Introduction to Fluid Mechanics
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Lecture – 22
Concept of different flow lines

Now, what we will do? We will try to see that what are the conceptual lines which are important for quantifying these visualizations and for that we will learn certain concepts. So, the first concept that we will learn is something which you have heard of earlier and that is the concept of a stream line.

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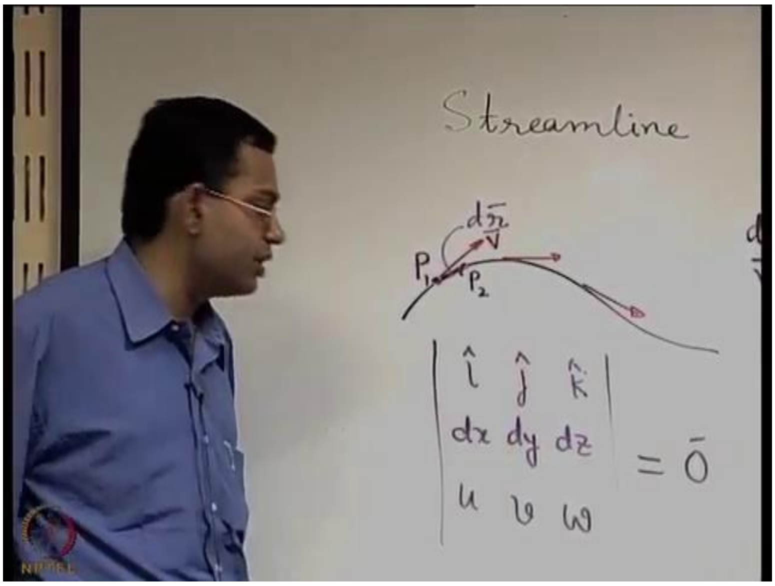

So, we are discussing about some conceptual paradigms which help us in visualizing fluid flow. So, when we say stream lines, how do we define stream lines? Stream lines are imaginary lines in the flow field these are not existing in reality so, imaginary lines. What type of imaginary lines? These are such lines that at an instant of time tangent to the stream line at any point represents the velocity vector at that point. So, if you have a stream line like this say. So, when you have a stream line like this; you may have tangent to it at different points. And, these tangents are representatives of the velocity vector at these points. One important concept that we miss many times is that it is defined at a particular instant of time; that means, at different times you may get different stream lines. Only when the flow field is not changing with time that is a steady flow you get same stream lines at all instants of time otherwise you may get different stream lines.

Now, to express the stream line in terms of some equation. So, this is a line, this is the locus. So, it should be expressible in terms of certain equations. Let us try to see that how we can express that, towards that we will first recognize that let us say that there is a point P or P_1 located on the streamline. The fluid particle at a particular time, the fluid particle is located also here it is coincident with this point. When the fluid particle is coincident with this point then after some time the fluid particle has come to a different point and so on. We are not bothering about the motion of the fluid particle, we are just bothered about say 2 points which are located on the streamline which are quite close to each other. Say P_1 and P_2 not that P_2 represents the local the location of the fluid particle at a different time not like that, just it is another point on the streamline which is very close to the point P_1.

The vector P_1P_2 let us say we denote it by a change in position vector dr and let us say that V is the velocity at that particular point. When we give it a name dr we have to keep in mind that it is very small right and it is differentially small; it is as good as writing delta r as Δr tends to 0. So, when $\Delta r \to 0$ then what is the status of the points P_1 and P_2, they are almost coincident. When they are almost coincident; that means, P_1P_2 then represents tangent to the stream line at the point P_1. So, what is the tangent? Tangent is the limit of a chord in the limit as the gap the distance between the 2 points becomes infinitesimal. So, in the limit P_1P_2 becomes tangent to the stream line at the point P_1.

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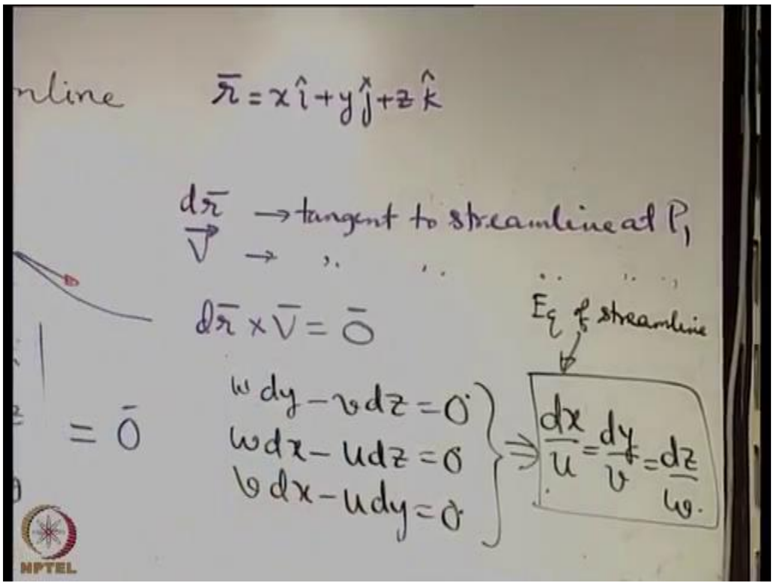

So, dr in that differential limit is tangent to the stream line at which point at P1. By definition V is also tangent to the stream line at P1 that is the definition of the stream line; that means, these two are parallel vectors. If these two are parallel vectors their cross product should be a null vector. So, you have $d\vec{r} \times \vec{v} = \vec{0}$. So, we can write dr and V in terms of components; you have $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x y z. So, when you write this cross product it is possible to write it in a determinant form.

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ u & v & w \end{bmatrix}$$

We can easily see that it boils down to 3 scalar equations for each for the x y and z components. So, what are these scalar equations?

$$\left.\begin{cases} wdy - vdz = 0 \\ wdx - udz = 0 \\ vdx - udy = 0 \end{cases}\right\} \Rightarrow \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

This the locus that we are looking for, you can easily obtain the locus by keeping in mind that are functions of position like x y z and also time, but when you are considering a streamline you are freezing the time.

So, at a given instant of time so, that does not become a variable in this case. So, x y z are the variable. So, you if these are substituted as functions of x y z you may integrate these to find out the locus that is very straightforward; later on we will work out some examples to illustrate that how we can do that. So, this is the concept of a streamline. Now, related to this concept there are certain other terminologies again; sometimes they are confusing because, streamline is more commonly used. Those are not very commonly used, but those are sometimes more fundamental and more relevant and the streamline. So, we will see the next example that is called as a streak line. So, what is a streak line?

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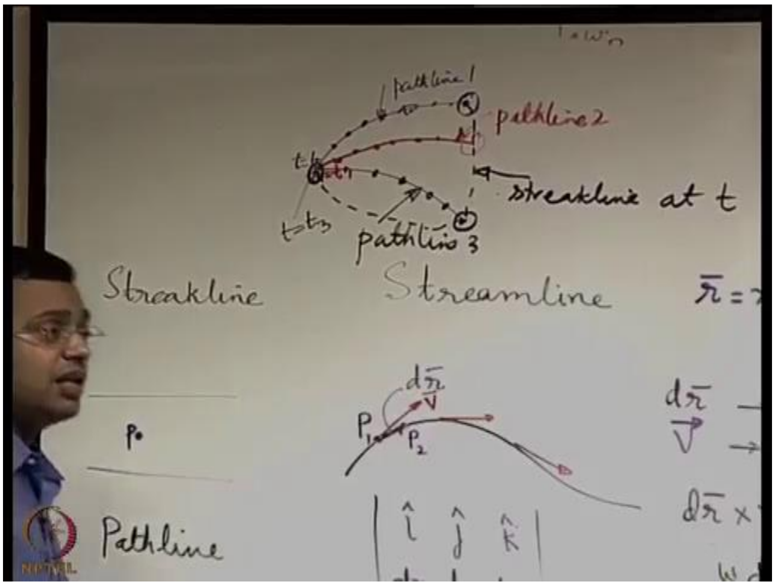

Let us say you want to visualize a flow, we will try to identify the concept from where it has come. So, when you want to visualize a flow say, this is the flow field you want to visualize a flow. A very common technique is what, you take an injection syringe in that injection syringe you take some colored dye; say a blue colored dye a common name of a blue color dyes called as thymol blue. So, you have taken a thymol blue looks like the ink. So, when you have taken that blue color dye and say that you are trying to put that blue color dye inject that blue color dye through this point P. So, the blue color dye is coming here through an injection syringe.

So now, you are going on injecting the dye here. So, what is happening whatever fluid molecules or fluid particles which are passing through this point at different instants of time they are illuminated by the dye? And so, wherever they go that tag of illumination remains. So, when you get an illuminated line it is at a particular time then what does it represent. It essentially represents locus of all fluid particles which at some earlier instant of time passed through this point of dye injection that is how they were colored by the dye. So, when you see a colorful line in the flow field it represents that locus and that locus is called as a streak line.

So, what is the streak line? Streak line is the locus of all fluid particles which at some instant of time at some prior instant of time all of which had passed through a common point. Mathematically we stop here, but physically we understand that common point is the point of dying action. So, that is called as a streak line. Let us try to conceptually draw a streak line let us say that this is a point at which dye is injected. So, when a fluid particle passes through these points say at time equal to t_0. So, when at time equal to t_0 the fluid particle passes through this point, the fluid particle then undergoes a locus. So, what is this locus we introduce a third line which is called as a path line.

Path line is the simplest and the most trivial concept to understand. It is the description of a flow from a Lagrangian viewpoint. So, it is the locus of an identified fluid particle. So, if you identify a fluid particle how it moves the path traced by that is called as a path line, that is very simple and trivial requires no explanation. So, when we want to draw different path lines see at say time t equal to t_0 you have one fluid particle which pass through this point of dye injection, that fluid particle at subsequent instants of time it is passing through different points. So, this is the path line 1 say. What is that path line 1? Path line 1 is the path line of a fluid particle which pass through the point of dye injection at time equal to t_0.

Let us say there is another fluid particle which has passed through this at time equal to t_1 and let us say that this red line represents it locus. So, this is something which was injected at t equal to t_1, the path may be different because it may be an unsteady flow. So, it is possible that the velocity field has changed with time. So, when just change with time the particle may be forced to move along a different locus. So, this is path line 2. Let us consider the third path line maybe for completeness let us say that we have a third path line like this which corresponds to that injection here at time equal to t_3. And, again the path line is different because the velocity might have changed at different points with time. So, this is path line 3. Now, say we are bothered about at time equal to t say now.

So, at time equal to t that particle which passed earlier through this point at t_0, say now it is here that particle which pass through this point at t_1 is now at here. And, that particle which pass through this point at say t_3 is here. Right now at time equal to t say there is one more fluid particle that is injected just here, if it is a continuous process. So, the locus of all these which at some earlier instants of time pass through the point of dye injection that locus is now the streak line at time t. So, you can clearly demarcate between streak line and path line. Let us take the example of a very special case, but a very interesting case. What is that what is that special case? That special case is for a steady flow.

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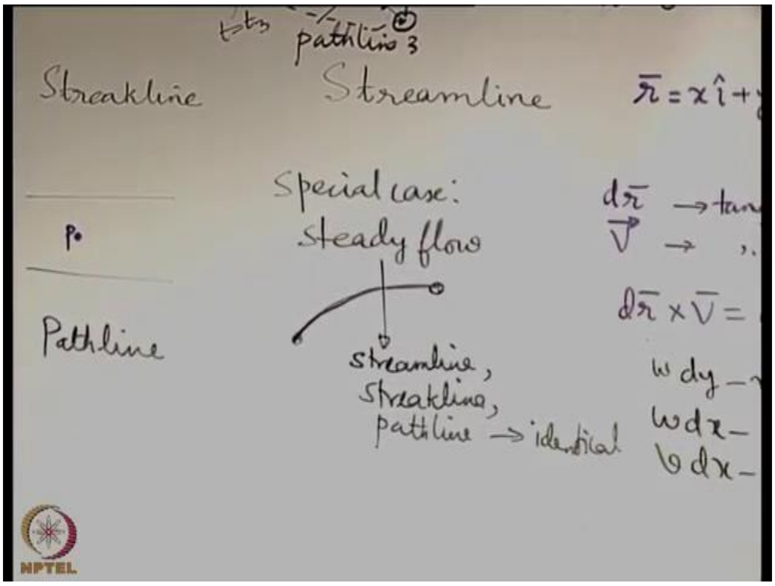

So, when you have a steady flow then let us try to draw these path lines. So, first let us draw the path line of that particle which pass through this point of injection at t equal to t_0. So, let us say that path line is this one. Now, another particle which pass through this at t equal to t_1 that will also follow this line because, with time the velocity field has not changed. So, it will be constrained to follow the same line. So, it will follow this line. Similarly, the third one that is in the one injected at t equal to t_3 that will also be constrained to follow this line. And what is the specialty of this line? This line is the locus of the fluid particle so; that means, at some time tangent to this line represents the direction of the velocity vector.

So, we can understand that this is also a stream line, these also the path line and again this is also the streak line because, whatever are the locations of fluid particles those are always constrained within this line. So, we can say that for steady flow, stream line, streak line and path line are identical that is one very important concept that we should remember; that in this case you have stream line, streak line and path line are identical they are identical. So, whenever we visualize a flow let us look into some example maybe some images through stream lines or streak lines.

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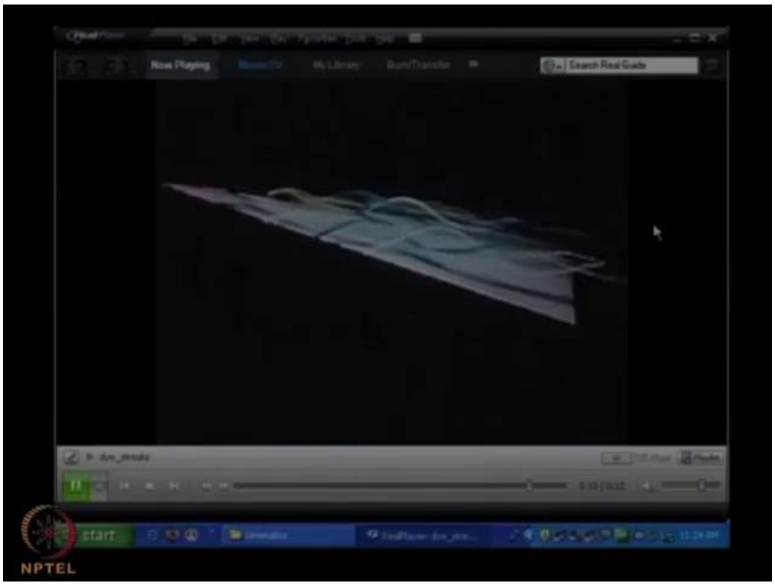

So, let us say that we see first this example, you can see these are streak lines. So, dye streaks which are which are injected at the tip you can see that now at different instants of time they are forming different colored images. And, if you track individual one then it is like a path line.

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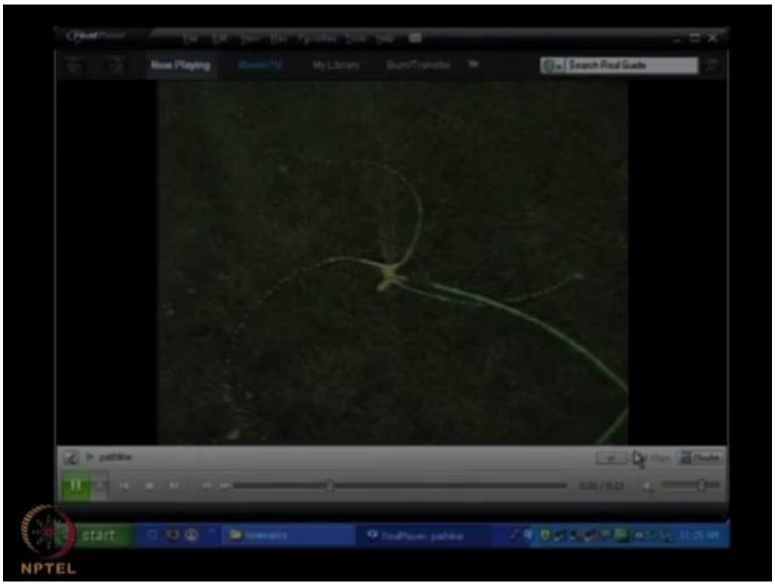

Now, if you see let us look into a path line example, see this is like a lawn sprinkler. So, many times it is used to sprinkle water in a garden. You can see that if you track the water droplets you can clearly see that the path what they are following. So, it is something like a visual representation of a path line. Let us look into a stream line example.

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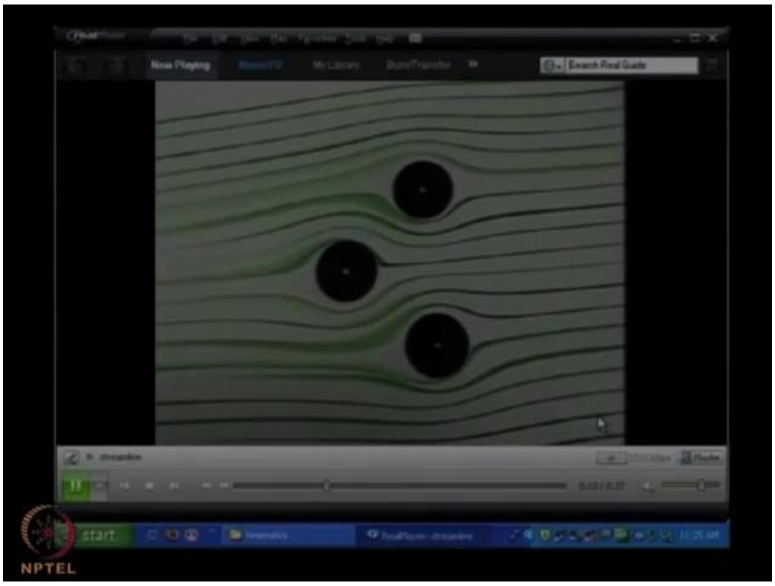

So, these are 3 cylinders in a steady flow and you can see that this green colored dye is giving an appearance of a stream line.

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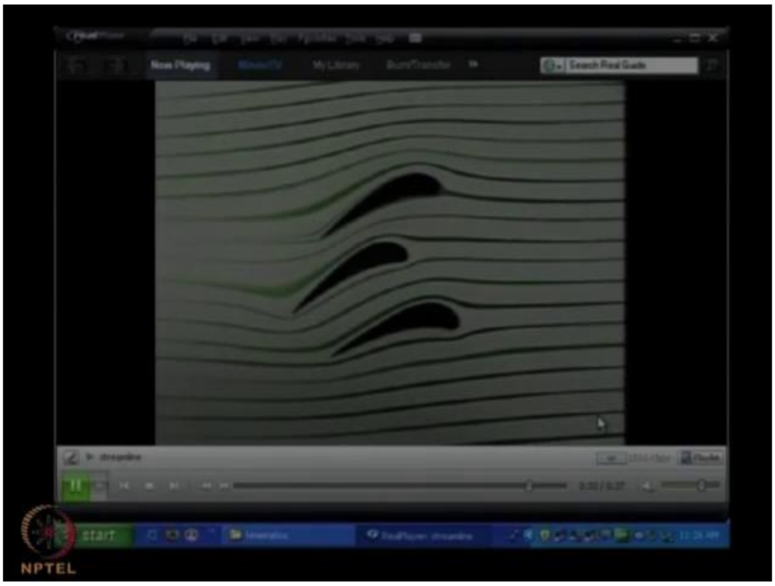

Fundamentally this is actually a streak line, but because it is a steady flow stream line, streak line, path line these are all identical. So, these represent different streak lines or different stream lines or different path lines whatever you say if it is a steady flow; if it is if it is an unsteady flow these will represent streak lines rather than stream lines. So, you can clearly see that these visualizations of fluid flow that we have seen as concepts these are may also be visualized in experiments.

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Many times we are interested to construct the velocity vector. So, see the example of the boat that we saw earlier and you may construct such velocity vector. So, it is not a direct visualization, but you may do it in two ways by post processing the visualization of the particles which are injected into the fluid or by doing a computer simulation. And, sometimes these may be equivalently compared computer simulation of course, then idealization because you are using certain boundary conditions certain properties which might not exactly prevail in reality.

But, sometimes it gives a very important idea of how the fluid flow is taking place and it is used for advanced designs also. So, this is known as Computational Fluid Dynamics or CFD. So, that is that is a separate area of research altogether where the whole idea is to computationally solve the equations of fluid flow to get a picture of the velocity field. With this understanding we will try to quickly work out an example to illustrate the concept of these lines stream line, streak line and path line. So, an example we will consider a 2 dimensional velocity field as an example.

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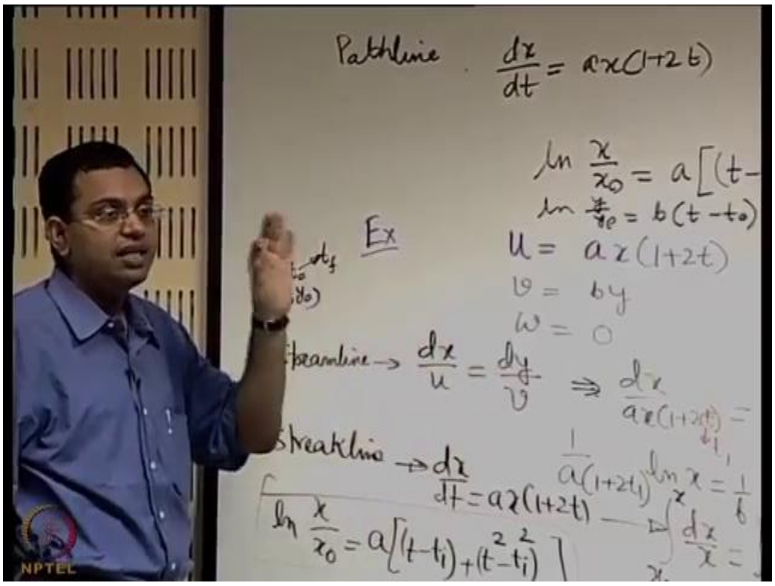

So, let us say that our velocity field has these types of components $\begin{array}{l} u = ax(1+2t) \\ v = by \\ w = 0 \end{array}$. So, it is a 2 dimensional field usually whenever you have a velocity field we call it 1 dimensional, 2 dimensional or 3 dimensional depending on the number of independent velocity components that you are having. So, you are having two independent velocity components, it is a 2 dimensional velocity field; a and b are some parameters and x , y are the coordinates t is the time. And so, a has a and b have certain dimensions with adjust these so, that you get the dimensions of velocities at the end. So, these are dimensional parameters, but constants let us say we are interested to find out the equation of stream line at a time say t equal to t_1.

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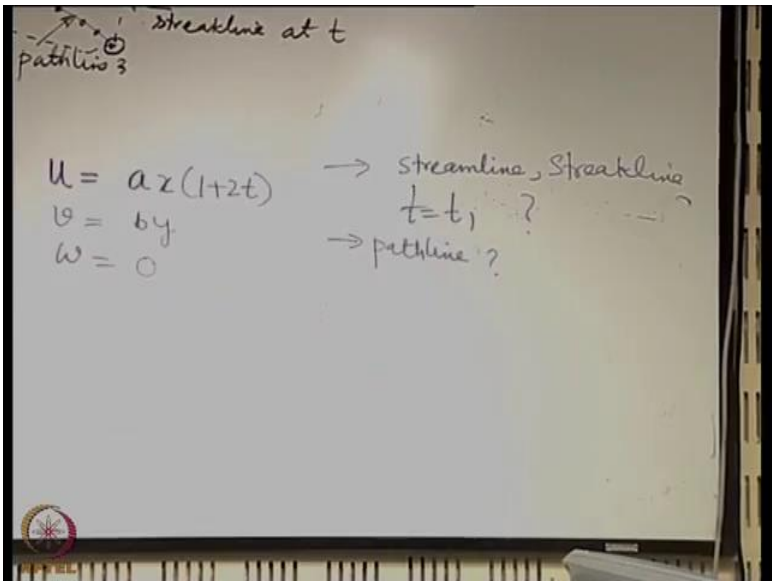

We are interested to find out stream line and streak line at time equal to t 1 that is one objective, the other objective is to find out equation of path line. So, to find out equation of stream line that is the easiest part let us do it first. So, you have $\frac{dx}{u}=\frac{dy}{v}\Rightarrow\frac{dx}{ax(1+2t)}=\frac{dy}{by}$

$\frac{dz}{w}$ is not relevant because it is a 2 dimensional flow. So, dx so we are talking about first streamline.

So, when you are considering a streamline you are freezing the time at the instant that you are considering. It is clearly an unsteady flow so, at different times you will get different streamlines. So, you can integrate this and what you will get $\frac{1}{a(1+2t_1)}\ln x=\frac{1}{b}\ln y+c$. How do you evaluate the constant of integration? You must be given a point on the streamline right say the stream line passes through some point. So, when you are given that the stream line passes through that point from that you can find out c. That is if you know that at time equal to t 1 whatever stream line you are drawing there is one point on it. So, that point when substituted x say x 1 and y equal to y 1 will give the value of c. So, that will give the equation of the streamline if you arrange it properly and in a compact form.

Let us consider the streak line. To consider the streak line you have to remember one thing that this is the velocity; that means, if a fluid particle is injected at a point it will also represent its rate of change of position. That means, you will have $\frac{dx}{dt}=ax(1+2t)$ where, x represent the x component of displacement of a fluid particle which is subjected to this velocity field. Remember fluid particle is inert to the velocity field. So, whatever the velocity field is the field is imposing on it to do it will do that. So, this is what it is imposing. Now, it is possible to find out how x changes with time. It is straightforward, but conceptually not that straightforward. To understand why it is conceptually a bit more involved we will parallelly write the equation of the path line.

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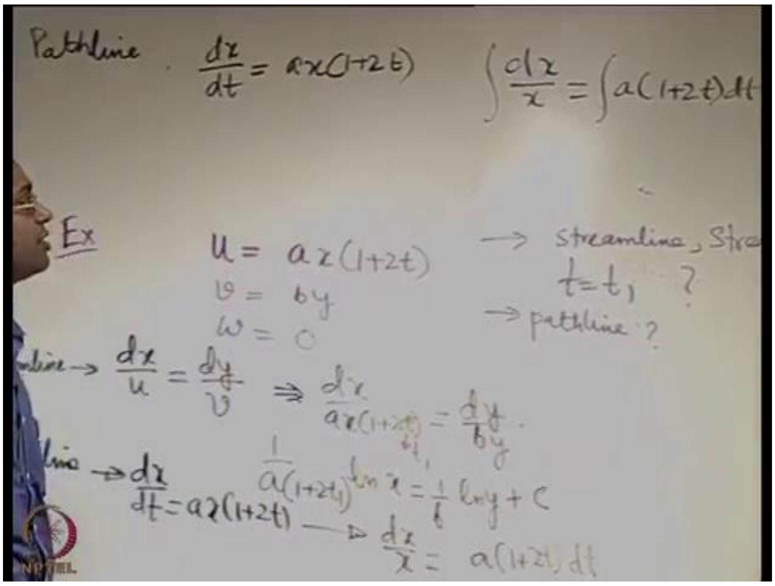

So, let us write the path line. This streak line is not yet complete, but we will draw a parallel analogy with the path line and see where is the difference. So, for the path line again you see path line is what; it is the locus of the fluid particle. So, for path line also there is no need to believe that it should be something different than $\frac{dx}{dt}=ax(1+2t)$ similarly $\frac{dy}{dt}$. Now, the difference in approach comes in the concept by which we integrate these two. So, when we integrate this one say we integrate the equation of the streak line. $\int_{x_0}^{x}\frac{dx}{x}=\int_{t_i}^{t}a(1+2t)$.

Let us say that you are injecting the dye at some point x_0,y_0 ; this is a point of dye injection. The dye injection starts at t_0 and the time the dye injection ends at t_f the final time. So, this is the interval over which the dye injection takes place and the time that we are bothering about

t_1 is something in between t naught and t_f. When you are considering the streak line you have to keep in mind one important thing that importance will be clear when you write the integration here. So, when you write the integration here at time equal to; now here the time=t_0.

Because, you are finding the locus of the particle when you are talking about the path line. So, when at time equal to t_0 $x=x_0$ at time=t, x = x. What is the difference between these two? See look carefully into the limits of t. So, when you say this is t_0 this is the fixed t_0 ; that means, you are finding the locus of a particle which at time equal to t_0 passed through the point of dye injection. Here you are dealing with a variable t_0 because you are dealing with locus of all particles which at different instants of time pass through the point of dye injection. So, t_i is a variable which may be anything between t_0 and t_f . So, this is a variable limit.

So, this actually needs to be eliminated we do not know what is this; only what we know is that t_i has to be between t_0 and t_f. But, it is not a specified time; this is a fixed specified time and that is how you are going to find the path line. So, similarly when you so, when you write this in terms of x. So, you have $\ln\frac{x}{x_0} = a[(t-t_1)+(t^2-t_i^{\,2})]$ right where, t_i is a variable. Fortunately, you will also get another equation involving y.

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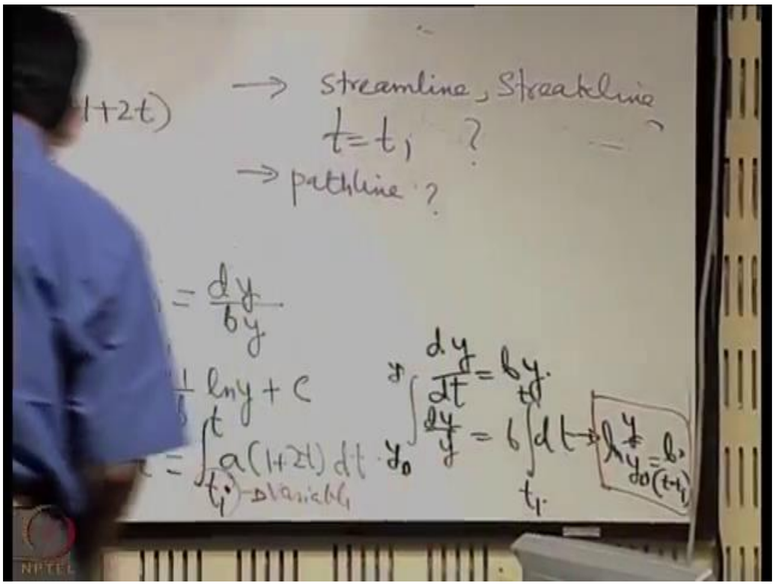

So, you can write similarly that $\frac{dy}{dt} = by$. So, $\int_{y_0}^{y} \frac{dy}{y} = b\int_{t_1}^{t} dt \rightarrow \ln\frac{y}{y_0} = b(t - t_i)$. So, you have an expression here which involves t_i , you have another expression here which involves t_i and you can eliminate t_i from these two to find out the relationship between x and y.

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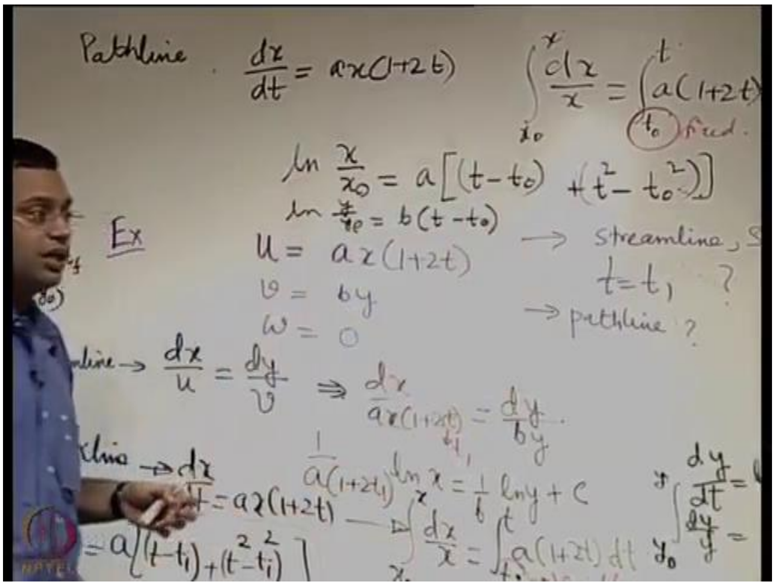

Here that is not necessary because, here you can straight forward write this. So, you can write

$$\ln\frac{x}{x_0} = a[(t - t_0) + (t^2 - t_0^{\,2})]$$
$$\ln\frac{y}{y_0} = b(t - t_0)$$

. Here what is the variable? Here actually t is a variable parameter because at different time it will have different position. To get that locus it is it may be convenient even you may write it in a parametric form, but conceptually you may eliminate t to write the locus y as a function of x whereas, to write this you have to eliminate t_i . So, here t_i is a variable whereas, here t is a variable.

So, conceptually it looks very similar, but there is a subtle difference and that subtle difference needs to be appreciated in the context of streak line and path line; stream line it becomes more or less straightforward. So, I hope you get the distinctive comes between stream lines, streak line and path line and how to find out the equations given a flow field ok. We stop here today.

Thank you very much.