Introduction to Fluid Mechanics

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Lecture - 26
Problems and Solutions

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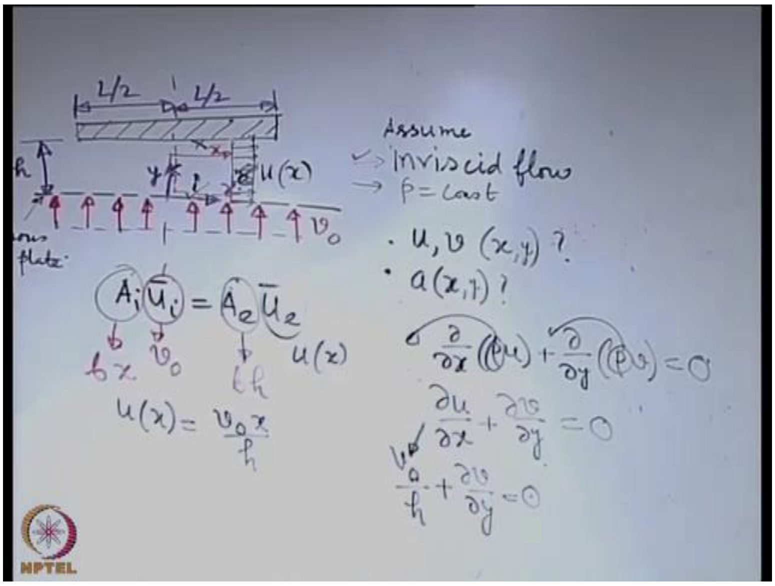

Now, say the top plate is a rectangular plate with the dotted line representing the axis of symmetry. The bottom plate is a very special plate. It has some holes; we call it a porous plate. So, it has some pores or holes and the idea of keeping these holes is to blow some fluid. We will see that it is not just a mathematical, mathematically defined idealistic problem. Many times, it is something which is followed in technology.

Let us say that this is a heated electronic chip and you want to cool it. So, it is possible that you blow air through porous plate, which goes into the chip and tries to keep it cooled. So, it is not a very hypothetical type of a situation, but the way we will look into this particular problem is abstracted from that any specific application but more into the fundamental that what goes behind this.

Let us say that this is a uniform velocity with which it enters say we call it v_0 . So, this is a porous plate. The gap between these two say is h, the length maybe this is $\frac{L}{2}$ and this is $\frac{L}{2}$. And let us say that we have x coordinate like this and y coordinate like this. We make an assumption that it is an inviscid flow that is given, assume inviscid flow. What is your objective? Your objective is to find out the velocity components u and v as functions of x i.e u,v(x) and what is the acceleration of fluid at a given x or maybe at a given (x , y).

So, when we say x may be let us make it generalized say, u,v(x,y) and a(x,y).

So, let us first physically try to understand that what is happening. Whenever you are solving a problem, we can of course, start putting equations but that is not always necessary. First, you have to understand. So, what is happening? Some fluid is entering. Now, the fluid cannot leave through the top because of what is it because of no slip condition, no; because no slip ensures that it has no tangential component, but it simply its a no penetration condition because it cannot just penetrate through the wall and go out, because it is just a fully covered solid wall. So, no matter whether it is slip or no slip, you can you cannot actually penetrate it and go out along y. So, only way this fluid can move in a steady condition is it can move sideways. So, whatever fluid enters, now maybe half enters right and half enters the left.

So, what we can say is that we may write a gross overall mass balance. So, when you write a gross overall mass balance, what we have to keep in mind that $\rho_i A_i \bar{u}_i = \rho_e A_e \bar{u}_e$. So, here let us say that ρ is a constant. So, that is another assumption that we make ρ equal to constant. So, when we make the assumption of ρ equal to constant, it is we have to just consider the area times the average velocity is same as what enters is same as what leaves. So, we have to fix up a control volume to write that expression because you require specific surfaces.

So, let us say that we consider a control volume like this which is say located its end face is located at a distance x. So, its local x coordinate is x. So, with respect to this control volume, what are the phases across which fluid flows?

One is the most straightforward is the bottom phase, yes through these fluid flows. Another straightforward is the top phase through which fluid does not flow.

So, if it tends to go to the right with a velocity say plus u, similarly it will have tendency some fluid particle look at the same position to go to the left with -u. Net effect is that at this axis of symmetry of no u is 0 because it is just like balanced from what you. So, whatever enters, it has a balancing effect of going to the right and to the left. So, there is no net flow across this.

So, when you have there is no net flow across this, there is only. So, this one has a net flow. So, when we say i and e, maybe this is the surface i and this is the surface e. $A_i \bar{U}_i = A_e \bar{U}_e$

To find A_e assumption of inviscid flow is important. So, if you do not consider inviscid flow, then you have to know what is the velocity profile in between you have to integrate that to get the average velocity. But, when you are given that is inviscid flow, your inherent assumption is that it is a uniform velocity profile like this. So, u is not changing with y. It is locally changing with x, but along y, it is uniform because it is inviscid. You can see that with inviscid flow, you cannot impose no slip boundary condition because if it has to be uniform.

So, it is not no slip here at the wall, but no penetration at the wall that is sufficient for solving this problem. So, no slip is not a necessary condition here and in fact, it will contradict if you say that is an inviscid flow, you cannot have no slip and in slip and inviscid, no slip and inviscid simultaneously. So, if there is some slip, so this velocity. So, it does not vary with y. So, we can say that it is like u just a function of x from the inviscid flow consideration.

$$A_i\bar{U}_i = A_e\bar{U}_e \Rightarrow bxv_0 = bh\frac{v_0x}{h}$$

So, if the fluid has a viscosity that will be propagated from the wall to the inside and it will in effect try to slow down the fluid elements which are close to the wall and as you go more away and more and more away from the wall, the velocity will be more and more.

So, the velocity profile in that case will have like a 0 value at the wall and then increasing away from the wall. But if you have no viscous effect, then the effect of wall is not propagated in the fluid. Fluid does not know that there is a wall and therefore, it tends to maintain a uniform velocity and that is why for in inviscid flow, you have such a kind of uniform velocity profile.

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{v_0}{h} + \frac{\partial v}{\partial y} = 0$$

$$v = -\frac{v_0 y}{h} + f(x)$$

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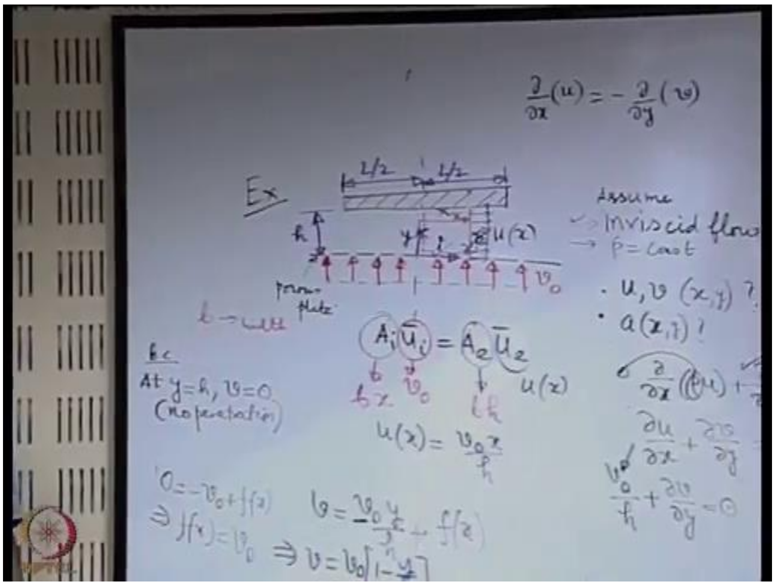

At y=h, v = 0(no penetration boundary condition)

$$0 = -v_0 + f(x)$$
$$\Rightarrow v_0 = f(x) \Rightarrow v = v_0\left[1 - \frac{y}{h}\right]$$

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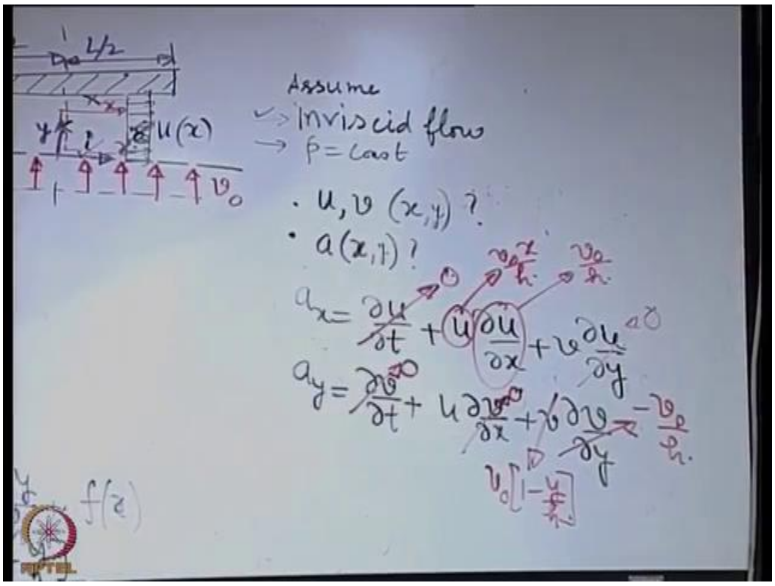

So, $a_x = \cancel{\frac{\partial u}{\partial t}}^{0} + \cancel{u}^{\frac{v_{0x}}{h}} \cancel{\frac{\partial u}{\partial x}}^{\frac{v_0}{h}} + v\cancel{\frac{\partial u}{\partial y}}^{0} = \frac{v_0}{h}$

$$a_y = \cancel{\frac{\partial v}{\partial t}}^{0} + u\cancel{\frac{\partial v}{\partial x}}^{0} + \cancel{v}^{v_0\left(1-\frac{y}{h}\right)}\cancel{\frac{\partial v}{\partial y}}^{-\frac{v_0}{h}}$$

$$a_x = {v_0}^2 \frac{x}{h^2}$$

$$a_y = -\frac{{v_0}^2}{h}\left[1-\frac{y}{h}\right]$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

So, this is the acceleration at a point. So, if there is a fluid particle located at some x comma y, that will locally be subjected to that acceleration because locally the fluid particle and the flow behaviour is identical. So, you can clearly see that although the velocity components are not functions of time, still you get an acceleration.

That is what is the important implication that we get from an already and approach because the entire acceleration component has horizon because of the convective component of acceleration because of the variation of velocity with respect to position if it is not because of change in velocity due to change in time.