**Introduction to Fluid Mechanics Prof. Suman Chakraborty Department of Mechanical Engineering Indian Institute of Technology, Kharagpur**

## **Lecture – 15 Manometers**

We continue with our discussions on fluid statics. And, what we will first do today is we will see some other mechanisms by which you can measure the pressure of a fluid.

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In the last class we say the example of a barometer; today we will first see the example of a manometer. Since this concept is well known to you we will briefly recapitulate it through an example. Let us say that there is a pipe and there is some fluid which is entering the pipe there are two different sections say marked by A and B and we are interested to find out what is the difference in pressure across these two. To do that what we will do?

We will be considering a u tube connected across these sections. This u tube will be the so called manometer and it will try to measure the pressure difference between these two. To see how we how it actually does, let us say that this device contains some other fluid, of course there is a fluid which is passing through the pipe say that fluid is water. Now, the fluid which is a part of this u tube manometer it is something different because it is the difference in density of these two fluids which will dictate the principle of operation of this device. Let us say that that fluid is mercury as an example. Mercury is a very common fluid for manometers and we just take that as a specific example.

Now, when the fluid is flowing from A to B which side you expect to be of higher-pressure A or B? You expect A to B of higher pressure. We will see later on that it is not always true that fluid will flow from higher pressure to lower pressure, it will definitely flow from higher energy to lower energy. And here other components of energy being unaltered as an example like the kinetic energy and potential energy therefore, the pressure differential in the sole driving parameter for the fluid flow to take place.

If A has a pressure greater than B, and if we have some fluid here which is which is not water, but some other fluid say mercury on which side you expect it to be more depressed on A side or on B side? So, wherever the pressure is more it will depress it more. So, let us make the sketch accordingly, and let us say this is mercury and let the difference in the level in the two sides or the two limbs of the manometer be  $\Delta H$ .

What is our objective? Our objective is to find out what is the difference in pressure between A and B. So, we want  $P_A-P_B$  to be evaluated. To do that we will just consider some reference dimension; let us say that this is say  $Z_1 \&$  this is  $Z_2$ , the difference between these two of course is  $\Delta$  H. What fundamental principle we will use here? We will use the fundamental principle that in the absence of any other body force when the fluid is at rest within this manometer. The pressure in a connected system is same along the horizontal line it, does not change. So, if you mark these points say C and D you must have the pressure at C and pressure at D as same.

So,  $P_c = P_A + Z_1 \rho_w g = P_B + Z_2 \rho_w g + \Delta h \rho_{wg} g$ . It is because of a combination of some column of water and some column of mercury.

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P_A - P_B = \Delta h \rho_{wg} g - \Delta h \rho_{wg} g = \Delta h \rho_w g \left( \frac{\rho_{wg}}{\rho_g} - 1 \right)
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what is  $\Delta H$  this you can experimentally measure, the difference in the height of the two limbs you can clearly find out what is the difference in pressure between two points. The factor that is dictating your result is the relative density of one fluid over the other, and of course, not only that what is the density of the original fluid that is also important. So, this is in a very simple way what is the working principle of a manometer.

You may have manometers in series connected from one point to the other point and if you want to find out the pressure difference between two points what you may do is you may start with a point and traverse along the manometer. When you are traversing along the manometer along the same horizontal level if you have the same connected system you may consider the pressures to be equal. So, you may neglect whatever is there below that, and that is how you may go from one point to the other calculate the pressure difference, go on calculating the pressure difference this is like coming down and going up like snake ladder game.

So, you come down somewhere find out what is the increasing pressure, again go up find out what is decreasing pressure so on, along whatever fluid you are traversing you have to consider the density of that corresponding fluid for calculating the pressure differences. You can clearly see that these devices although we say that these are pressure measuring devices, but that is a very loose way of looking into this because these actually measure pressure difference not really absolute pressures. So, whenever you have a fluid you expect that that there may be different pressures at different points and this mechanism is only trying to find out pressure at one point relative to the other not in a very absolute sense.

The other important remark is if you see that the resolution of this device strongly depends on what is this  $\Delta H$  because if this is very small there can be a lot of reading error and that will magnify the error in the description of error in the determination of what is  $P_A - P_B$ . So, if this quite large or magnified it may be easier for us to read with greater accuracy. To do that sometimes people use inclined tube manometers. To understand the working principle we will again take an example.

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In an inclined tube example, it need not always be a u tube, it is basically having a measuring tube. It may be a collection of measuring tubes may be u tube, but the axis of the tube is inclined with the vertical it is not vertical. So, let us say that you have a tank like this and you have a tube connected to it in this way. Initially everything was exposed to atmosphere and let us say, that this red colored dotted line represent the initial height both in the inclined limb as well as in the tank. Let us say that capital D is the diameter of the tank and small d is the diameter of the tube, assume both are circular cross section.

Now, let us say that you apply a pressure differential that is you apply a higher pressure from the top, if you do that this level will be depressed it will come to a new location let us say it comes to this location. When it comes to this location obviously, there is a drop in height and where this extra liquid will go it will climb up, in the incline; it will come say to some height like this.

And you therefore, now have a net difference in the level as say L. L is the new level difference earlier there was no level difference, but because of the application of pressure differential say  $\Delta p$  the fluid in the deep tank has gone down, the same volume has gone up along the inclined tube and the level difference now is L. This is inclined difference and that is what you can read, if you graduate with lines on or markers on this inclined tube you can read the length very easily just like scale. Let us say that you have the angle of inclination as theta with the horizontal.

Now, one important principle that may guide you that what should be the corresponding rise, if you have length up to this much say  $L_1$ , you can clearly relate  $L_1$  with say the distance by which the level has gone down in the big tank let us say that that is  $\Delta$  h. Since the big tank this may be small since this is of smaller cross section the change will be large. So, we can say from the conservation of volume that 2  $\pi D^2$  $\frac{1}{4}$  =  $L_1 \times \frac{1}{4}$  $\Delta h \frac{\pi D^2}{4} = L_1 \times \frac{\pi D^2}{4}$ . So, this from the consideration that the volume at state 1 is same as the volume in state 2, that is what is the principle that is guiding this very simple equation. So,  $L_1=$ 2 2  $h\frac{D}{\cdot}$ *d*  $\Delta h \frac{D}{r^2}$ .

Now, what is the difference in pressure now between the two levels? It is that difference  $\Delta p$ because if one end is exposed to atmosphere then the other end is subjected to a pressure of say difference of  $\Delta p$  then that should give rise to the difference in level.  $\Delta p = (L \sin \theta) \rho g$ .

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L = L_1 + \frac{\Delta h}{\sin \theta}.
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 So,  $\Delta p = (L \sin \theta) \rho g = \left(\Delta h \frac{D^2}{d^2} \sin \theta + \Delta h\right) \rho g = \Delta h \rho g \left(\frac{D^2}{d^2} \sin \theta + 1\right).$ 

You may of course replace  $\Delta H$  with L<sub>1</sub> again because at the end it is not  $\Delta H$  that you hare measuring its  $L_1$  or  $L$  that you are measuring. So, you can write it express in terms of say  $L$ . So, either way I mean mathematically you can express either in terms of delta h or in terms of L, but when you come to the final expression it is more convenient if you express in terms of L because that is what you can experimentally read, more clearly and that is the whole objective of keeping it inclined.

So, when you keep it inclined you can see that the for the same vertical height you can get more inclined distance and pressure differential is dictated not by the inclined distance, but by the vertical height. So, for the same vertical height you will have the same pressure difference, but with more inclination you can have more inclined length and that is how you may have a greater length for greater readability or for better readability for the pressure difference measurement.

If you have say the same system, but with a vertical tube say with water as a fluid then that will be equivalent to some  $h_w \rho_w g$ . So, the *w L h* is an indicator of the sensitivity of this device that  $h<sub>w</sub>$  for a vertical rube being very small, but for an inclined tube the corresponding L which

shows the same pressure difference will be quite large. So, it adds to the sensitivity of the device. So, *w L h* is also an indicator of the sensitivity of the device.

Now, it is not, but the manometer that is normally used for measuring pressure differences. If the pressure differences are not very large sometimes there are inexpensive means of measuring pressure differences, and one such example is a pressure gauge. So, pressure gauge it is an indicator of pressure at a point relative to some other reference pressure. Very often that reference pressure is the atmospheric pressure and therefore, it implicitly many times reads the gauge pressure. We have earlier defined the gauge pressure is the pressure relative to the atmospheric pressure at that location.

So, we see an example with a demonstration. You see pressure gauge. The example that we will be seeing here it is known as a bourdon gauge.



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So, if you look at this one, we will see the example once more. But if you see that it has a tube and the tube is connected to a mechanical arrangement, and one end of the tube is fixed and the tube is of say elliptical cross section, it is a deformable tube. When fluid enters the tube then what happens, when fluid enters the tube the tube gets deformed and the section of the tube tends to get more circular as compared to the elliptical one.

So, what is happening is, there is a fluid that is entering the tube one end of the tube is fixed, the other end of the tube is moving or movable as the fluid enters the tube there is a deformation in the tube and that deformation is being read by a dial indicator. So, the deflection of the indicator in the dial gauge it gives an indicator of the deformation of the tube and the deformation of the tube has taken place because of application of a pressure differential. Earlier it was 0 deformation or a base state deformation now with application of a pressure difference there is a change in deformation from that. So, if this is calibrated then one may calibrate the deformation as a function of the applied pressure difference it is just like calibrating a spring because of application of a force.

So, it is just like a spring mass type of arrangement. So, this name of this device is bourdon gauge. So, that is an interesting or a simple way of measuring pressure. The more advanced ways of measuring pressure are not these ones, the more advanced ways are by utilizing principles of certain things known as transducers. So, what the transducers do they convert maybe one form of signals mechanical form of signal into an electrical form of signal. So, there are certain piezoelectrical pressure sensors.

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So, what do these do? Say piezoelectric pressure sensors, so these are made up of materials which when subjected to a pressure difference converts that into an electrical voltage signal. So, that electrical voltage signal is read by a convenient mechanism and that is how you may have a digital output which does not directly show the pressure difference, but it shows the corresponding electrical voltage that is developed and if the pressure difference is calibrated with that voltage then it is possible to very accurately describe that what is the pressure difference that actuated that voltage. One obviously, needs to calibrate this type of device, but once it is calibrated it may be very very accurate. So, it is based on the transformation of signal in one form say mechanical form into a signal in another form like the electrical form.

In a summary what we may say that we have discussed very brief about the different pressure measuring devices starting from the simple barometer to manometer, pressure gauges, and pressure sensors in piezoelectric form. With these pressure sensitive devices what we may experimentally find out is what is the value of pressure at a point maybe relative to some reference.