Introduction to Fluid Mechanics
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Lecture – 12
Surface Tension- Part- I

So, the next fluid property that we are going to learn is Surface Tension, and what surface tension can do and what it cannot do. First let us look into some images before we go on to the more mathematical description of the surface tension.

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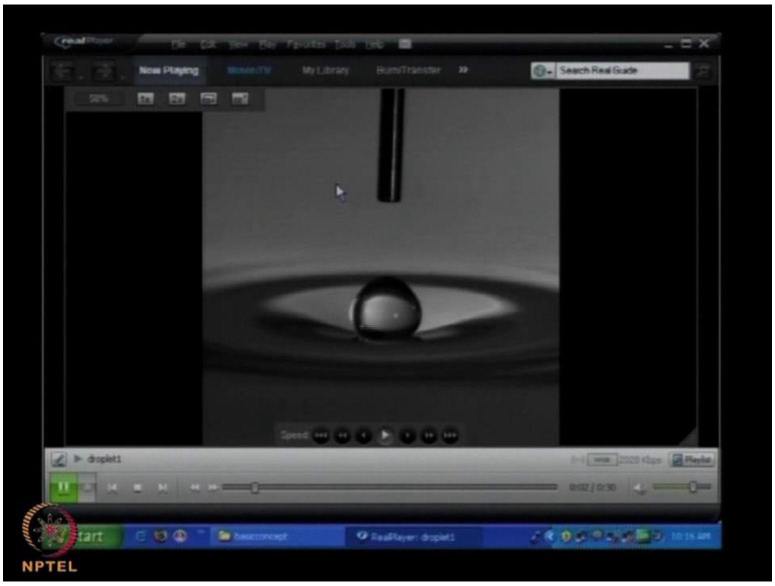

So, first see this example. So, this example is like you have a fluid droplet us which is interacting with another fluid, and see that what type of behaviour is taking place. And many times it looks like a magical behaviour and surface tension really can create magical behaviour. So, it can create instabilities in jets and droplet us and this type of instability is very very common.

I will tell you that qualitatively why should you have.

It seems you are liking this very much, but I am not sure that you will be liking the mathematical details which go behind this very much. So, that is what we are trying to do.

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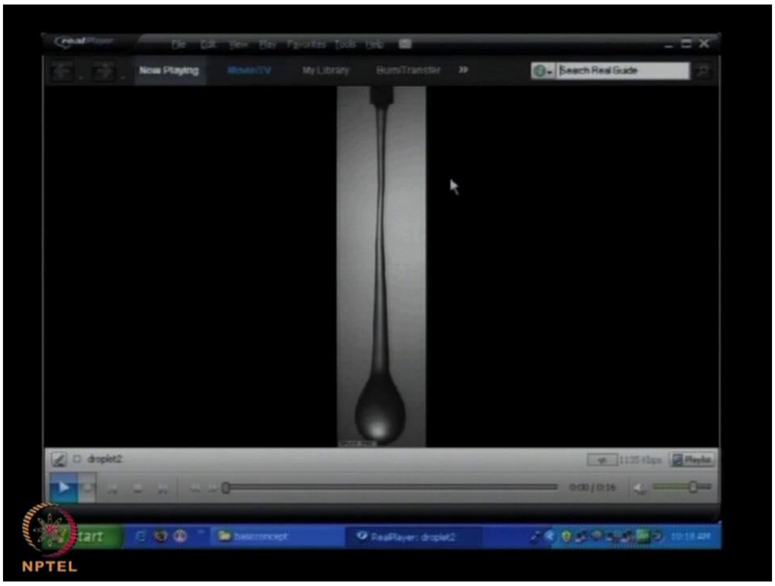

So, we look into second example. Some of the examples later on may appear to be even better than what you saw earlier. So, this is breaking up or kind of making of a liquid column. So, there is a liquid column it is getting broken up and the reason is that see what are the forces which are acting here one is viscosity, viscous force another is surface tension force we will come across that. And there are competitions between these two. So, if the viscous resistance is overcome may be surface tension driven instabilities can break it up or tear it up into nice pieces.

And the third example, this is like a train of droplets and you see the droplets are like, these are not like rigid spheres. So, they are continuously deforming and you see the way in which these droplets are moving. So, initially one big droplet and then you see that there are trains of droplets of different sizes and they are continuously interacting with each other. So, at least you can appreciate that this is beautiful, but this is a very complex physical phenomenon. It is not such a simple phenomenon; it is not like that there is a rigid valve falling from the top towards the bottom. And fluid mechanics therefore, is something which is fascinating, but it is not as trivial as sometimes mechanics of simple particles.

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So, we will see may be one more example. You see the type of pattern that is being created by a dye on a surface. So, we will just play it once more to wrap this visual display up and go to see that what is the fundamental that goes behind, ok.

So, let us therefore, with this motivation try to understand that what is this surface tension all about and what is its implication.

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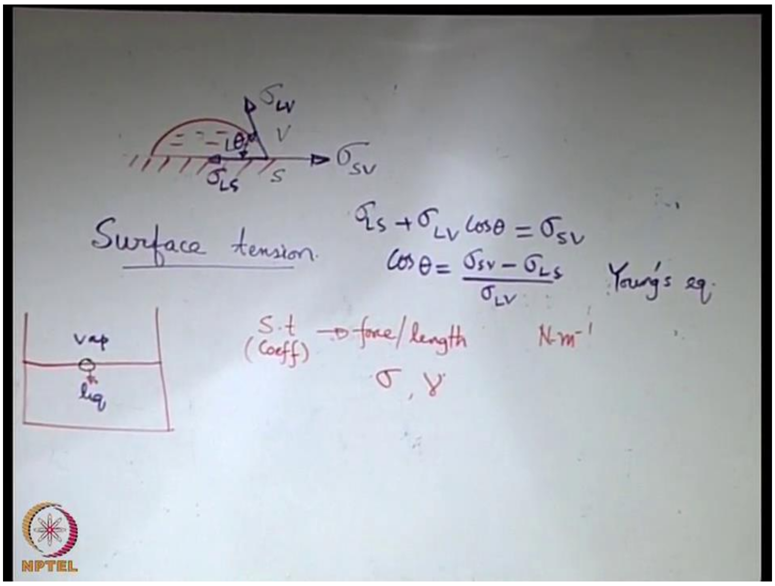

So, we go to the description of Surface Tension. Again, this is a very involved topic we will try to develop very elementary qualitative feel of what it is about. Let us say that you have a container, in the container there is an interface between so called vapor and liquid very common, you may have water and water vapor, and liquid water and water vapor, and there may be an interface.

Now, what we are trying to do, we are trying to focus our attention on what happens to the molecules which are there at the interface. So, let us identify some molecule which is there at the interface. Let us say this molecule is sitting on the interface, and what are the forces to which this molecule is subjected let us try to investigate that. Clearly you can see that, when there is a molecule at the interface there are surrounding molecules, and this surrounding molecules on one side there is vapor on another side there is liquid.

Vapor is obviously much less dense than liquid. And what you expect? You expect that which side will be pulling this molecule, mode liquid side or vapor side. The liquid side will be pulling it more. So, it is institutively expected that this molecule will have a net resultant pool that is acting on it. When there is a net resultant pool that is acting on it then there would have been a great chance that this molecule get will get dissolved in the liquid, but it does not happen like that because an interface is always formed. So, there is something that make sure that this interface is always formed. What is that something that we will now understand.

So, one important concept that we can appreciate is that at the interface there is a resultant force on the molecules which are there at the interface, despite that resultant force the molecule still holds its presence at the interface. That means, it has some additional energy or effective energy by which it can by virtue of which it can sustain its position at the interface by overcoming this net interaction and that energy is known as surface energy.

And whenever you have a surface energy this surface energy is also manifested in form of a force because it appears that the molecules here are in a sort of a state of tension. So, when they are in a sort of a state of tension because of this net pulling and pushing that particular force which is responsible for keeping it in tension is also known as surface tension. So, surface tension and surface energy are quite related. And typically, whenever we express surface tension we express surface tension as force per unit length. What is that length? The length is the perimeter length on which this force is acting.

So, surface tension therefore, is a force per unit length. So, in SI unit it will be Newton per meter. So, this is not the surface tension force, but this is surface tension coefficient. So, when we say surface tension we loosely say surface tension, but actually it is surface tension coefficient. The surface tension force of course, is this times the length on which it is acting that is understandable. Typically we use two symbols to denote this either σ or γ . These are the common symbols which are used to designate surface tension.

Now, if we want to see that what how the surface tension keeps a system in equilibrium, let us take an example of a droplet or a part of a droplet sitting on a solid surface. So, this is liquid, and the right side say this is vapor; so this is liquid vapor and this is solid. So, you can see that at the interface between this there is a triple junction that is created. You have a place where you have a sort of contact between liquid vapor and solid.

Now, if you want to see that, what is the equilibrium that sort of keeps this in perspective or keeps this in equilibrium then we can see that you have a force in this direction. This force is because of the surface tension between or maybe we let us just show it in the opposite sense, sense will automatically come out if we write the equilibrium but let us just show it in a opposite sense. Let us say that we show it like this because just to appreciate that it is an element intention. So, this is because of the interaction between which two phases?

Solid and vapor. So, let us give it a name, let us give it a name σ_{SV} , S for solid, V for vapor. So, whenever we are talking about the surface tension coefficient it basically talks about two different phases which are forming an interface and that is where the surface tension comes into the picture. If we are thinking of this one, this is between liquid and vapor. So, surface tension is tangential to the interface. And we can give it a name σ_{LV} you could also write σ_{VL} and it is just the two phases which are important not the order in which you write is important. And regarding the liquid and solid you have an interface here maybe σ_{SV} . There is an angle between these two say θ which is known as the contact angle.

If you write the equilibrium along the horizontal direction then you can clearly see that you can write $\sigma_{LS} + \sigma_{LV} \cos\theta = \sigma_{SV}$. So, whenever considering an elemental area that or an elemental length that elemental length is cancel from all sides, so only the σ 's remained. It is basically a force balance, but it looks like a surface tension coefficient balance because the corresponding length that is on which this acts like its may be a unit length like that and that gets cancelled. So, you can see that you get $\cos\theta = \dfrac{\sigma_{SV} - \sigma_{LS}}{\sigma_{LV}}$.

So, if you know what are the surface tension coefficients between two phases taken at a time then from that you can estimate the contact angle. This contact angle this is known as a static contact angle of course, if it is dynamically evolving then the contact angle may change and this is known as Young's equation; just, a simple equation that relates the contact angle with the surface tension coefficients at equilibrium.

Now, the question is, is it like is it the only condition for equilibrium or is it something different. To understand that we will try to consider a more involved situation, when you have say a sort of element like this and say you are stretching the surface. This may be a small element of the surface of a droplet and say we are trying to stretch it.

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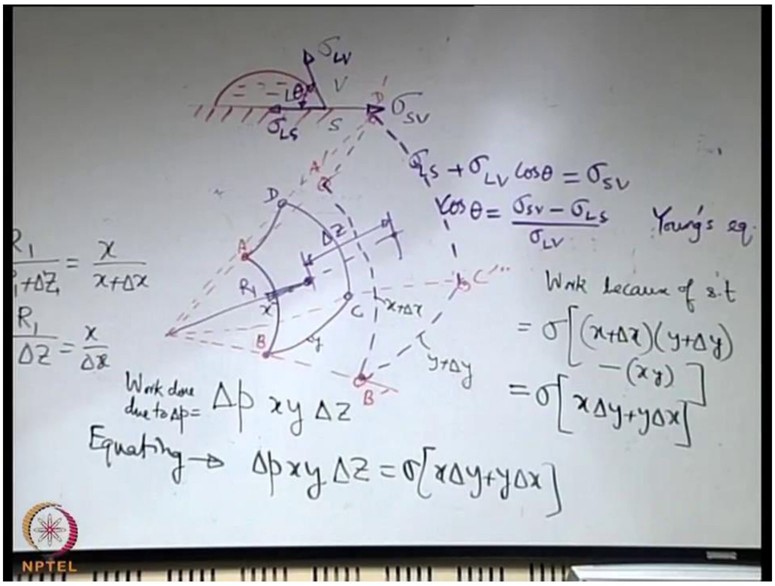

So, let us see that what are the kinematics or what are the even the kinetics of the stretching. So, kinetic aspect we will forget for the time being and we will just concentrate on the forces which are responsible for the stretching and the geometrical change which are responsible for that.

So, to do that we will just draw some sketch this kind of thing we are trying to draw to see that as if there is a stretching which takes say the points A , B to new locations A' and say B'. Similarly if you draw the same type of radial lines the other points which are forming the boundary of this they will also go to different locations. So, it is possible that you get a new location for the other points that is C and D because of the stretching. So, you can get a C' and D' let us you will try to locate some C' and may be some D'.

So, we have a deformed element, but may be similar in terms of the geometrical characteristics because we have used a sort of stretching and it comes to a new configuration. You can clearly see that this element is made up of actually two different types of lines one sort of parallel line say AB and CD, another sort of parallel line like BC and AD. Therefore, we say that it has two different curvatures in two different planes. So, this, like AB and CD let us say they have a particular curvature and AD and BC they have a different type of curvature.

So, let us say that we are calling this radius of curvature as R_1 and let us say that with respect to this R_1 now there is a displacement. So, when we say R_1 we basically mean up to the centre, so you have to imagine the, this is like a surface which is a curved one. So, as if these goes to the centre of this one and then from this so maybe you just stretch it like this and then from here to here let us say that this is Δ Z is the displacement.

Let us say that originally the dimensions of these curved surfaces where like x and y, now x becomes x $+\Delta$ x and y becomes y$+\Delta$ y. So, we can write from the similarity of the entire geometry that $\frac{R_1}{R_1+\Delta Z}=\frac{x}{x+\Delta x}\Rightarrow\frac{R_1}{\Delta Z}=\frac{x}{\Delta x}$

Now, we are interested to see what are the forces which are acting on it say what stretches it let us say there is a pressure differential between the outer and the inner of this. So, this is a membrane. So, the membrane has a difference in pressure from the outer and the inner. Let us say that difference in pressure is Δ p, when you have a difference in pressure of Δ p that gives rise to a force. What is the force? That acts on an area xy, this is almost like a rectangle these are small elements because if you take big elements the local radius of curvature change has to be taken into account. So, this is just magnified for clarification, but this is small.

So, Δ pxy is the resultant force because of pressure differential between the inner and the outer surface of the membrane and Δ pxyΔ Z is the work done because of the pressure difference. So, this is work done due to delta p and that contributes to the surface energy. So, this should be the corresponding work because of the surface tension. So, what is the corresponding work because of the surface tension? That is the surface tension coefficient times the change in area.

So, the work done because of surface tension, let us say σ is the surface tension coefficient between the two phases interacting here that times the Δ a. So, work= $\sigma[(x+\Delta x)(y+\Delta y)-xy]=\sigma[x\Delta y+y\Delta x]$

This is basically work associated with stretching of a surface and this pressure differential is creating this displacement it has undergone a stretching the surface has some energy now to sustain that and retain its form. So, that is by virtue of the surface tension.

Now, if we equate $\Delta p xy \Delta z = \sigma[x\Delta y + y\Delta x]$

So, $\Delta p = \dfrac{\sigma}{\Delta z}\left[\dfrac{\Delta x}{x} + \dfrac{\Delta y}{y}\right]$

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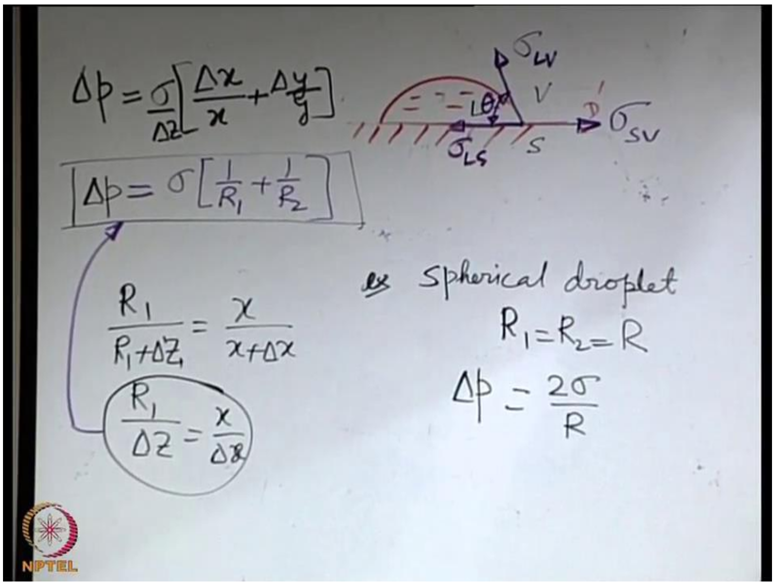

So, now if you use this relationship $\dfrac{\Delta x}{x} = \dfrac{\Delta Z}{R_1}$. So, this relationship can be utilized. Similarly if you write in terms of y it will be R_2 that is the only difference where R_2 is the other the radius of curvature for the other elements. So, this can be simplified by taking help of this and similar expression as $\Delta p = \sigma\left[\dfrac{1}{R_1} + \dfrac{1}{R_2}\right]$.

So, if you have an element of an interface in equilibrium then this is how you can relate, the pressure differential across that, with the surface tension coefficient and the radius of curvature or the radii of curvature of the elements that are constituting the surface. We can take examples as special cases which are convenient examples to take; we consider first spherical droplets or maybe a spherical bubble.

So, if you consider a spherical droplet then what is the situation? If you have a spherical droplet as an example. So, when you have the interface of the sphere it has same radius of curvature at all points. So, for a spherical droplet of radius R you have $R_1 = R_2 = R$ and therefore, you have

$$\Delta p = \frac{2\sigma}{R}$$

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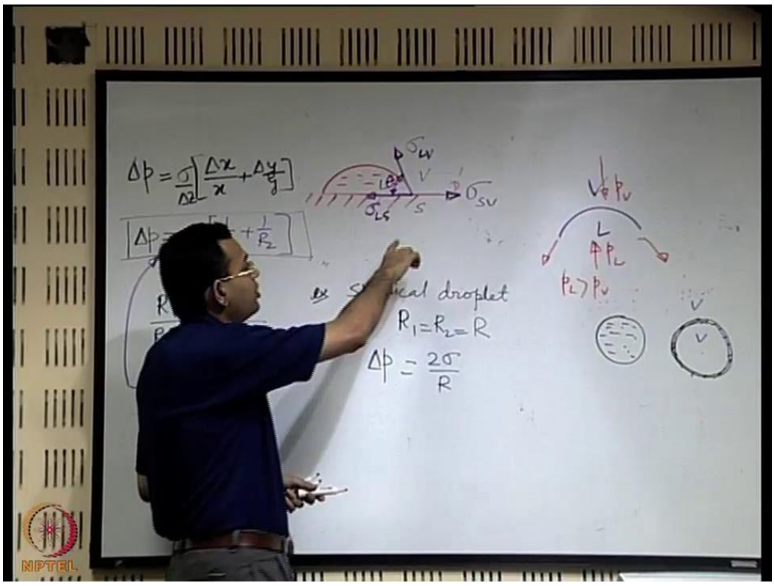

In place of this droplet if you have a sort of a bubble, say. What will be the difference between the droplet and the bubble? See droplet is like you have this full thing liquid and outside ambient maybe vapor. If you have a bubble, so you have a thin layer of liquid here, and you have something outside and something inside.

So, you have here say a vapor, then you have here also a vapor. So, you have basically two vapor liquid interfaces to consider, one is as you jump from the inner vapor to this liquid line and then you jump from the outer liquid to the outside the vapor, outside location that is the vapor. So, you have two interfaces which are formed because of this one. So, maybe this should be multiplied twice to get the net pressure difference.

So, you can see that depending on the physical situation this needs to be adjusted, but this basically talks about that if there is one interface across that interface if there is a pressure difference then how can that pressure difference be related to the surface tension coefficient. Qualitatively if we try to understand that if we have say such a interface let us say that you have vapor on one side you have liquid on another side. Now, can you physically tell that on which side the pressure should be more if the interface looks like this vapor or liquid?

So, if you think about the surface tension it is acting something like this. So, when you have a pressure from this side you have a pressure from this side say pL you have a pressure from this side pV. See in the surface tension is already acting is having its component downwards, so the upward component should be more strong to overcome that; that means, pL should be greater than pV in this example. So, when we are talking about delta p we are talking about the magnitude the difference between pL and pV, but out of these which one is more that should come from your physical understanding of the problem, that is quite clear.

The other point that we will mention here is that in a very nice way I introduce this equilibrium to you, but you must have seen or you if you have noticed it carefully we have not really shown the equilibrium in the y direction. It should come to your mind that yes very nicely we have seen the equilibrium along x, but if you think about equilibrium along y there is only a vertical component; there is nothing else to balance it, then the droplet should go off take off from the surface we have never seen it just like taking it off like that.

We are talking about the surface and interface. So, it is an equilibrium of the interface when we are talking about it is not just the weight, but some interaction between the surface and the fluid molecules which are which are manifested in forms of different intermolecular forces. All intermolecular forces are not together brought in the category of surface tension. So, on an effect the net effect is there is a normal reaction just like what you have as a normal reaction on a block on a plane in a very similar way. What is normal reaction? It is a manifestation of some molecular scale interactions on a larger scale.

So, that type of normal reaction is also there we have not drawn it here explicitly, but that type of normal reaction is something that takes care of these type of interaction. So, we should keep in mind that this is one of the situations where you are having a statement of equilibrium, the normal component we are not always keeping in view but that also has its role to play.

So, the two conditions of equilibrium one is this one and of course the other is the expression for $\cos\theta$ that we have derived, but we have to remember that these are necessary conditions for equilibrium, but not sufficient. That means, that these conditions may be more fundamentally derived by minimizing the surface energy of the system. So, a system any system in equilibrium minimizes its energy so that is a stable sort of configuration.

So, if we express the surface energy and set out its partial derivative with respect to say R and θ to 0 then we will get the corresponding expression for equilibrium. But that these expressions do not automatically ensure that the second derivative is positive; that means, it ensures zero gradient, but it does not ensure that it is a minimum. Therefore, these are necessary conditions for equilibrium, but not sufficient. So, even if these conditions are satisfied still you may get interesting instabilities in the droplets and bubbles and so on and some of those pictures that we have already seen.

So, maybe we wind up here for the day and we will just see one very small movie to wind up the study for the day. So, we just play it again and see that, what effect surface tension is creating here, and maybe that is enough for the day and we will continue with that in the next class.

Thank you.