Introduction to Fluid Mechanics

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Lecture - 17
Force on a surface immersed in fluid-Part-II

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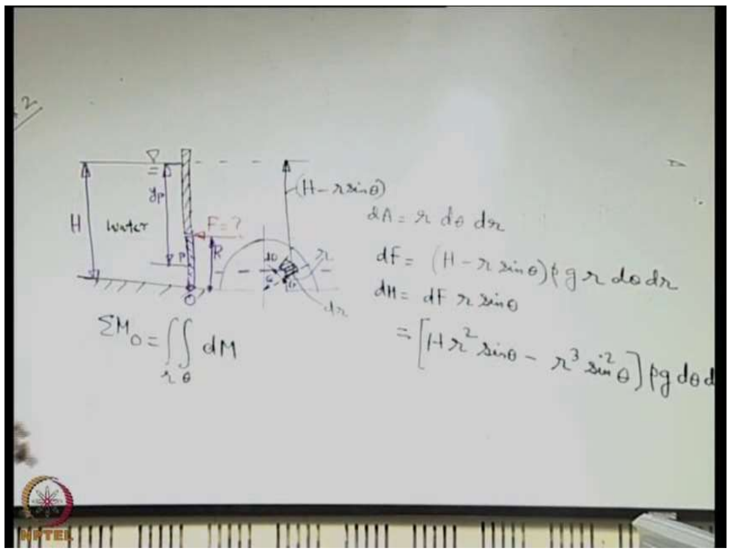

We continue with our discussion on forced on a plain force on a plain surface submerged in the fluid and we take up the same example that we took in the last class and we will try to find out the force on the surface by direct integration without going into the standard expression for force. So, here if we recall what is the objective of this problem is to solve that what is the force that is required to keep this gate stationary and to do that what we required is to find out the moment of the distributed forces acting on this semi-circular plate with respect to the hinge point O that was the one of the objectives for writing the equation of the equilibrium with respect to the rotational equilibrium. So, to do that what we can take we can take a small element.

So, we can consider a small element shaded like this, what is the specification of the small element it is located or centred around r, θ; that means, we consider that this is located at a radial location of r and angular location of θ the angle subtended by the element is dθ, the radial location is small r which is a radial location of the element, the radial width of the element is dr. So, what is the differential area that is being represented by the element?

dA=r $d\theta dr$. What is the force that acts on this element due to pressure, what is the pressure acting on this element? It is related to the local height of the element, what is this local height?

$dF = (H - r\sin\theta) grd\theta dr$, dM=dF $\sin\theta$ $=(Hr^2\sin\theta - r^3\sin^2\theta)\rho g d\theta dr$, $\sum M_O = \int_r \int_0 dM$

So, you may evaluate the integral with respect to r first or θ first it is irrelevant. Let us say that you want to evaluate the integral with respect to r first. So, you have first the integral with respect to r so, when you have integral with respect to r , r varies from 0 to R.

$\sum M_O = \int_r \int_0 dM = \rho g\left[H\frac{R^3}{3}\sin\theta - \frac{R^4}{4}\sin^2\theta\right]d\theta$. One will get an expression for resultant moment of the distributed force with respect to O in terms of H R and of course, ρ and g and this resultant moment is equal to what FR. All other forces acting through it will pass through o. So, they will have no moment with respect to o. So, this is just like these are the two counteracting moments. So, this example shows that whenever you have force on a surface it is not necessary that you have to go by the formula that we have derived, you may as well take a small element consider pressure distribution over the small element and find out what is the resultant force due to that pressure distribution resultant moment and so on by fundamentally integrating it over the entire area without going into the formula. Next what we will consider is force on a curved surface.

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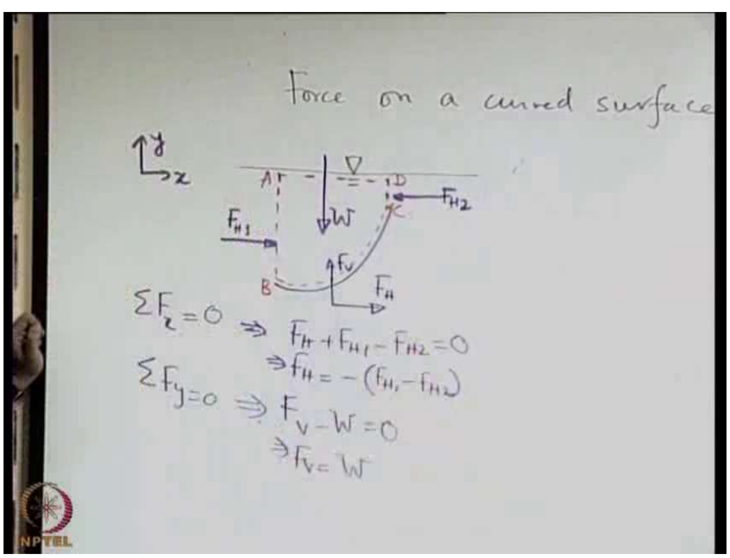

Now, can you tell me that what is the fundamental difference or what you expect the fundamental difference to be as compared to the force on a plain surface how you expect force on a curved surface to be different conceptually; force on a curved surface again what is this surface this is emerged in a fluid at rest so, that we are not repeating.

So, the surface may be something like let us say the surface is something like this. Again if you see the surface is actually something like this and there is some fluid on one side and that fluid is exerting some force on the surface. So, this is again edge view of the surface. Earlier we were concentrating on force on a surface like this is a plain surface, now it is a curved surface. So, you can understand the geometrical difference. That is fine fundamentally in terms of basic mechanics how do you expect these to be different as compared to that for force on a plain surface.

Force direction changes as you move from one point to the other point; that means, it is not a system of parallel forces. So, the plain surface has to deal with a system of parallel forces. So, you can treat it like a scalar addition problem or a scalar summation problem whereas, in this case it is having the pressure always acting normal to the boundary normal to the boundary is a direction that is changing from one point to the other and therefore, the resultant force is being dictated by a varying direction of the surface. So, you no more have a system of parallel elements which is giving rise to the resultant.

So, whenever you are adding here that to make the resultant force it has to be a true vector addition it is not just by adding it up in a scalar way integration is nothing, but summing up the individual components and you could see that very simple integration could give the same result for force on a plain surface, for a curved surface therefore, fundamentally the principle is the same that is you take element of the curved surface you find out what is the force acting on it the force acting on it will be normal to it, you may break it up into components horizontal and vertical components in this way for each of the elements you can find out the horizontal and vertical components algebraically sum them up and make the vector addition.

So, this is a something which is very trivial. Now, it is possible sometimes to reduce the calculation a little bit by taking some help from the concept of force on a plain surface and let us see how. Just consider that there is a fluid column like this, this is a volume of fluid which we are considering to be located within the projected part of this curved surface. So, in this curve ends of the curved surfaces are projected to the free surface whatever volume is contained within that that content volume is enclosed by this dotted line. Now, let us say that we are interested on the forces which are acting on this volume of fluid. So, this volume is now having 2 plain surfaces, 3 plain surfaces, but forget about the top surface just for the time being consider the side surfaces.

So, if you consider the volume say A, B, C, D the surfaces AB and CD are plain surfaces and let us see that what are the forces which are acting on these plain surfaces. So, we are essentially trying to draw a free body diagram of the volume element which is enclosed by this dotted line. What are the forces which are acting on this. So, when you have the left face there is some fluid towards the left of it that exerts the force due to pressure. What will be the direction of that force let us say we set up coordinates like this x and y.. So, we call it a horizontal force assuming that x is the horizontal direction.

Let us say that F_H or F_{H1} is the horizontal force acting from the left towards this element. Similarly, if you have some fluid again on this side there will be a horizontal force say F_{H2} acting from the other side towards this. Remember this force is due to pressure it is compressive in nature so whatever is the fluid element look at the other side it is having a tendency to compress it and that is how the senses of these vectors are there. These are the horizontal components of the forces on the sides, what are the additional forces on this element? It should have his own weight. So, whatever water or fluid is contained here that will have its weight say W.

There is a reaction between the surface and the fluid and that reaction again is likely to have two components; one is the horizontal component and one is the vertical component. Let us say that it has a horizontal component F_H and a vertical component is F_V. These are the components of the reaction forces exerted by the curved surface on the fluid. Now the fluid is in equilibrium.

So, when the fluid is in equilibrium, $\sum F_x = 0 \Rightarrow F_H + F_{H1} - F_{H2} = 0 \Rightarrow F_H - (F_{H1} - F_{H2})$

$\sum F_y = 0 \Rightarrow F_V - W = 0 \Rightarrow F_V = W$. From this apparently very simple calculation we come up with a very interesting result. That is if you have a curved surface stilll whatever forces which are acting on it you may resolve it into 2 components, for the horizontal component it is basically the resultant force on the horizontal projections or on the projections of these curves ends of the curves on a vertical plain or so called horizontal components of the forces on vertical projections of the end of the curve; that means, if you have a curve like this, when you consider the end this end when it when you consider its projection the projection is just like this so, we are basically having to consider a horizontal component of force for the left, a horizontal component of force for the right and the difference of these two is actually giving the horizontal component of the net force. For the vertical component it is just the weight of the fluid that is being contained within this extended volume. So, if you somehow can calculate the weight of the fluid and that is as good as calculating the volume of the fluid because then you can use the density to calculate the weight of the fluid.

So, whatever fluid is contained here within this dotted line weight of that fluid is the vertical component of the force and whatever is the horizontal component of force or whatever is the force because of pressure on the projections taken from the sides of this curve that contributes to the resultant horizontal force. And remember that these are the forces exerted by the curved surface on the fluid we are interested in the other thing the opposite thing that is what is the force on the curved surface. So, by Newton's third law those are just negatives of this one so, minus F H and minus F V are the forces which are exerted by the fluid on a curved surface. Let us take an example to see that how we calculate it.

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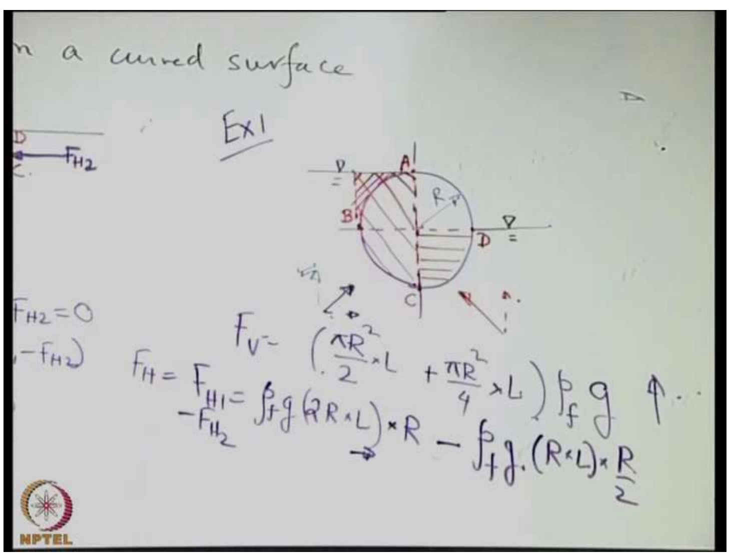

Let us say that we have a long circular cylinder and you have the free surface of the fluid in this way. So, on the left side the this is the free surface on the right side this is the free surface and solid material is a circular cylinder its length is perpendicular to the plain of the board. We are interested to find out what is the resultant horizontal and vertical component of force exerted by the fluid on this cylinder.

So, what we will do is we will give some names of or we will give some markers to important parts of the surface say the 4 important points A, B, C and D and we will consider the forces acting on this curved surface one by one. So, first let us consider the force acting on AB. First let us consider the vertical components of forces then we will do the horizontal component. So, for vertical component what we want we should raise projections from the end of the surface till it reaches the free surface. Whatever is the volume of the fluid contained within that it is the weight of that.

So, for the part AB it is like this next let us consider the part BC. So, when you consider the part AB and you know the vertical component of force is the downwards or upwards, it is downwards, it is clear that the pressure is being exerted in such a way that its vertical component is downwards. Now consider BC. So, for BC what we do again we raise the projection from one end it is like this, from C we raise the projection up to the free surface. So, whatever is the volume that is contained within this. So, what volume is contained within this, these are the volume that is contained between BC and its projected parts up to the free fluid surface. So, this is an imaginary volume of fluid right and what will be the direction of the vertical component of force acting on it upwards or downwards how do you make it out just see that if you have BC like this you will have the pressure acting on it in this way its vertical component will be upwards.

So, it is such a distributed pressure over BC. So, look into the fundamental origin that will give you the guideline whether the resultant forces upwards or downwards on that part of the surface. Now when you consider these two together you can see the common part which is shaded once it has come downwards another it has come upwards. So, they have cancelled out. So, what remains is the fluid equivalent to the volume of half of the cylinder for this half part. So, what is the vertical component of force acting on say ABC it is nothing, but the equivalent to the weight of the volume of half of this cylinder, weight of what weight of the fluid equivalent to the volume of the half of the cylinder.

So that means, as if it has displaced a fluid equivalent to its volume and that is exerting it an up thrust. So, this is nothing, but the Archimedes principle that you have learnt in high school physics. So in effect what is happening what when the solid is being immersed in a fluid it tends to displace a volume of fluid which is equivalent to the volume which is immersed and that tends to exert an up thrust and that up thrust is nothing, but same as the weight of the displaced volume of fluid by that particular volume of solid. Now if you come to CD So, for CD how you calculate the force vertical component of the force. So, you extend it up to the free surface. So, for this part the free surface is up to the level shown.

So, it is nothing, but the volume of this much. So, is it upwards or downwards it is upwards. So, if you have the pressure acting in this way, its vertical component will be upwards. So, the resultant is upwards with what magnitude if R is the radius of this cylinder. Volume is $\left(\frac{\pi R^2}{2}L+\frac{\pi R^2}{4}L\right)$, $F_v=\left(\frac{\pi R^2}{2}L+\frac{\pi R^2}{4}L\right)\rho_f g$. So, this is the resultant vertical component of force. How can you calculate that what is the location through which the resultant of these force passes. So, it will definitely be passing through the centroid of the displaced volume and then it boils down to the calculation of the displaced centroid of the displaced volume, we are not going into that you can do it by simple statics. How do you calculate the horizontal component of force? So, again you have something in the left and something in the right. So, when you have this ABC, on ABC what is the horizontal component of force? Let us say that is F_{H1} just following this symbol and on CD there is a horizontal component of force say that is F_{H2}. What is it projected area on which you are considering that because this is now equivalent to force on a plain surface. So, 2R is the height and L is the length. So, its projection on the side view is 2R is its height because 2R is the height of the cylinder and L is the length perpendicular to the plain of the figure.

$F_{H1}=\rho_f g(2R\times L).R$. H_c is the location of the centroid of the projected area not the curved area. So, it is the location of the centroid of the projected area from the free surface. F_{H1} is toward right because again you can see that its horizontal component is towards right.

$$F_H=F_{H1}-F_{H2}=\rho_f g(2R.L).R-\rho_f g(R.L).\frac{R}{2}$$