Introduction to Fluid Mechanics
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Lecture – 33
Bernoulli's equation – Part – II

We will continue with the Bernoulli's equation about which we were discussing in the previous class.

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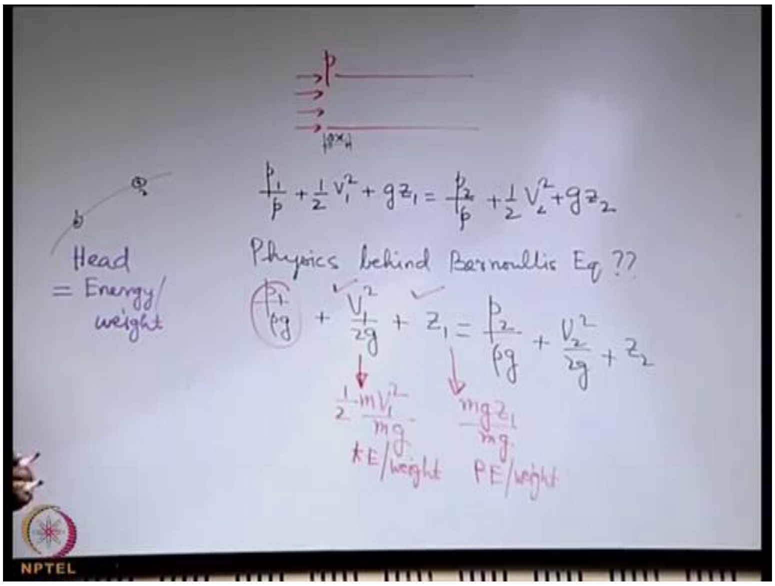

So, the Bernoulli's equation as we have seen is taking the form

$$\frac{P_1}{\rho}+\frac{1}{2}V_1^{\,2}+gZ_1=\frac{P_2}{\rho}+\frac{1}{2}V_2^{\,2}+gZ_2$$

Density is constant and we derived it along a stream line, where we it requires no other restriction, but if you want to apply between any 2 points who are not necessarily located on the same stream line then it has to be irrotational flow or maybe a special case, when the cross product of the velocity with the vorticity that is perpendicular to the line element, that is being chosen.

Now, when we come to this form; so let us say that we are considering it along a stream line. There are points 1 and 2 and this equation is valid. After all, we are not mathematicians; we are not just bothered about that here there is a equation where we can plug in values to get a numerical answer. What is even more important for us to appreciate that what is the physics behind the Bernoulli's equation.

So, that is what we will try to learn because once we appreciate the physics properly, we will be perhaps able to utilize this equation in certain cases where this equation may not be exactly valid, but in a in a somewhat approximate sense. To understand the physics it may be better to appreciate the physical consequence of each and every term in the equation. These terms may be written in different ways. This is one standard way of writing it, but in engineering sometimes what we do, we divide all the terms by g and write it in this form.

$$\frac{P_1}{\rho g} + \frac{1}{2g} V_1^2 + Z_1 = \frac{P_2}{\rho g} + \frac{1}{2g} V_2^2 + Z_2$$

$\frac{V_1^2}{2g} \Rightarrow \frac{1}{2} \frac{mV_1^2}{mg}$ represents kinetic energy per unit weight.

$Z_1 \Rightarrow \frac{mgZ_1}{mg}$ is potential energy per unit weight.

So, you can see that these terms are therefore, representatives of energy per unit weight. Energy per unit weight in fluid mechanics is known as head. So, that is a term given as head = energy per unit weight.

Let us say that you have a pipe and fluid is trying to enter the pipe. When the fluid is trying to enter the pipe, let us say that the pressure at this inlet section is p. The fluid is having a particular velocity, it is entering the pipe. Now, see this is a flowing system. So, there is a pressure in the fluid, this pipe is not that this pipe is like in vacuum. So, you have a continuous flow going on like this. So, this is filled with say water and water is continuously entering and leaving like that.

Now, if the water which is entering the pipe has to penetrate over a distance, it has to do that in presence of the pressure and therefore, it has to do some work. Let us say that it undergoes a displacement of Δx; why we are considering just a small displacement? Because we will consider that this pressure is remaining constant over the displacement. It may be a variable pressure. So, we will consider only a small displacement over which the pressure is supposed to be a constant. The whole idea is based on that we are interested to calculate the work done in presence of pressure.

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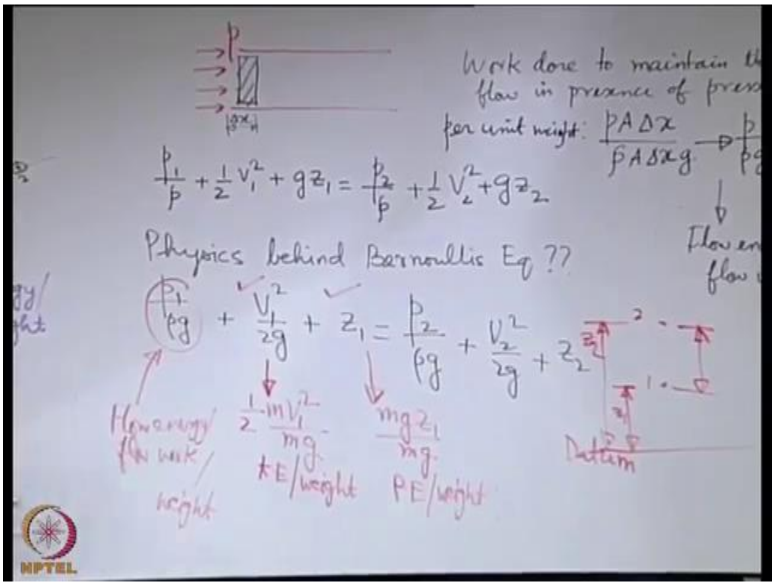

So, if A is the area of cross section of the pipe, then what is the work done to maintain the flow in presence of pressure? In presence of pressure, let us say we want to find it out. So, to do that, we will first consider the small displacement.

Per unit weight: $\dfrac{PA\Delta x}{\rho A\Delta xg} \rightarrow \dfrac{P}{\rho g}$

It represents the work done to maintain the flow in presence of pressure. So, if there is no flow, then this term would have been absent. So, the fluid must possess this additional or the fluid must be capable of transferring or transmitting this additional energy through its motion so that it can overcome whatever pressure is there and still it can maintain the flow.

So, this extra energy the fluid must possess to maintain the flow in presence of pressure. This is known as flow energy or flow work.

Now, the question is - Is this energy being possessed by the fluid or what? To understand that let us consider an example; say you are there in an airport and there is a conveyor belt. Now a suitcase is put on the top of the conveyor belt. The suitcase moves from one place to another place because of the motion of the conveyor belt and the conveyor belt just as an analogy consider it like a fluid flow. So, it is like moving the suitcase from one place to another place. So, that suitcase is like something which is put on the fluids just like some energy.

So, you have some total of these three energies that is somehow being there in the flow and it is getting transmitted from one place to another. So, it is never possessed. Therefore, the

Bernoulli's equation essentially says physically that the sum total of the flow energy, kinetic energy and potential energy per unit weight remains conserved as it is transmitted from one point to another in the flow field. It is not possessed; it is just transferred. So, the flow is just acting like a medium which is not holding the energy, but which is transmitting or transferring the energy from one place to another place. This is like a sort of statement of conservation of mechanical energy.

But we will see that under restricted cases, it may also take different forms; not that this is the only form that is there for the conservation of mechanical energy. So, we have now sort of clear picture on the significances of different terms in the Bernoulli's equation.

The next point is that whenever you are talking about energies in different terms, you must have a reference like when you say potential energy, you have a datum with respect to which you are calculating the height z. So, this z is not in an absolute sense it does not make any make any big significance because eventually in this equation $Z_1 - Z_2$ that is what is important and that is independent of the choice of the reference.

So, if there are two points 1 and 2 say this is 1 and this is 2. So, this difference in the height between 1 and 2 vertical height is that what is important. So, if you have this as z_1 and say this has z_2 with respect to some datum; say this is the datum, then it independent of the choice of the datum $Z_2 - Z_1$ remains the same. But still if when you want to prescribe it in an absolute sense, you require a datum for or a reference. For the kinetic energy term, there is a velocity and you require a reference for that.

So, there should be a reference frame with respect to which you are prescribing this velocity. Typically this reference frame is a reference frame at rest. So, you are writing here the absolute velocity and this reference frame for pressure is also very important. We have already discussed when we were discussing the statics of fluids that you can prescribe pressure also with risk with reference to something. If we prescribe with risk reference to the atmospheric pressure, we call it a gauge pressure. So, you can as well prescribe the pressures in the different terms in terms of a reference pressure and then you can substitute as gauge pressure.

Important thing is whatever reference you use for postulating the different terms, it should be same in the left hand side and right hand side. That is very common and obvious conclusion.

Now, we will see that what could be the other variants of these Bernoulli's equation or more fundamentally the Euler's equation.

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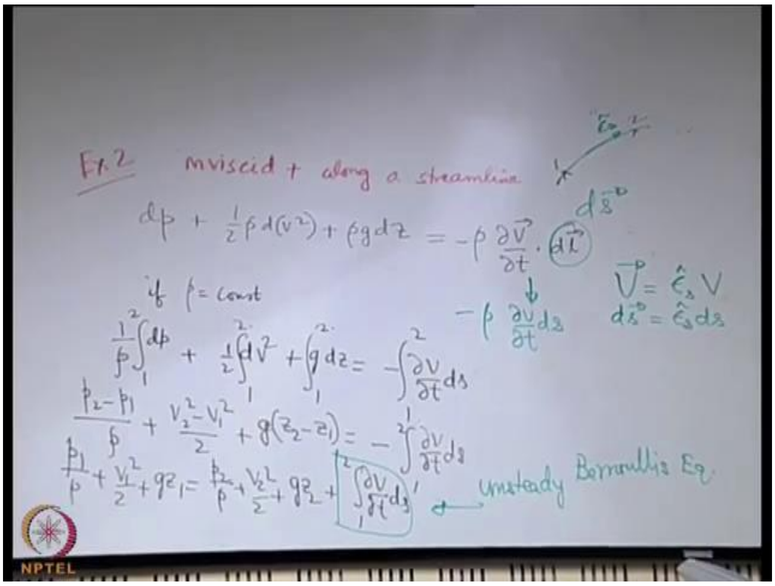

So, in the example 2, we will keep all our previous assumptions as valid that is inviscid flow and flow along a streamline. So, we will consider inviscid plus along a streamline.

Inviscid + along a streamline $dp + \frac{1}{2}\rho d(V^2) + \rho g dz = -\rho \frac{\partial \vec{v}}{\partial t}.d\vec{l}$. Here $d\vec{l} = d\vec{s}$

Now, when we are considering the streamline v is already oriented along the streamline.

$$\vec{V} - = \hat{\varepsilon}_s V$$
$$d\vec{S} - = \hat{\varepsilon}_s ds$$

If ρ =const.

$$\frac{1}{\rho}\int_1^2 dp + \frac{1}{2}\int_1^2 d(V^2) + \int_1^2 g dz = -\int_1^2 \frac{\partial \vec{v}}{\partial t}.d\vec{s}$$

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1) = -\int_1^2 \frac{\partial v}{\partial t} ds$$

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + gZ_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + gZ_2 + \int_1^2 \frac{\partial v}{\partial t} ds$$

So, now, it also can be an unsteady flow. This is known as unsteady version of the Bernoulli's equation. We will work out some problems subsequently to illustrate the use of this one. So, what is clear is that whatever term, we have dropped because of steadiness, now that term has appeared and it is just creating an extra effect and the meaning of this term is quite clear because it gives a variation of the effect of the variation of the velocity with respect to time.

Now, we will take a third example when we consider irrotational flow. So, let us take the third example.

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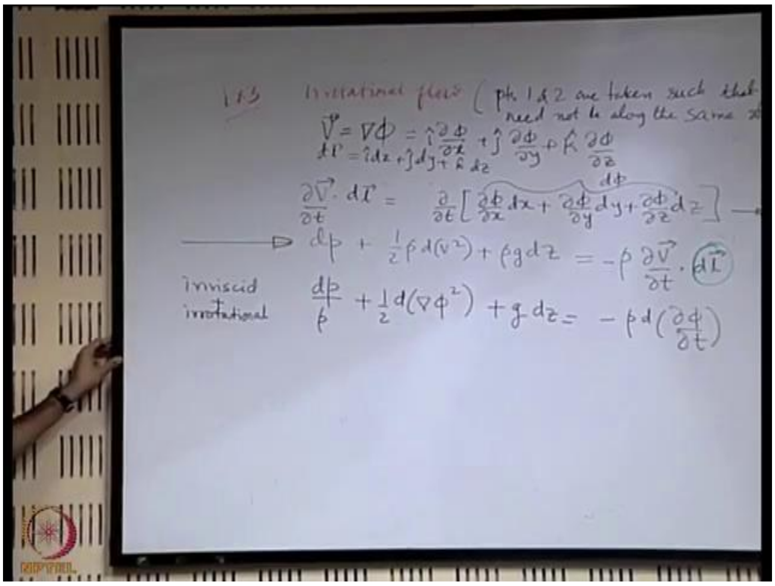

Example 3: irrotational flow; so, when you take the example of irrotational flow let us see that what happens to the equation. So, when you have the irrotational flow, the thing is that $d\vec{l}$ we are not writing as ds because when we are writing irrotational flow, we are keeping in mind that if points 1 and 2 are taken such that they need not be along the same streamline.

So, if they need not be along the same streamline the consequence is that the equation is just like this equation what is there in this example 2, but we will not substitute $d\vec{l}$ with $d\vec{s}$; we will just keep $d\vec{l}$ as it is. But when it is an irrotational flow, we know that the other term where there was a vorticity vector that term will become 0 not only that you can write V as the gradient of a scalar potential which is the velocity potential that we discussed.

$$\vec{V} = \nabla\phi = \hat{i}\,\frac{\partial\phi}{\partial x} + \hat{j}\,\frac{\partial\phi}{\partial y} + \hat{k}\,\frac{\partial\phi}{\partial z}$$

$$d\vec{l} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\frac{\partial\vec{V}}{\partial t}.d\vec{l} = \frac{\partial}{\partial t}\left[\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz\right] \rightarrow d\left(\frac{\partial\phi}{\partial t}\right)$$

$$\frac{dP}{\rho} + \frac{1}{2}d\left(\nabla\phi^2\right) + gdz = -d\left(\frac{\partial\phi}{\partial t}\right)$$

This is the Euler's equation in terms of the velocity potential. So, this is valid for inviscid plus irrotational