Introduction to Fluid Mechanics

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Lecture – 34
Bernoulli's equation-Part-III

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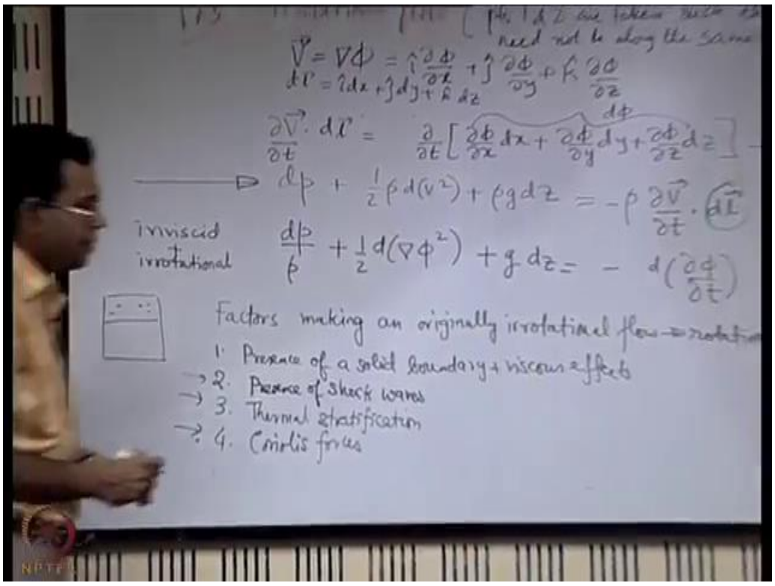

Now, you can see that there is some special requirement inviscid and irrotational. Now, there is very important and interesting relationship between these two, fundamentally we could try to answer these questions. If there is an irrotational flow is it true that it has to be inviscid and if it is an inviscid flow is it true that it has to be irrotational. Remember these are not very simple questions to answer and we will try to look into very basics of looking into these issues.

Let us say that you have an irrotational flow, say there is a free stream which is having an irrotational flow; that means, it has null vorticity vector. Now, the question is there any agent that can make the flow from irrotational to rotational. So, will the rotationality be preserved. So, there are certain factors which can create a situation such that an irrotational flow, the flow which was originally irrotational now becomes rotational. So, what are those factors? So, the factors making an irrotational flow and originally irrotational flow to a rotational one. One of the important factors is presence of a solid boundary and viscous effects. Presence of a solid boundary is there in many wall bounded flows and viscous effects are common for fluids with some substantial viscosity.

Even if the flow was originally rotational physically it will not be able to retain its irrotational state. That means, although you may start with an irrotational assumption the viscous flow assumption will not hold that irrotational state physically. We will see that mathematically it will not be able to; will not be able to reflect this directly in such an elementary level. It is possible to look into that mathematically, but not in such an elementary level. But physically we have to at least appreciate that, if it was irrotational there is no guarantee that eternally it will remain irrotational. And the factors which disturb that irrotationality one of the factors is the viscous effect in the flow; presence of wall boundedness.

One of the other important factors is presence of shock waves. Shock waves are created by situations in highly compressible flows when there is a abrupt discontinuity in the fluid properties. So, there is like a wave front across which there is a jump in all the properties of flow and that takes place with a condition, that across that there is a change in state from a supersonic to a subsonic flow. So, a Mach number greater than 1 to a Mach number less than 1.

So, the thermal stratification means there is a thermally stratified layer that is being created because, the density gradient is being created by the temperature. So, hottest ones are there at the top and cooler and cooler ones are at the further bottom. But, created by the change in created by the temperature gradients prevailing in the system and that also in a direction oriented against the gravity that is known as thermal stratification. So, if you have such stratified layers then it is possible that makes the flow rotational from irrotational. Then other forces like there may be Coriolis force is present.

So, Coriolis forces or Coriolis effects can create a rotationality in the flow if it was originally rotational. The earth when it is rotating it has a Coriolis effect. So, there are rotationalities in the ocean currents which are predominantly created by the Coriolis effects. So, if the earth was stationary, that is the hypothetical case to think it might be possible that that was irrotational.

But, because of the Coriolis effects being present that is converted to a rotationality effect. So, there are many factors these things just show that these are very natural factors, these are not any artificially imposed factors on the system. And these natural factors have a tendency to create a rotationality in the flow.

So, we cannot ensure that if we have a irrotational flow as a reference case or as the undisturbed flow that will remain as irrotational. But if it is inviscid and then if we consider that the effects

2, 3 and 4 are not there in a system; then if it is inviscid and irrotational origin and it will remain irrotational forever.

The message that the solid boundary is there cannot be propagated through the fluid; viscosity is that messenger which propagates the presence of the solid wall into the fluid. So, if the viscous effects are not there the fluid will be dumb in responding to the presence of the wall. And then an flow which is originally rotational we will retain its irrotationality that is one of the very important understandings.

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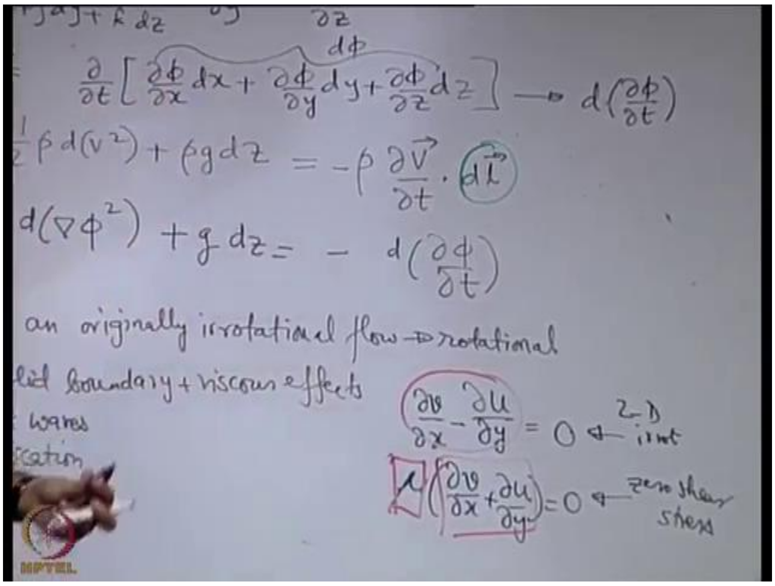

So, $\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0 \leftarrow 2-D$ irrotationality

So, for a Newtonian fluid $\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)=0 \leftarrow$ Zero shear stress

We have seen such an example where you have a fluid element which was originally of a particular orientation. It does not change anything angularly, it just gets stretched along one direction and reduced in length along the other direction that example we saw in the previous class. But that is a very special case; in general if you have an irrotational flow.

Irrotational flow is also called as a potential flow because, velocity potential exists in irrotational flow.

We found that it is a conservative velocity field. Because, the velocity because the field vector field is conservative we could write it as a gradient of a scalar potential. Now, when you have a conservative field physically it means that they are negligible dissipations in the system; just like if you have a conservative field as a gravity.

Velocity field is not exactly like a force field, but you may think it analogously because, it is also a vector field. So, in a velocity field what could create a disturbance in the conservativeness is the presence of a dissipation and that dissipation is through the mechanisms of viscosity. So, if viscous effects are strong then physically it may not help in retaining the flow field as a conservative field. So, physically it might be very common that if it is in irrotational, if it remains if it wants to remain irrotational it has to be inviscid. Viscous effects will create sort of dissipations in the flow just like what friction does in a force field.

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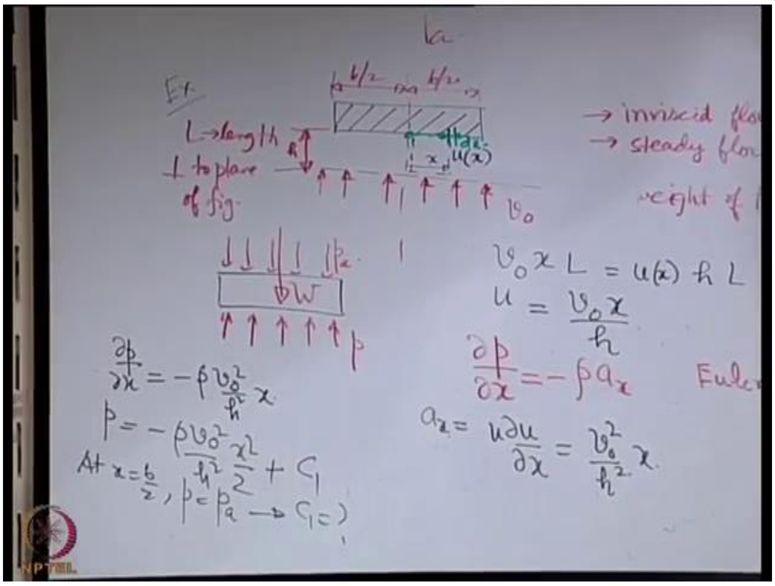

We refer to this example as such to work out a problem in the context of fluid kinematics. So, there is something like plate a rectangular plate and there is a bottom plate, there are holes in the bottom plate through which fluid say air is blown like this.

we have a uniform velocity say v_0 with respect to which air is entering through these pores and let us have a coordinate system like this where this is symmetrically located with respect to the plate.

So, let this be $\frac{b}{2}$ and this is $\frac{b}{2}$. Let us say that L is the length of this plate perpendicular to the figure. The gap between these two, let us say the gap is h and our assumption is that it is inviscid flow and let us say steady flow. We are interested to find out, what should be the weight of this plate to keep it in such a position.

So, from this given consideration we have to find out what is the velocity field. So, if you recall that we earlier considered like at a distance say x from one end and we found that what is the rate of flow entering and rate of flow leaving. This control volume which is marked by the dotted lines. So, the rate of flow that was entering is v_0x, now the length perpendicular to the plane of the board is L. This is the volume flow rate and what it leaves here, let us say u is a function of x because of the assumption of inviscid flow u does not vary with y.

$v_0 xL = u(x)hL$

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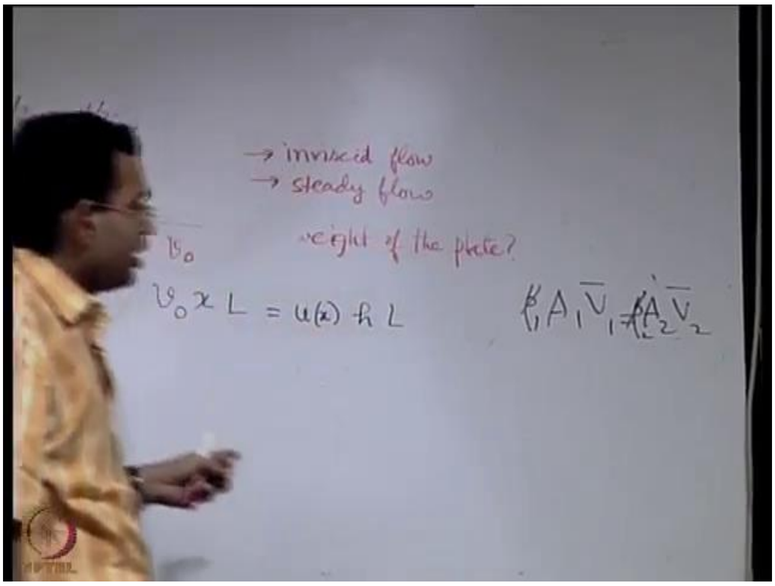

$A_1V_1=A_2V_2$ is valid when 1 and 2 are the two sections that we are looking for over which we are having equivalent constant velocities V_1 and V_2. So, if they are not constant these have to be replaced by the average velocities over the sections.

$\rho_1 A_1 \overline{V_1} = \rho_2 A_2 \overline{V_2}$ This was derived by dropping the unsteady term in the continuity equation and integrating the remaining terms in the continuity equation; that means, the only assumption was it was steady flow. So, unsteady term goes away. So, when it is steady flow it need not be constant density. If ρ_1 & ρ_2 are the same then it becomes $A_1V_1=A_2V_2$.

So, it has two assumptions; one is the steady flow another is ρ is a constant because, you have cancelled or maybe whatever function of low is there in the left hand side same function is there in the right hand side. It is a ρ is a function of something else a time so, left hand side and right hand sides the same function so, they are cancelled out. But, ρ cannot be a function of time because you have already considered a steady flow. So, it cannot be a function of time.

So, it is just like a constant which is same in the left hand and in the right hand side that is how these two got cancelled. And when we say ρ equal to constant the other thing again I am going to hammer on you keep in mind ρ equal to constant is a special case of incompressible flow.

But, incompressible flow does not require ρ to be a constant. This is often like even in some of the best of the textbooks this confusion is retained. So, we will see that when assumptions are written for a problem it is written that incompressible flow; well incompressible flow can be handled without requiring ρ to be a constant.re So, whenever we consider ρ to be a constant we specially specifically we will say that ρ is a constant, that is what is our assumption. Incompressibility is not good enough to ensure that ρ is a constant. But if ρ is a constant it has to be incompressible ok. Now so, this is something that we have already derived and let us write the velocity u as a function of x.

$$u = \frac{v_0 x}{h}$$

If you consider the sort of free body diagram for the plate or if you want to think it as a chip. So, there is a pressure distribution from the bottom, there is a pressure distribution from the top which is because of the atmospheric pressure.

Let us say that it is entirely surrounded in a uniform atmospheric pressure which say is p atmosphere which is along all the sides except the inside part. Now, because of this difference in pressure let us say this is p. So, if p is greater than P_{atm} there will be a up thrust on it and that should be balanced by the weight to keep it in equilibrium. That means, if we find out what is the resultant force due to pressure on this chip that will give us an insight on what is the weight.

Euler's equation of motion along x: $\frac{\partial P}{\partial x} = -\rho a_x$

acceleration along x: $a_x = u\dfrac{\partial u}{\partial x} = \dfrac{{v_0}^2}{h^2}x$

$$\frac{\partial P}{\partial x} = -\rho\frac{{v_0}^2}{h^2}x$$

$$P = -\frac{\rho {v_0}^2}{h^2}\frac{x^2}{2} + C_1$$

At x=$\dfrac{b}{2}, P = P_a \rightarrow C_1 = ?$

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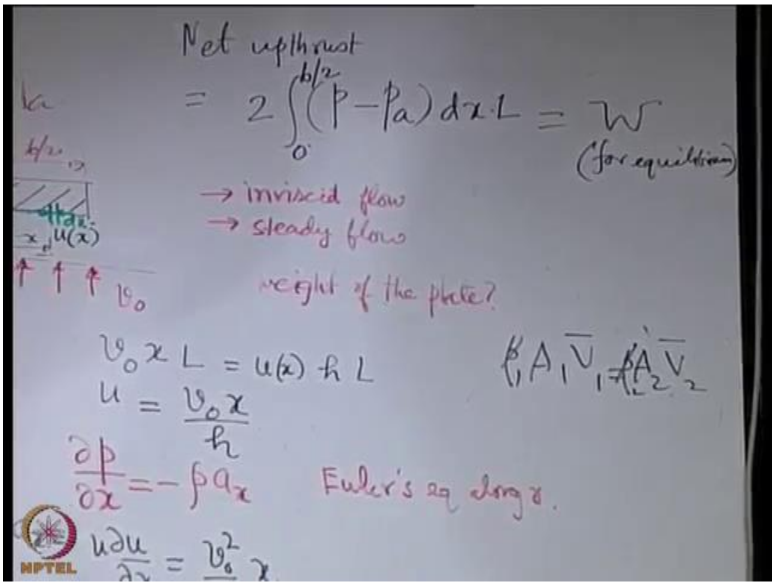

Net upthrust= $2\int_0^{\frac{b}{2}}(P - P_a)dxL = W(\,for$ equilibrium)

We will just create a small introduction for that we will write the Euler's and the Bernoulli's equation in terms of a different coordinate system that is a streamline coordinate system.

So, why we are interested to write it in a streamline coordinate system because, we know that along a streamline under certain conditions these equations are valid. So, if we write it for 2 points along a streamline it may be very convenient, if we use the streamline coordinates.

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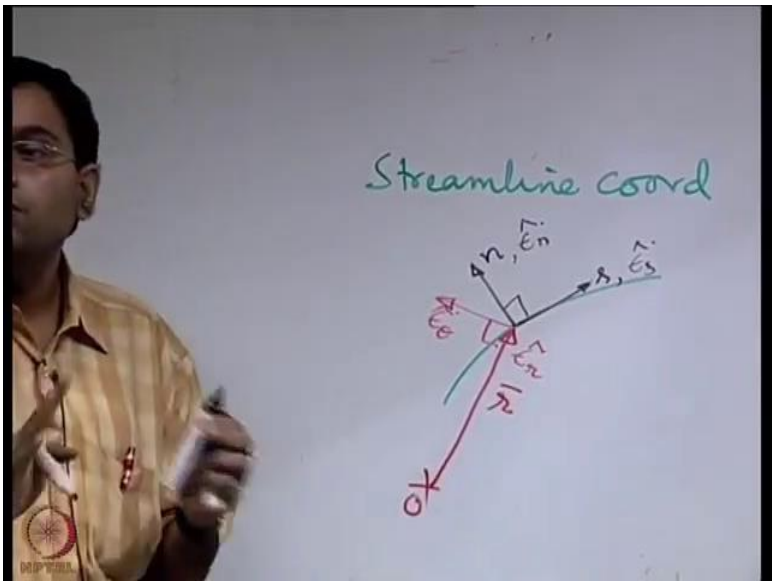

So, if you have a streamline we consider tangential to the streamline as s and normal to the streamline as n. Many times there is a confusion between this coordinate system and the coordinate system that is used in a cylindrical polar coordinate system. So, we will try to avoid that confusion from the very beginning; say if you are using a polar coordinate system. So, if you have this as the origin or the pole then it is represented by one radius r, this is the radial direction and perpendicular to that θ direction. So, we have unit vectors along this as ε_r & ε_θ and unit vectors along this as ε_s & ε_n.

Both of these are orthogonal systems. This is not a tangential direction; only for a circular geometry normal to the radius is the tangent, but not for all types of curves. So, this is fundamentally called as radial and cross radial direction.

So, ε_r is the unit vector along the radial direction, epsilon theta is the unit vector in the cross radial direction which is perpendicular to that or orthogonal to that whereas, these are sort of tangential and normal direction.