Introduction to Fluid Mechanics
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Lecture - 16
Force on a surface immersed in fluid-Part-I

When we are discussing about fluid statics, one of our objectives will be that to find out because of that pressure what is the force that is acting on a solid surface, the solid surface may be a plane one or may be a curved one.

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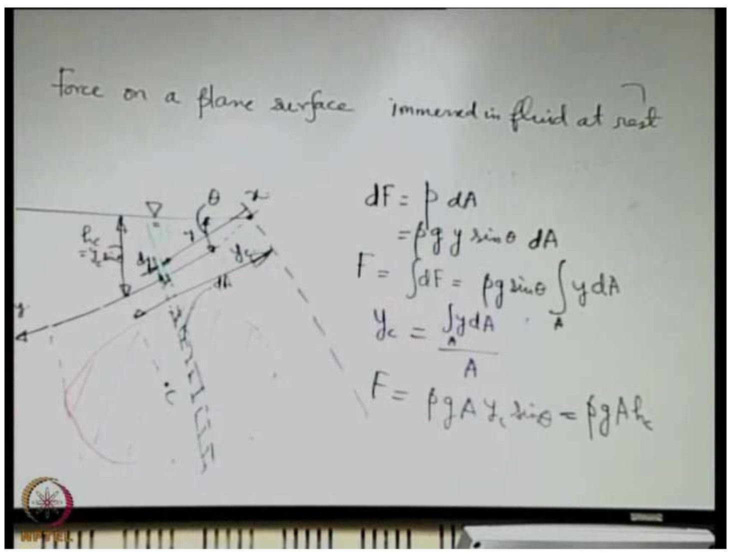

We start with an example of force on a plane surface. What is the special characteristic of the plane surface that is such a surface that we are considering that is immersed in fluid at rest. We will try to make a sketch of the arrangement. Let us say that this is the free surface of the fluid that means on the top there may be atmosphere, in the bottom there may be water as an example. There is a surface the edge view of the surface is like this it is a plane surface.

So, this is a plane surface. And when you are seeing its projection in the plane of the board, it looks like just the edge view that is the line. So, whenever we draw a line, to keep in mind that it is it is some kind of arbitrarily distributed flat surface, the edge of which is represented by this line. So, such a surface is there in a fluid, and we are interested to find out what is the force on this surface because of the pressure distribution in the fluid. With that objective in mind let us try to maybe draw the other view of the surface. It just may be very very arbitrary. So, it is a plane surface of some arbitrary geometry. So, let us say that this is the section of the surface when looked parallel to it.

We will set up certain coordinate axis, let us say we extend this and it meets the free surface at this point. So, we will call this as y-axis and maybe an axis perpendicular to that as x-axis. Our problem is actually a very simple problem. It is a problem of finding out the resultant of a distributed force, because pressure distribution gives rise to a distributed force. Why it is a distributed force, because pressure varies linearly with the height, different elements of the plate are located at different heights from the free surface. So, it is a distributed force. The advantage of handling with a plane surface is that this distributed force is a system of parallel forces.

So, if you have for example, if you consider a small element at a distance y from the axis, and let us say the thickness of the element is dy. So, what is the force due to pressure that acts on this element? Let us say that dF is the force that acts on this element due to the pressure distribution this is the fluid at rest. So, what is dF, dF is the local pressure on the element times the elemental area let us say that the elemental area of which we are talking that can be represented in the other view completely let this be dA. It just corresponds to this dy. So, $dF = ydA$.

dF = $\rho g y \sin\theta$ dA. So, $F = \int dF = \rho g \sin\theta \int_A dA$

Pressure at the free surface is the reference pressure say if the water was not there at the bottom. Just consider this example. Let us say that this is a surface now there is no water, say it is surrounded by air from all sides. If it is in equilibrium; that means, air pressure is cancelling the effect from all the sides together. The net effect is that the sum total force is 0. So, whatever force is acting on the surface is because of the difference from the atmospheric pressure.

So, whenever you are calculating the resultant force on a surface, remember that you are implicitly dealing with a gauge pressure not the absolute pressure because the atmospheric pressure if atmosphere was existing; otherwise it would have kept it in equilibrium. Now, you are having some pressure over and above that. Why atmospheric pressure any other pressure if there is a uniform pressure acting on a closed surface, we will see that later on. That if you have uniform pressure acting on a closed surface then the resultant force of that is 0 that may be proved by a very simple mathematical consideration that you have a distributed force which is always normal to the boundary, then integral of that over a closed boundary 0 if the if the intensity of that pressure is uniform throughout.

So, if you have if you want to find out what is the resultant force, you may eliminate that common part. And consider only that part which is over and above that that is why we are considering only the water effect. Now, you can clearly see that what does $\int ydA$ represent?

First moment of area gives the centroid of the area. $y_c = \frac{\int_A ydA}{A}$. $F = \rho g A y_c \sin\theta$. Let us say that the centroid is somewhere here. So, what we are talking about we have some distance y_c. And this height which we may give just a name say h_c, this is $y_c \sin\theta$ which is the vertical depth of the centroid of the plane surface from the free surface.

$F = \rho g A h_c$. Now, this gives the resultant force, but since it is a distributed force, we also need to find out what is the point through which the resultant of the distributed force passes, so the point of application of the distributed force. To find out that let us say that there is a point P, we give it a name P. And say that P is the point over through which the resultant of this distributed force passes.

So, what we can do to find out what is the location of P location of by location of P, we mean the y coordinate of the point P. So, our objective is to find out, what is the y coordinates of the point P through which the resultant of the distributed force due to pressure passes.

For that we will just use the very simple principle which we have learned in basic statics that if you consider an axis with respect to which you take the moment of forces, then the moment of the resultant force with respect to that axis is nothing but the summation of the moments of the individual components of that forces with respect to the same axis. This is known as Varignon’s theorem and we will try to use that for finding out the location of y_p.

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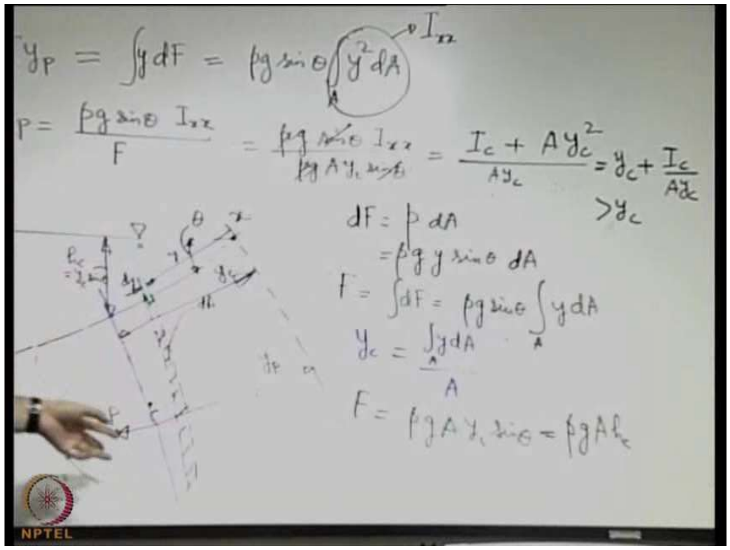

So, if F is the resultant force and the moment that we are trying to take is moment with respect to the x-axis, then Fy_p gives the magnitude of the moment of the force F. This is same as the sum of the moments of the individual components. So, individual component is like you have a dF that is an individual component. So, what is the moment of dF with respect to x, so that is $dF \times$ y. So, the total moment is $\int ydF = \rho g \sin\theta \int y^2 dA$ this is the second moment of area. Sometimes also loosely called as moment of area moment of inertia, just by virtue of a similarity with mass moment of inertia, this is not really a fundamentally moment of inertia, it is better to call a second moment of area.

So, we can write this as the second moment of area with respect to the x-axis, I_{XX}

$y_P = \dfrac{\rho g \sin\theta I_{XX}}{F} = \dfrac{\rho g \sin\theta I_{XX}}{\rho g A y_c \sin\theta}$, I_{xx} represent the second moment of area with respect to some arbitrary axis, it is more convenient to translate that to an axis which is parallel to x, but passing through the centroid. And because centroid is a reference point with respect to a particular surface; and to do that we may use the parallel axis theorem to translate it to c.

So, if we consider an axis which passes through c and parallel to x. With respect to that axis we can write that I_{xx} is nothing but I_c where by c, we mean this axis which is passing through c and parallel to x that axis is totally visible from the view parallel to the surface. So, $y_p = \frac{I_c + Ay_c^2}{Ay_c} = y_c + \frac{I_c}{Ay_c}$. This point P is given a special name in consideration of fluid statics and that name is center of pressure. So, center pressure is the point through which the resultant of distributed force due to pressure passes that is known as center of pressure. We can clearly see from this expression that y_p is greater than y_c because y_p equal to y_c plus a positive term, that means, the center of pressure in terms of depth lies below the centroid right and that is a very important observation.

So, the two things that we learned from this simple exercise, one is to find out what is the resultant force on a plane surface which is immersed in a fluid due to the pressure distribution, and where is the point through which this resultant force acts. We will consider a simple example to begin with to demonstrate that how we may calculate this.

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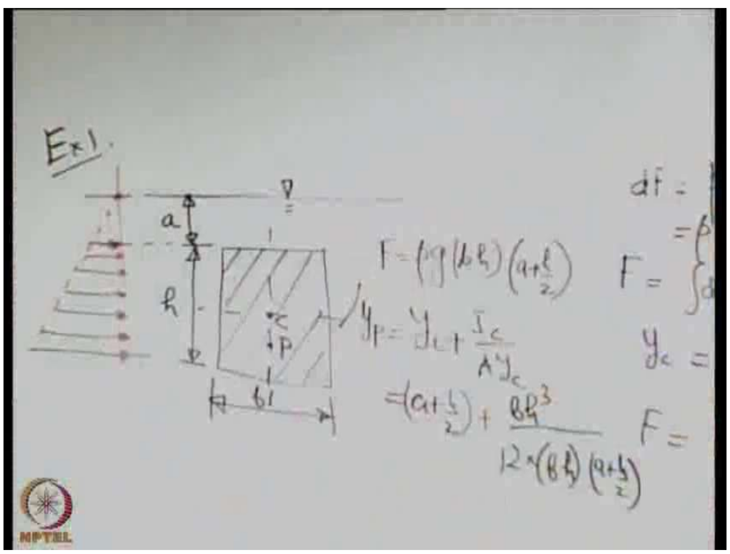

Let us say that you have a surface which in its sectional view is like this. And this is the vertical surface immersed in a fluid. So, this is the fluid. And this subject is a or like this object is a rectangular section with the dimensions b and h. What is the resultant force due to pressure acting on this, let us give a dimension of this say a, this is example 1. The question is, what is the resultant force on this shaded surface because of pressure.

So, take an example like this. So, this is like a surface water or fluid some other fluid is acting on it from one surface. And you are interested to find out what is the resultant force because of pressure distribution on this. So, this is a special case of the inclined situation. So, the inclined situation was like this. Now, we had made it vertical. So, it is also an inclination with theta equal to 90^0. So, for such a surface, now if you want to find out what is the resultant force that acts on the surface, $F = \rho g(bh)\left(a+\frac{h}{2}\right)$, So, you have to now figure out first that is it second moment of area with respect to this vertical axis or this horizontal axis.

So, try to recall that when this was the inclined plate, the second moment of area was taken with respect to this axis right. So, now, when it is vertical, you have a second moment of area with respect to this axis and that is translated to the centroidal axis, so that means, the correct axis should be this one. $y_p = y_c + \frac{I_c}{Ay_c} = \left(a+\frac{h}{2}\right) + \frac{bh^3}{12\times(bh)\left(a+\frac{h}{2}\right)}$. If you want to find out or if you just make a sketch of how these force is distributed, let us try to make a sketch of the pressure distribution. To make a sketch of the pressure distribution, what we note the pressure varies with the depth. So, here the pressure is 0 which is the reference pressure not 0 in absolute sense, but relative to atmosphere. And then it will linearly increase with the depth.

So, at this height, this will be the pressure; at the bottom height, this will be the pressure. And it is a distributed force like this which varies linearly with the height. And you can clearly see that the area under this loading diagram we will eventually give you what is the force. These kinds of examples you have already gone through in basic engineering mechanics and you can verify it for the case of fluid at rest very very similar.

Now, what is the state of stress in which fluid at different depths they are the fluid elements at different depths they are subjected to. To consider that we will refer back to the stress tensor which we introduced earlier when we were discussing with the traction vector I am trying to relate the traction vector with a stress tensor.

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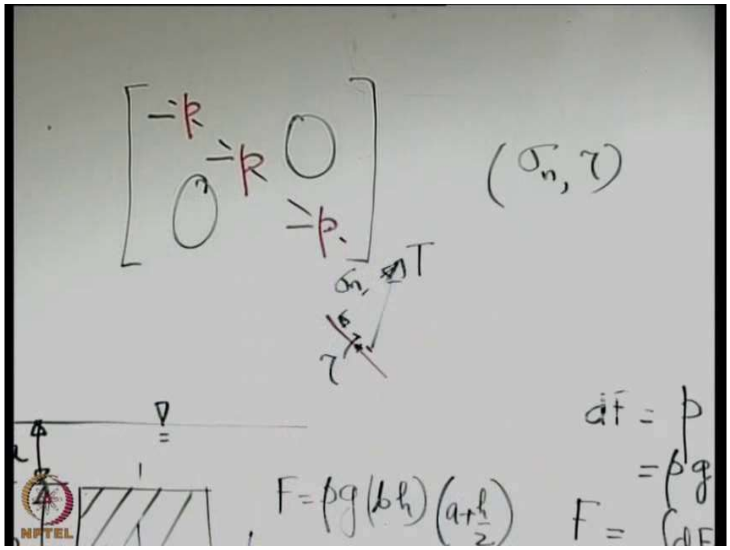

So, whenever you have a fluid element at rest, let us say that we are interested to write the six independent components of the stress tensor. So we have the diagonal elements and off diagonal elements. If you recall the diagonal elements represent normal components of stress, and the off diagonal elements represent the shear components of stress. So, what will be the diagonal components? They will be zero because it is fluid at rest. So, it is not subjected to shear, because with shear fluid will deform. So, these are all zeros.

When you come to the diagonal element, you have only the state of stress dictated by the normal component which is just pressure, and that acts equally from all directions. So, that all the three components will be -p,-p,-p, $\tau_{11}, \tau_{22}, \tau_{33}$. And the reason of putting minus is obvious the positive sign convention of normal stress is tensile in nature whereas, pressure by nature is always compressible.

Now, let us say that we have the task of drawing a Mohr circle of distribution of state of stress. If you recall what is the Mohr circle, so if you consider that there is an elemental area which has an inclination say θ with respect to some reference that has a resultant force, and that resultant force is given by the traction vector components per unit area. You can decompose it. Let us say the traction vector component is like this. You can decompose it into two parts; one is a tangential component, another is the normal component. Let us say we call it σ_n and let us say we call it τ .

So, depending on how you orient the area, you will get different combinations of σ_n & τ. If you draw the locus of that, then that is what constitutes the Mohr circle right. So, Mohr circle gives you the visual appeal or a visual feel of what is the state of stress of different locations, or at different locations based on the choice of different choice of orientation of the area, or may be at the same location with different choice of orientation of the area. So, when we are drawing up one particular Mohr circle, we are concentrating on one particular point what the orientation of the area to get the feel of the normal and tangential components of the forces on that.

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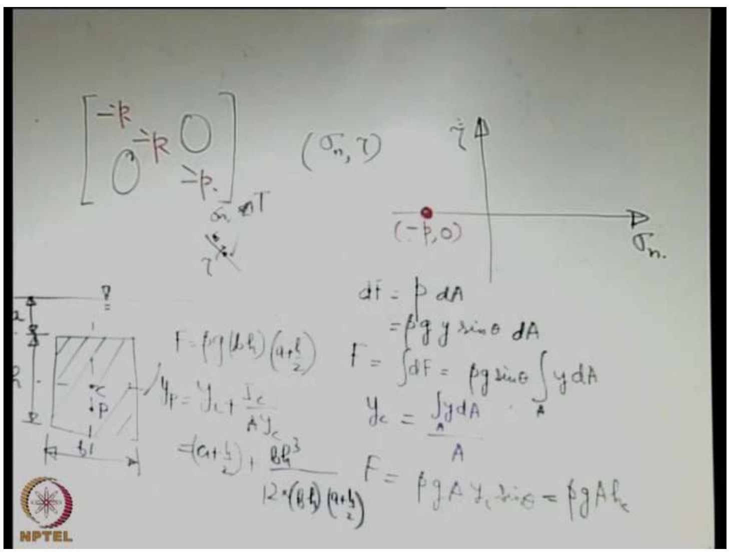

So, we are having two axis, one is σ_n and another is τ. Our locus is or our objective is to find the locus of σ_n versus τ or τ versus σ_n. So, how will the Mohr circle look like for such a state of stress this state of stress is a very unique one. And this is known as hydrostatic state of stress the reason is obvious it represents a hydrostatic physical situation.

So, how will the Mohr circle look like, see at a particular point you have the normal component of stress that is because of pressure and it acts equally from all directions. That means, if you change the orientation of the plane, σ_n will not change, σ_n will be unique. And what will be τ, τ will be 0. That means, no matter whatever plane you choose σ_n = -p and τ = 0. So, the locus of all states of stress converts to a single point with coordinate (-p,0). So, the Mohr circle becomes a point say (-p,0). So, this is just a point not a circle, I have encircled it, but just like to show that it is a point ok. So, this is a very important interesting limiting case, when the Mohr circle swings to a point signifying that there is no change in state of stress with change in orientation.

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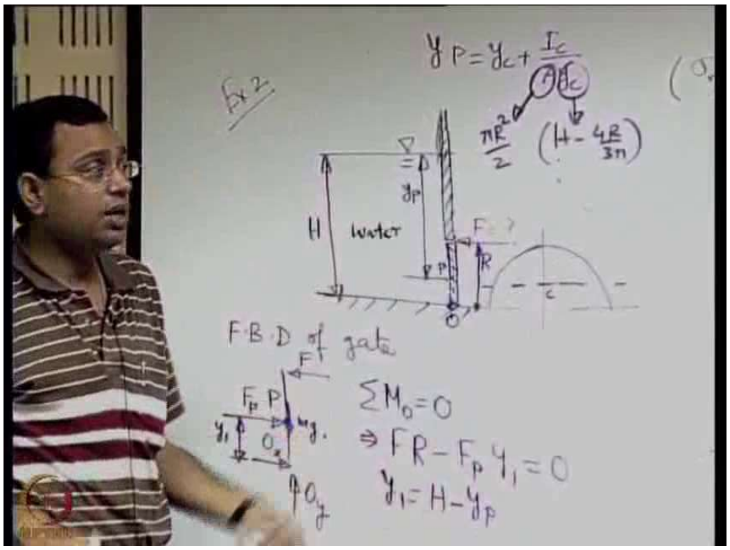

Next we will consider another example on force on a plane surface. Example 2, we will make this example a bit more involved than the previous one. Let us say that there is again a free surface, there is some fixed structure there is a base and there is a gate like this. So, it is something like a sewage gate. So, on one side, there is water and it is so that this water is being confined in a particular place, and this gate has a tendency to move because there is a resultant force of water from the left side.

And there must be some mechanisms or holding mechanism by which this gate is kept at rest at equilibrium. So, there is some force which is acting on the gate, say the force is this F by some support or whatever, there is some force F which acts on these to keep it in equilibrium. This gate may be of different shapes and let us take an example with the gate shape as a semicircular one like this. So, this is the section of this gate.

So, on these gate, there is a force due to water. And that force you may calculate by considering some dimensions, one of the dimensions say this is H. Let us say that the radius of this semicircular gate is R. And the density of the fluid of course, is given and g. This gate is hinged at this point say O.

So, this is a situation where if you want to find out what force F should keep it in equilibrium, you must calculate what is the resultant force due to pressure acting on the gate, this is a plane surface shape of the plane is a semicircle, but it is still a plane surface. And you may use the formula for force on a plane surface. So, let us try to draw the free body diagram of this gate. So, our objective is that what should be this force to keep the gate in equilibrium.

So, we draw the free body diagram of the gate. You have a force F. What other forces you have, you have the hints reactions say let us say O_x and O_y , the two components. This has its own weight, so some mg. And the force due to pressure distribution in water let us say F_p, which passes through the center of pressure. So, for equilibrium, the resultant moment of all forces with respect to O should be 0, $\sum M_O = 0$. The reason of choice of O as moment center is obvious it eliminates all unknowns except the F that we are interested to find out.

$$F.R - F_p.y_1 = 0 \ , \ y_1 = H - y_p \ , \ y_c = H - \frac{4R}{3\pi} \ , \ A = \frac{\pi R^2}{2}$$

I is second moment of area with respect to an axis of a semicircular thing which passes through its centroid

But we will see that this derivation is not necessary because we will try to avoid this root of this formula based determination of this. And we will just do it from the fundamental method of integration by which you find out the resultant of a distributed force the resultant moment of distributed force and so on. And the entire reason is that there are certain simple areas for which we may remember the expressions for the second moment of area with respect to the centroidal axis quite easily, but it is not so convenient for many complicated areas. This is not complicated as such, but even for that we should not tax our brain by remembering that I mean that is not a very special information that we should remember.

So, what we will do is in the next class we will try to see we will keep this problem in mind, we will see the alternative way by which we will be solving this problem. You can of course, solve this problem by substituting the value of I_c here expression. And this expression is given in the appendix of the textbooks in statics. So, you can find out the expression or you may even derive it if you want, and just substitute it to get what is y_p; and from that you can get F. But we will see that whenever possible and whenever convenient it may also be all right if we just find it out by simple integration of the distributed forces, so that we will do in the next class. Let us stop here.

Thank you.