\partial s} - \rho g\frac{\partial z}{\partial s} = \rho\left[\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \right]$$

So, then the partial derivative and d will become similar because v is a function of s only provided v is not a function of time. But otherwise we have to keep in mind that if v is a function of time also this partial derivative nature has to be maintained. So, for unsteady case one has to retain it like a partial derivative

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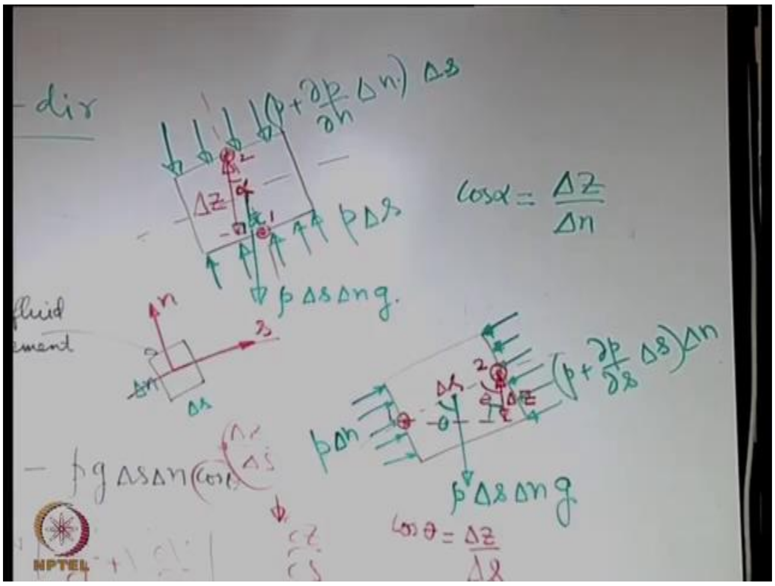

So, equation of motion so along n direction. Usually we are happy to write it along the s direction. Because we feel that that gives us enough information because; flow is along that s direction. But along the n direction something interesting also is possible and let us try to look into such possibilities.

So, we will now write all the force components with an understanding that we are interested for the forces along the n direction. So, let us represent those forces in the n direction. $P\Delta s$ and $\left(P + \frac{\partial P}{\partial n}\Delta n\right)\Delta s$ are the forces.

Again this has it is own weight so one has to consider the weight of the fluid element and when you consider the weight of the fluid element. Now we are not bothered about the left and the right face we are bothered about the bottom and the top face. Let us say again that the centers of these faces are located at a vertical elevation difference. So, this is say 1, this is 2, and there is a difference in vertical elevation or the height between the points 1 and 2 say that is given by Δ z.

So Δ z represents the vertical elevation between the two faces which are contributing to the forces along that direction. Weight of the fluid element is $\rho\Delta s\Delta n g$.

$$\cos\theta = \frac{\Delta z}{\Delta n}$$

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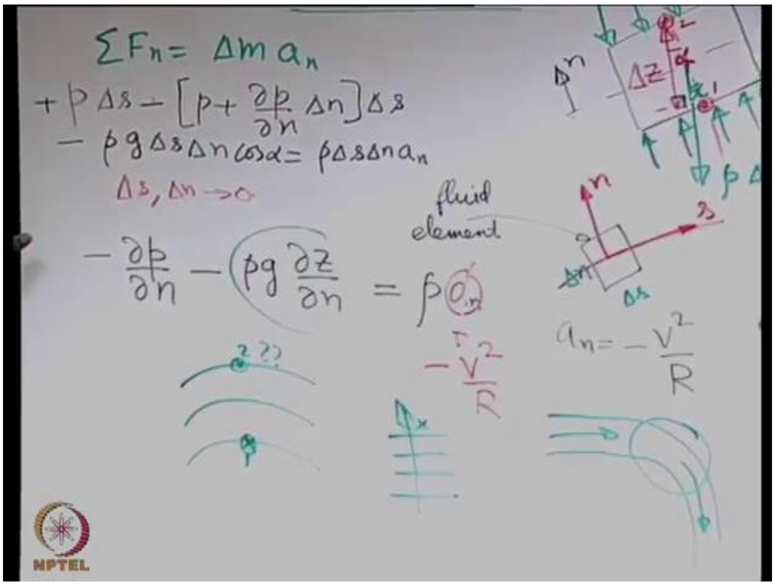

$$P\Delta s-\left[P+\frac{\partial P}{\partial n}\Delta n\right]\Delta s-\rho g\Delta s\Delta n\cos\alpha=\rho\Delta s\Delta n a_n$$

$$\Delta s,\Delta n\rightarrow 0$$

$$-\frac{\partial P}{\partial n}-\rho g\frac{\partial z}{\partial n}=\rho a_n$$

$$a_n=-\frac{v^2}{r}$$

It says that if there are curved streamlines there will be a pressure gradient across the streamlines. So, let us say that you have curved streamlines like this. When such curved streamlines are possible let us say that you have a pipe bend.

So, there is a pipe which was carrying fluid like this. Now the fluid is changing it is direction it is moving in a different direction. It is very much possible and very common pipes have bends to change the direction of flow. So, in the pipe bend if you consider the streamline streamlines are like oriented along the direction of the flow. So, you have curved streamlines let us say that this entire bend is in the horizontal plane.

Say there is a pipe which is bend in a horizontal plane; so, then just for simplicity so that this term is not there. So, when that term is not there you can clearly see that the pressure gradient along the normal direction should be given by $\frac{V^2}{r}$. So, if the stream lines have radius of some radius of curvature.

And if you know the pressure at say this point 1 you should be able to predict what should be the pressure at this point 2. If you know the radius of curvature of these stream lines. If the stream lines are such that they are parallel to each other then the radius of curvature tends to infinity. That means there is no crosswise pressure gradient because of the curvature so this is only the curvature effect.

So, because of curvature of the streamline there is no crosswise pressure gradient. There might be crosswise pressure gradient because of a change in vertical elevation. But here we are considering it in the horizontal plane so that is not coming into the picture. So, this n component of the equation or motion is very important because it gives a pressure gradient because of the curvature of the stream lines in the cross stream direction which; you cannot get from the equation of motion from the s direction.