Introduction to Fluid Mechanics
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Lecture – 11
Problems and Solutions

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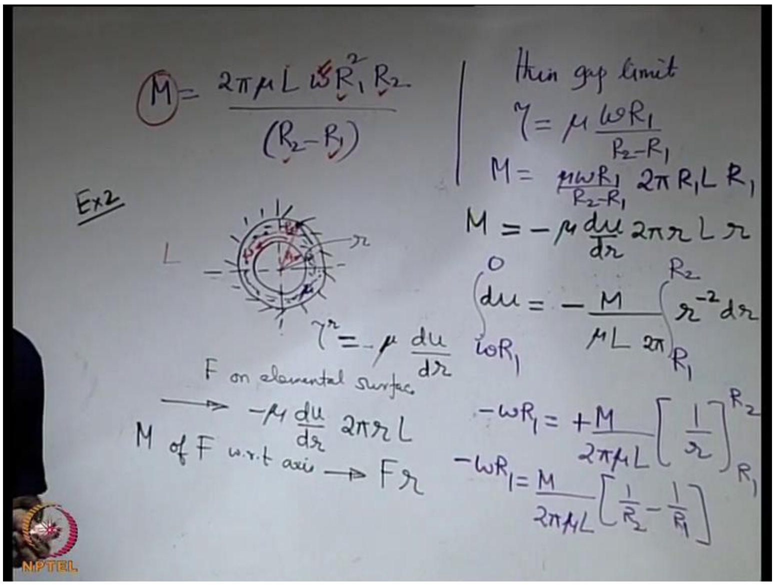

We continue with our discussions on viscosity. So, let us consider second example. Let us say we have concentric cylinders. And the specialty of this system is that maybe one of the cylinders is having a relative motion with respect to the other. So, let us take an example where the outer cylinder is stationary outer one is stationary, and the inner one is rotating with a particular angular speed omega. There is a fluid that is present in the gap between the two cylinders this kind of visual example that we have seen in our previous lecture.

This type of example is important in many ways. I will give you two examples for application of these types of situation. One is if you consider an industrial application. In industry there are shafts cylindrical shafts, which transmit power from say one point to the other. So, think about the inner cylinder like a shaft, so that shaft is rotating. And it is rotating, but it has to be housed in a certain position at the same time, if you constrain it with the direct metal to metal contact that will give rise to a lot of wear and tear.

So, there is a lubricating material, which is typically like an oil which separates that from a outer housing which sometimes is called the bearing. And the whole idea is to create a lubricating layer in between these two, which prevents wear and tear because of the metal to metal contact. So, the shaft bearing type of arrangement in industries is very common, and that is where you will find lot of application of this kind.

Even if you are looking from a more fundamental consideration of fluid mechanics, there is often a necessity to measure the viscosity of fluids. And this kind of arrangement may be utilized to measure viscosity of fluid that is the case where it is known as a rotating type viscometer. Viscometer for measuring the viscosity and rotating type, because of its particular nature of motion that you can easily appreciate.

We will try to see that, what is the physical situation that is going behind this type of example when the inner cylinder starts rotating, it will try to move the fluid with it because of no slip boundary condition. The fluid immediately in contact with that will be rotating with the same linear speed at different locations. . As you go radially outwards, let us say the inner cylinder has a radius of R 1 and the outer has a radius of R 2. So, as you go from $R=R_1$ to $R=R_2$ what you will find, we will find that the velocity in the fluid goes down. And the velocity is 0 at the outer radius $R=R_2$ that is also no slip boundary condition.

Now, because of the presence of the fluid, the cylinder which was which is rotating it is not rotating in an unhindered manner, it is being subjected to some resistance. It has to overcome that resistance and maintain its motion. So, it requires an external power to be imposed or so to say a torque to be there, which is continuously rotating it overcoming the viscous resistance.

And let us say that we are interested to find out, what is that torque or maybe power necessary to make it rotate with a uniform angular speed that is the objective of analyzing this. And therefore, if we apply that particular torque, and if we see that it is rotating with a uniform angular speed that may be measured by something like a tachometer, then it is possible to relate these two in terms of the viscosity of the fluid.

So, everything other everything else being measured from that expression, we should be able to evaluate what is the viscosity of the fluid that is the basic principle by which one may measure or evaluate what is the viscosity of the fluid that is there in between. Typically, this gap is very narrow, and we will see what is the consequence of that narrowness.

Now, let us say that we are interested about a section of the fluid, let us say at a radius at some intermediate radius say r, which is a local variables small r. Let us say that the length of the cylinder or both the cylinders is L, which is perpendicular to the plane of the board. And mu is the viscosity of the fluid, which is occupying the annular space.

So, when we consider at a location r, we have an imaginary surface of fluid, which is having a surface area of what $2\pi rl$ that surface of fluid is a surface on which there is relative resistance or there is there is a relative motion between the fluid layers one is towards the inner, and another is towards the outer. Whatever is towards the inner tends to move faster, whatever is towards the outer tends to move slower, so that is a location where there is a shear stress that is present, which is related to the rate of deformation.

So, if we want to write what is the shear stress at the radial location r or we should use τ^{r}. So, if we write this, then what would be its corresponding expression in terms of say Newton's law of viscosity? So, what is let us let us call it some $\tau^{r} = -\mu \frac{du}{dr}$.

And if we had used a coordinate of y, the only difference should have been that y is from the valve from the solid from the zero velocity valve towards the inside towards the inner one, so that y is just oppositely directed to r. So, whatever is $\frac{du}{dy}$ is just adjusted with a $-\frac{du}{dr}$ there is no other difference because, y direction is preserved for the direction which is from these zero velocity to the fluid. And this is the r direction is just opposite to that that is why these minus sign is there to adjust it.

And you may think also in also in a different way as you are increasing with radius you are having reduced velocity. So, this is negative if you want to make it positive, we want to adjust it with the negative sign that is just a matter of sign convention. But, we have to be consistent with the sign convention, whatever we are followed till now we will preserve that, so that is a shear stress.

If it is a Newtonian fluid, then what is the shear force which acts on these elemental surfaces? Let us say dF is the shear force on elemental surface, we have already identified what is the elemental surface that is the surface of the imaginary fluid with the dotted line as its radial envelope. So, $dF = -\mu \frac{du}{dr}$ or $F = -\mu \frac{du}{dr} 2\pi rL$.

So, this force is a tangential force. So, this force is like typically you will have these types of forces, which is tangential to these elements. So, this force will have a moment with respect to the axis of the cylinder. So, moment of F with respect to axis that is M=Fr, we are just writing it in a scalar form not bothering about the vector nature, because the moment vector is perpendicular to the plane of the board that we can understand very easily.

So, this is something now you have to understand physically what is happening, there is a particular power that is impose a motor is driving this that means, there is a torque that is being input to the system. And the same torque is transmitted across different fluid layers otherwise it will not be able to rotate with a uniform velocity. So, what it means is that if you call this as say M, then M is something which is a sort of an input. And it is balanced with the resistance movement that takes place at various sections, so that you have a particular number associated with that, and that is dictated by the input power of the motor.

So, $M = -\mu \frac{du}{dr} 2\pi r L r \Rightarrow du = -\frac{M}{\mu L 2\pi} r^{-2} dr$

Remember one very important thing this u is the velocity in the fluid. So, it has a variation from the inner to the outer. For the inner cylinder within the cylinder there is no variation in velocity, because it is a rigid body. So, of course linear velocity is varying, but angular velocity is the same. Now, the outer cylinder also is stationary, but in between there is a difference in linear velocity, because the fluid is deforming, it is not a rigid body. So, at the inner radius that is at r_1 what is the velocity?

So, $\int_{\omega R_1}^{0} du = -\frac{M}{\mu L 2\pi} \int_{R_1}^{R_2} r^{-2} dr$

So, $-\omega R_1 = \frac{M}{2\pi \mu L} \left[\frac{1}{r} \right]_{R_1}^{R_2} \Rightarrow -\omega R_1 = \frac{M}{2\pi r L} \left[\frac{1}{R_2} - \frac{1}{R_1} \right]$

So, $M = \frac{2\pi \mu L \omega R_1^2 R_2}{(R_2 - R_1)}$

But, if you neglect the variation of or you if you neglect the velocity profile variation from the inner to the outer, and assume that the gap is very thin, so that it is a linear profile, then what would be the difference in expression that you get? $\tau = \mu \frac{\omega R_1}{R_2 - R_1}$

So, $M = \mu \frac{\omega R_1}{R_2 - R_1} 2\pi R_1 L R_1$

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So, if it is a linear velocity profile that is taken, that is the velocity is varying from ωR_1 from inside to 0. So, for in a thin gap limit, you have $\tau = \mu \frac{\omega R_1}{R_2 - R_1}$

So, $M = \mu \frac{\omega R_1}{R_2 - R_1} 2\pi R_1 L R_1$

So, you can clearly see that as the difference between R_1 and R_2 tends to 0, these two expressions lead to almost the same thing. So, if the gap is narrow, then the second approximation will give you a very quick estimation of what is the situation. And from this odd the more involved expression, you can clearly see that if you now know what is the power input to the shaft, the power input is this Mω.

So, if you are having a careful experiment, where you are having the proper estimate of the power input as well as what is the angular velocity at which this inner cylinder is rotating, it will give you some good estimation of what is the viscosity of the fluid that is there inside, provided it is Newtonian. And that is how you may estimate the viscosity of an unknown fluid that is a fluid for which you do not know the viscosity.

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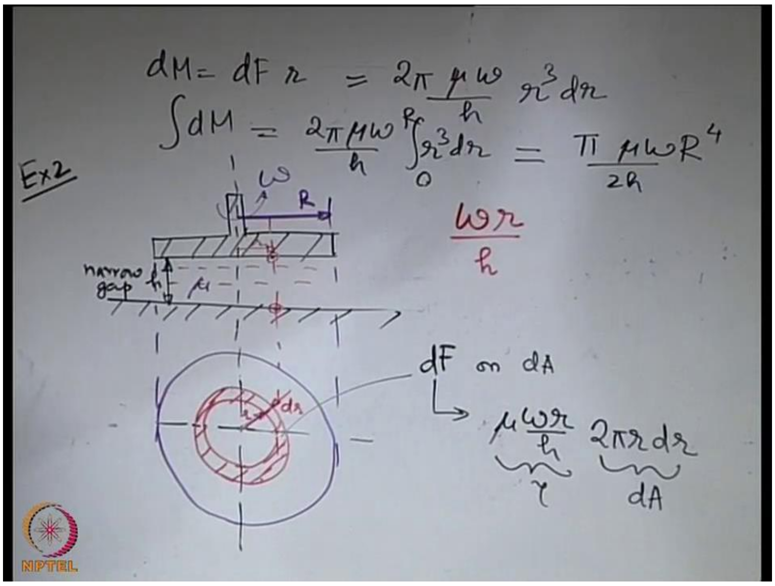

Let us consider a third example. We considered that there are two plates, for example two circular plates, so these are the sectional views. If you draw the other view, this will be a circle. So, if you would draw the other view of say the top plate, it will be something of a circular nature.

So, assume this is a circle. So, this is the other view of the plate, there is a fluid which is there in between and our object is again is to see that what is the torque or power required. If we want to rotate this one with a particular angular speed, the bottom one is stationary. Situation is quite similar to the previous one, so we should be able to work it quite quickly.

We have assumed that this gap is narrow, the viscosity of the fluid is μ. And let us say radius of the plate is the top plate is R, because the gap is narrow. Obviously, it is expected that we may approximate it with the linear velocity profile from the bottom to the top. But, a key factor here is that that linear velocity profile is now radially changing. So, if you consider say a particular radial section like this, here you have 0 velocity here, and what is the velocity that you will have here?

So, let us say small r is the local r, so ωr will be the velocity. Therefore, the velocity gradient at section r will be $\frac{\omega r}{h}$, and this is because of linear velocity profile assumption from the bottom to the top. So, if you take different radial sections, this will be different. So, if this was a constant, we could have easily calculated the shear stress by multiplying whatever constant it was with the total surface area of the plate that is being exposed to the fluid.

But, now this being a variable, we must take it as summations of constant over small elements. And that is how or that is why we have to choose small elements, and integrate over that elements. So, we take a small element at a radius r of thickness dr. So, whenever we are solving any problems, these are very common situations. Many times because of systematically practicing problems, we are habituated in taking elements in certain cases doing integration and so on but, many times we forget why we are doing it, and it is very important to keep in mind that why we should do it.

So, here since it is continuously varying, we are interested to obtain estimation for the shear stress of the shear force, which is our objective. And that shear force is locally varying, because the shear stress is locally varying. We should take a small element at least toward which it is a constant. So, within this dr it does not vary significantly, therefore it may be treated like a constant over dr.

So, if we consider this area, we can multiply the local shear stress with that area to get the local shear force. So, what is that local shear force? So, let us say dF; we call as local shear force on this dA. So, $dF = \frac{\mu\omega r}{h} \times 2\pi r dr$. So, with this as the shear elemental shear force, this elemental shear force will have a moment with respect to the axis.

So, $dM = dF.r = \frac{2\pi\mu\omega}{h} r^3 dr$

So, the total resistive moment should be $\int dM = \frac{2\pi\mu\omega}{h} \int r^3 dr$. No it is distributed over the entire surface. So, it is like it is when you consider a radial location, it is not a point it is like entirely distributed, and that distributed force has a moment with respect to the axis. So, it is like over the entire element. You can think it even more fundamentally do not does consider the $2\pi r$, but consider a small angular element with between $\theta \,\&\, \theta + d\theta$ and between r and r+dr.

And then if you integrate that from theta equal to 0 to 2π that 2π term has automatically been taken care of, so you should not take it take care of it doubly by considering 2π here also. So, this is a very simple expression. But, it again tells that like there can be situations of variable velocity profiles, and those may be taken care of in this way.

So, we can sum of with our studies on viscosity, we will study more effectively viscosity. Later on in one of our related chapters that is equations of motion of viscous flows. When we will learn viscosity effect more mathematically, but just now we can sum it up to see that here there is some highly viscous gel. And it is highly viscous gel is a being start and you can see that when it is being start, it tends to get broken and separated in parts. So, when it is doing that of course it is a highly viscous gel, and there is an important additional force that is coming into the picture, which is making it to behave in that type of way, and that force is nothing but a surface tension force.