Introduction to Fluid Mechanics
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Lecture – 10
Problems and Solutions

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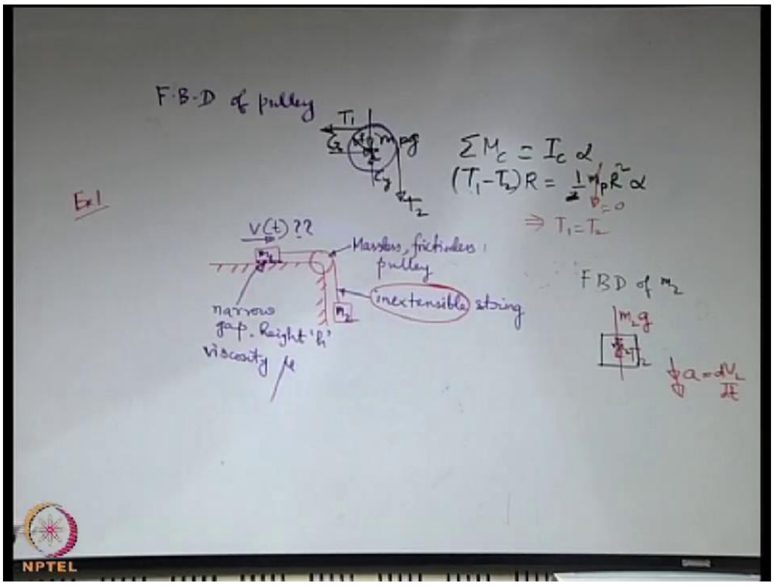

So, let us consider an example 1. Let us say that you have system with mass and pulley and spring the kind of system that you always love to deal with in mechanics problems. So, we start with such an example because it will have good transition from your earlier studies in mechanics to the mechanics of fluids, so where we make a change is like we put some fluids

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So, whenever we work out a problem it is important that we also try to get a physical feel of what is happening try to formulate in terms of the physical feel, try to get a sketch maybe of what is the variation and then it will be so easy to interpret whatever results you are getting whether they are correct or not, or whether they are having some physical sense or not. With this we will have a short break, and then we will continue with the discussions on viscosity and surface tension in the next part of the lecture.

Thank you.

say oil in the narrow gap between the block which is there on the plane and the plane. So, this qualitatively is a narrow gap of say height small h.

Let us say that mass of this is m1 and the one that is hanging is m2. We make further statements that it is this pulley is massless and frictionless. And also, this string is inextensible. Assume that the fluid which is there in between the block and the plane is Newtonian with the viscosity μ. Our objective say is to find out what is the velocity of this block of mass m1 as a function of time.

So, this is the question that we would like to answer this is a very simple problem, but we will try to go to as much fundamental depth as possible for such a simple problem. We may start with the expressions and you will see the expressions are very institutive we may write those easily, but we will start with the free body diagram of different elements which are present in the system to have a more thorough insight on what happens.

Let us draw first the free body diagram of the pulley. Most of the times you will never draw it but if required we should. The pulley is hinged to this surface. So, the center of the pulley at the center there is a hinge. Now, what should be the forces which are acting on this?

Student: Solid reaction

Normal reaction if you say normal reaction.

Student: Solid reaction

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Thank you.

By whom to whom?

Student: (Refer Time: 04:07).

One by one. So, first say we are talking about the hinge. So, what type of reaction you expect for the hinge?

Student: (Refer Time: 04:22).

So, there are contact forces it is something which is occurring in a plane. So, in general you will be having general force in a plane which may be resolved into two components. So, let us say that we are talking about two components C_x and C_y which are like arbitrary orthogonal components of the forces which are there at the hinge.

Tension in the string. So, here you have some tension T_1, here you have some tension T_2. For the time being let us forget about these assumptions which are been given mass less pulley, frictionless pulley, inextensible string, and then we are not exactly using the consequences of these in drawing the free body diagram. So, in the free body diagram we are keeping everything as general. So, let us consider that the pulley might have a mass also.

So, just to see that if it has of course, it has a mass when we mass less, we do not mean that it is literally mass less we mean that effect of that is not significant that’s all. So, whether the effect of mass is significant or not how will we understand. These types of statements are very important like mass less, frictionless, inextensible and so on. Most of the times you think that

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these are for beautification of the problem statements; and at the end you come up with the conclusion which sort of abstracts you from the path by which you have come to the conclusion.

For example, if we say T_1 equal to T_2. We will of course, see T_1 equal to T_2 all of you have learned from your mechanics that it should be like that. Now, can you tell that whether T1 equal to T2 is because of massless pulley, frictionless pulley inextensible string or what?

Let us see which one is the fundamental and which one is dominating. So, if you think that the massless is the thing that should be put in question so let us consider that it has a mass and see that what is the consequence? It will help us in understanding that what would be if it is massless let us say that m_p is the mass of the pulley. So, it has its weight also.

Now, what we do is we want to get an expression between T_1 and T_2. So, if we take moment of all forces with respect to see, the three C_x , C_y and mg, this get cancel it helps us in obtaining a relationship between T 1 and T 2. So, what is the that basic equation that we are looking for resultant moment of all forces with respect to an axis passing through C perpendicular to the plane of the board is the moment of inertia of this pulley with respect to an axis which is the same as the axis with respect to which you have got the moment times the angular acceleration

$$\sum M_c = I_c \alpha$$

So, if say capital R is the radius of the pulley, then we can write $(T_1 - T_2)R = \frac{1}{2} m_p R^2 \alpha \Rightarrow T_1 = T_2$

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Thank you.

You should try to understand it through a contradiction. Say you have encountered cases where you have a pulley with a string or a belt around that and there is a difference between these tensions. And the difference is given by say if it is an impending slip T1 by T2 is e to the power coefficient of friction into the theta angle of wrap these you have learned in the statics. So, now I mean here we are seeing something different right. So, what is the anomaly you think about it, I will ask you next time? Let us continue with this problem because for this particular problem this that is not going to be important so much I can tell you.

Now, we come to say the free body diagram of m 2 may be. So, you have the weight, then you have the tension which is T2 of course, T2 is equal to T1. And then if the system is released, what would be the direction of motion that you expect?

So, this is coming downwards and this is expected to go towards the right. So, understand the physics of this problem, when it is moving downwards it is tending to go towards right there is a resistance at this interface which does not want to make it go towards the right so easily. So, it is somewhat like a friction. And that friction here is sort of lubricated it is not a direct contact between two solids, but there is a thin film of liquid in between. So, that means if that is the case then if it comes down let us say it comes down with an acceleration a. So, if this comes down with an acceleration a which is you may say this a is like $\frac{dV_2}{dt}$.

So, when it comes down with an acceleration a you expect that m1 also moves right towards right with an acceleration a again by which assumption it is there?

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Thank you.

Inextensible string. So, inextensible string had a role to play here, massless pulley we have seen had a role to play here. And yes frictionless pulley also has a role to play here because the belt is being wrapped around the pulley and because of friction there may be a difference in the contact forces between the belt on the pulley, so that will be more apparent if you draw the free body diagram of the belt and see it is interaction with the pulley. So, when you have when you are thinking of interaction between the belt on the pulley that is not considered here and that is not considered implicitly because it is frictionless.

So, important interaction is through friction normal reaction will always get nullified even if it is considered because you are taking moment with respect to the centre. So, all these have been used in some in some way or the other. Let us draw the free body diagram of m1 which is the important matter so far as the understanding of viscosity goes.

So, if we draw now the free body diagram of m1. Now, you tell what are the forces which are acting on m_1?

There is some normal reaction, what is the origin of this normal reaction? This is not a direct contact between the solid boundary and the block.

There is some fluid is there and there is a pressure distribution at the interface between the fluid and the solid. So, the resultant of that pressure distribution give rise to this normal reaction. Of course, the same normal reaction we have drawn, but we have to understand that physically where it originates, then there is some resistance also. So, what is the resistance there is let us call it F just with analogy with the problems involving standard mechanics of solids. So, this F

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is sort of friction force, but here again the origin is not the direct contact between the plane and the block. So, what so what is the origin of this friction force?

Viscosity right. So, it is viscosity or viscous effects at the interface between the block and the fluid, so that should be expressible in terms of the shear stress. And the shear stress is expressible in terms of the rate of deformation through the Newton’s law of viscosity because we are assuming it is a Newtonian fluid. So, what we expect here is $F = \tau A$

So, let us say that A is the bottom surface area of the block, so that is another input to the problem. Now, only work that is left is to relate the τ with the rate of deformation. Now, the rate of deformation is something which takes place where which takes place if you look at the magnified view of what happens in this thin film. So, in this thin film, you see that there is a solid boundary at the bottom which is stationary; at the top there is a block that is moving. So, this is the block. So, this block is moving with a particular velocity. And this is like this is this block is idealized as a particle.

So, when you when you idealize sort of rigid body as a particle when you do not have rotational effects. The fundamental difference between a particle and rigid bodies are rigid body has rotation or it is capable of having different rotational components, but particle cannot rotate. So, this of course, we are not considering that this could rotate in this type of situation. So, it is just like a particle or a point mass. So, everything is moving towards the right with a particular velocity which is changing with time.

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Thank you.

So, let us say that this as the velocity V. So, when these are the velocity V of course, that is a function of time that you have to understand. Now, if you draw the velocity profile at any section which includes the fluid between the block and the plane. At the wall because of no slip boundary condition it is 0. Here what should be the velocity of the fluid.

It should be V that is also because of no slip boundary condition. So, no slip boundary condition is not zero velocity of the fluid, but zero relative velocity. So, if the solid moves with that velocity fluid will also move with that same velocity. Now, see the catch word again a very nice catch word is there, there is a narrow gap. So, narrow gap means this thickness h is so small; it is so small that these variation of velocity between 0 to V may be assumed as linear ok.

Now, if you see that this shear stress, let us now complete the expression for the shear stress, it is $\mu\dot{\theta}$. Rate of deformation could be different at different y locations, but if it is a linear profile it is same everywhere, because rate of deformation is related to $\frac{du}{dy}$ that we have seen. So, rate of deformation in such a case is $\frac{du}{dy}$, where u is the velocity. So, if u versus y is the straight line $\frac{du}{dy}$ is a constant.

So, $\mu\dot{\theta} = \frac{V}{h}$. So, remember what are the approximations or simplifications which have led towards to this. If this is not a narrow gap, it has to be what is the rate of change of u with

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respect to y at the interface between the block and the fluid not at anywhere, but since now it is a straight line it is as good as it is anywhere. So, the very important understanding is we are using that $\tau = \mu \frac{du}{dy}$, it is better to say that it is a partial derivative of u with respect to y, because u could be function of many other things.

Here of course, u is a function of time, but no space coordinate. So, still it is like a partial derivative if you write $\frac{du}{dy}$ which we wrote for the first time when we wrote such an expression just for simplicity in understanding, but it is in general of partial derivative. It assumes that there is only one component of velocity and the partial derivative comes from the fact that that one component of velocity itself could be a function of many things x, y, z, time like that. And it could itself be a variable.

And what is y? The important thing to keep in mind is that if you have a solid boundary y is the coordinate which is normally directing outwards from the solid towards the fluid. So, y is a genetic thing. It is not that any y axis, you define it is that $\frac{du}{dy}$. So, this y has a special meaning. This y is the axis which is normal to the surface into the fluid that we are considering. So, if you are having a different type of coordinate axis, then you have to adjust this with a plus or minus sign, so that you have to keep in mind that this is the orientation that we are considered for describing the Newton's law of viscosity that sanctity should be preserved. With that

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understanding now we write the equations of motion by Newton’s second law which should be straight forward.

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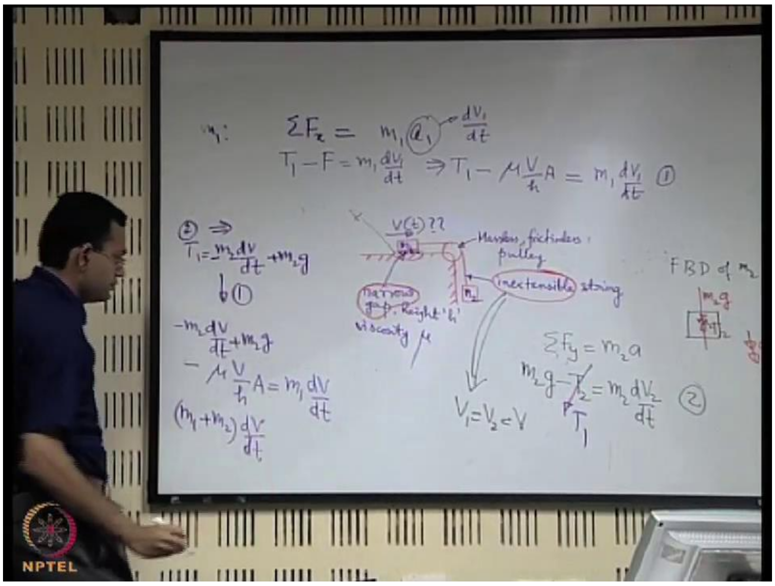

So, for the block m1 of course, along the y direction it is equilibrium. So, m=mg, $\sum F_x = m_1 a_1$ where $a_1 = \frac{dV_1}{dt}$. So, resultant force along x is $T_1 - F = m\frac{dV_1}{dt}$. So, $T_1 - \mu\frac{V}{h}A = m_1\frac{dV_1}{dt}$ let us say this is equation number 1. We write next the equation number 2, which should be for the mass m_2. So, let us draw the free body diagram of the mass m_2 or that is already there. So, we just write it is equation. So, it is now along y we are considering.

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Thank you.

So, this is coming down. So, $m_2 g - T_2 = m_2 \frac{dV_2}{dt}$. From the inextensible string, we can write $V_1=V_2=V$. And we have already seen $T_1=T_2$. So, if we call this as equation number 2, in equation number 2, we have this T 2 same as T 1. So, we can get what is the expression for T 1 and substituting equation number 1. Let us do that. So, from 2, we get $T_1 = -m_2 \frac{dv}{dt} + m_2 g$ and that we substitute in 1. So, $-m_2 \frac{dV}{dt} + m_2 g \Rightarrow -\mu \frac{V}{h} A = m_1 \frac{dV}{dt}$

So, you have $(m_1 + m_2)\frac{dV}{dt} = m_2 g - \frac{\mu VA}{h}$

So, you have $\frac{dV}{m_2 g - \frac{\mu VA}{h}} = \frac{1}{m_1 + m_2} dt$. So, you can integrate it with respect to time say

$$\int_0^v \frac{dV}{m_2 g - \frac{\mu VA}{h}} = \int_0^t \frac{1}{m_1 + m_2} dt$$

So, you can from this if we integrate, you will find an expression for V as a function of time. Of course, in an exponential form it will come because it will be in a logarithmic form the integral that will appear. So, it will be some function of time. And if it is released from rest, it should its velocity should increase with time or decrease with time?

Student: Increase.

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Thank you.

Increase right, because the m_2g that is falling now and making it to move towards the right. At the same time, there is a viscous resistance, so it will come to a sort of asymptotic state when it has a balance between these two forces and then there is no further change in velocity. So, without working it out may be the velocity versus time characteristic could be something like maybe like this. And let us say that it comes to a sort of steady state at some time which is which is large time theoretically tending to infinity practically large time. So, what should be this velocity?. So, can you tell that what should be this velocity?

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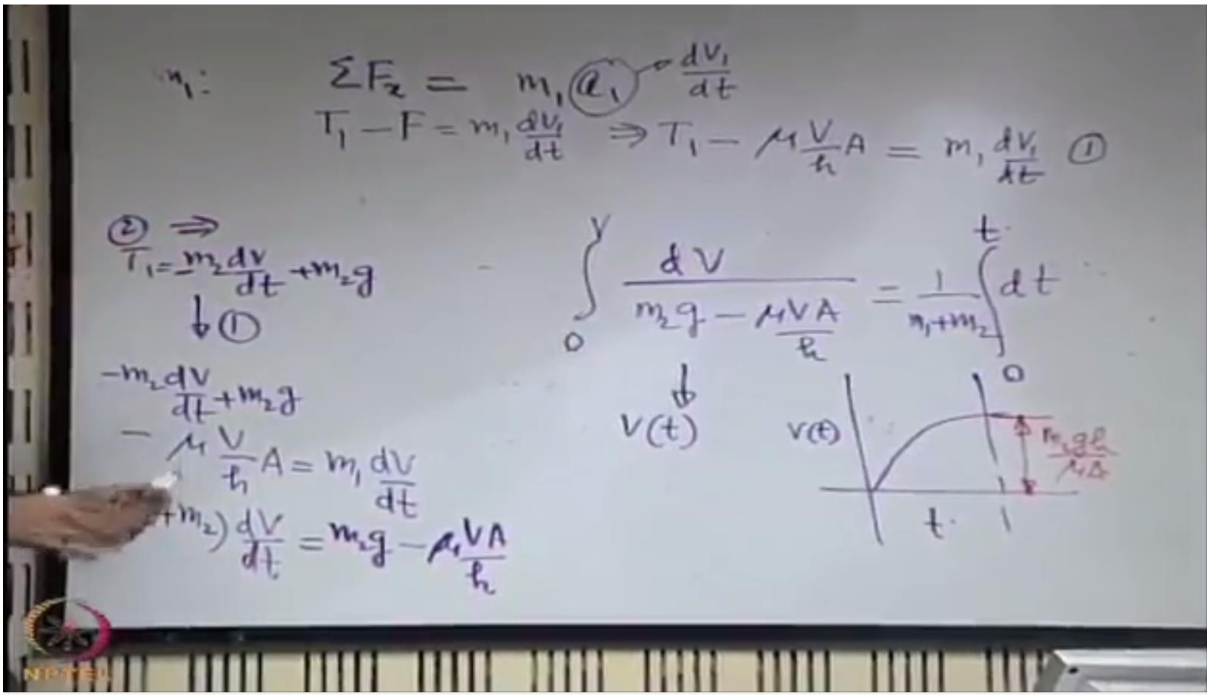

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