Kinematics of Mechanisms and Machines
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Lecture - 09
Grashof Criterion – Problems

In the previous lecture we have discussed about Grashof criterion, which tells us the presence of crank in a kinematic chain. Today I am going to discuss some Problems based on the Grashof Criterion.

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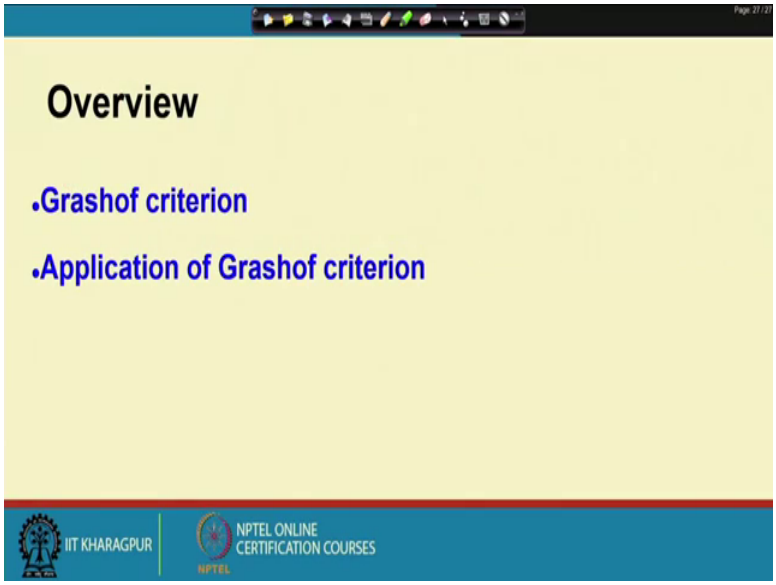

We will have a quick recapitulation of the Grashof criterion and its significance and then I will show you the application of Grashof criterion, in certain problems.

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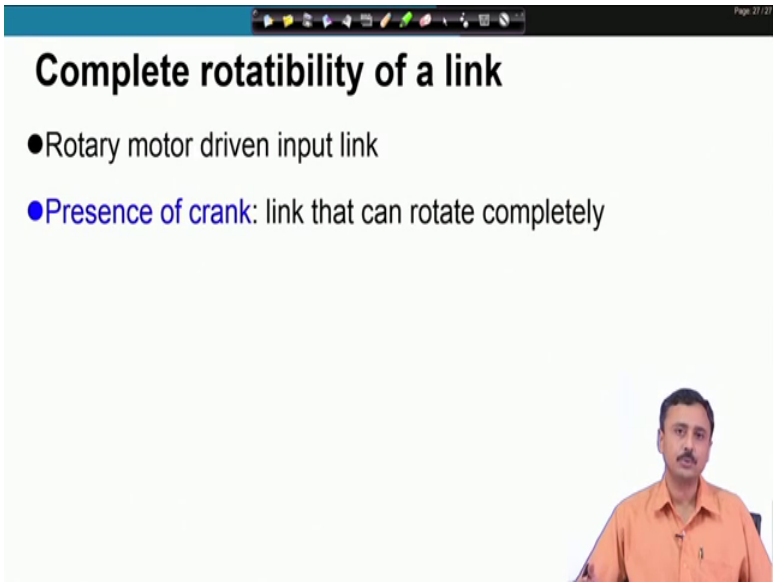

So, you can recall that in a number of applications mechanisms required or are driven by a motor which is rotating continuously. Therefore, we require one link of this mechanism which can rotate completely, this link is known as the crank. So, the presence of crank in a kinematic chain is an important issue.

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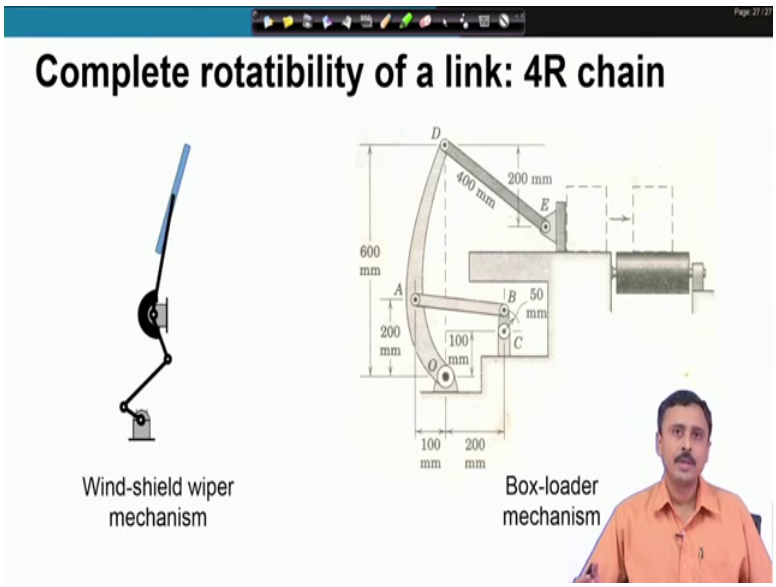

Why or in works what kind of applications do we require this complete rotatibility, here I have shown two examples the windshield wiper which is driven by a motor, it is continuously rotating and the wiper is oscillating. The other was the box loader

mechanism in which a motor is continuously rotating and the boxes are being loaded onto the conveyor.

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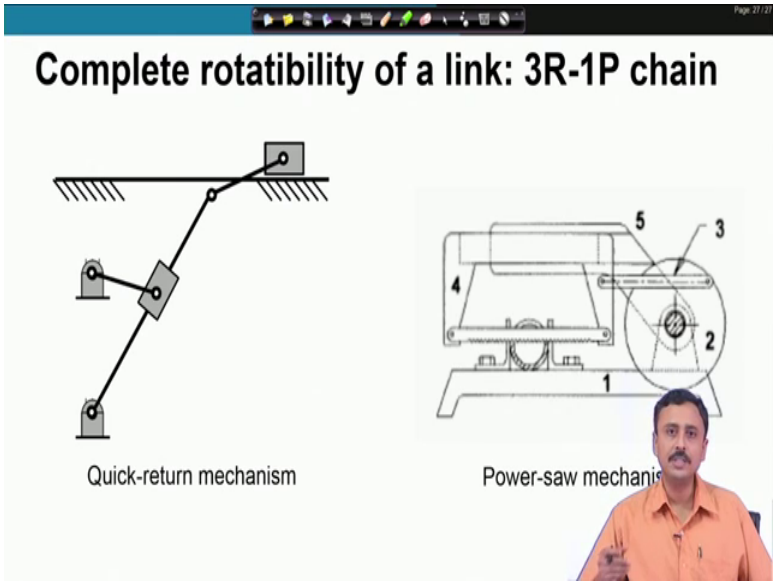

This is an application of the 3R-1P chain in which a motor is continuously rotating one link and the slider or the hacksaw is a oscillating. Therefore, in these all these applications we require one link of the kinematic chain to be a crank.

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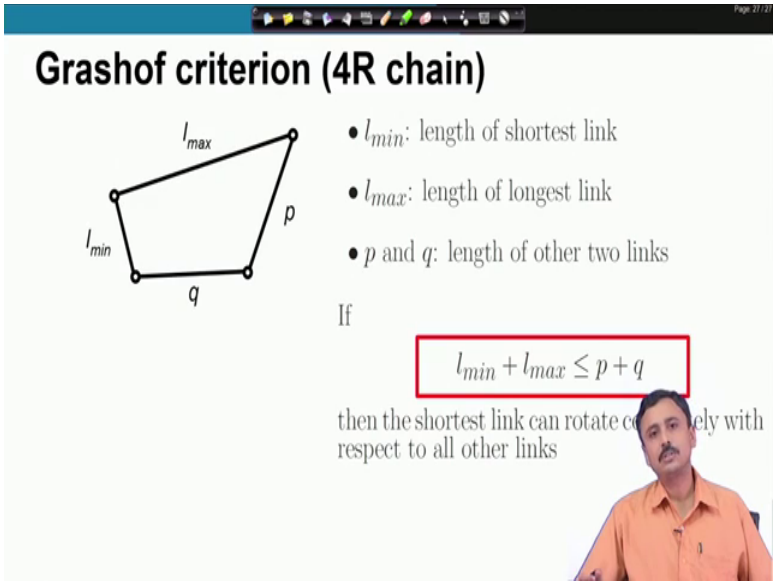

We had discussed about the Grashof criterion for a 4R kinematic chain and this is shown here, if the sum of the length of the minimum or the shortest link plus the length of the longest link is less than the sum of the other two links then the shortest link is a crank.

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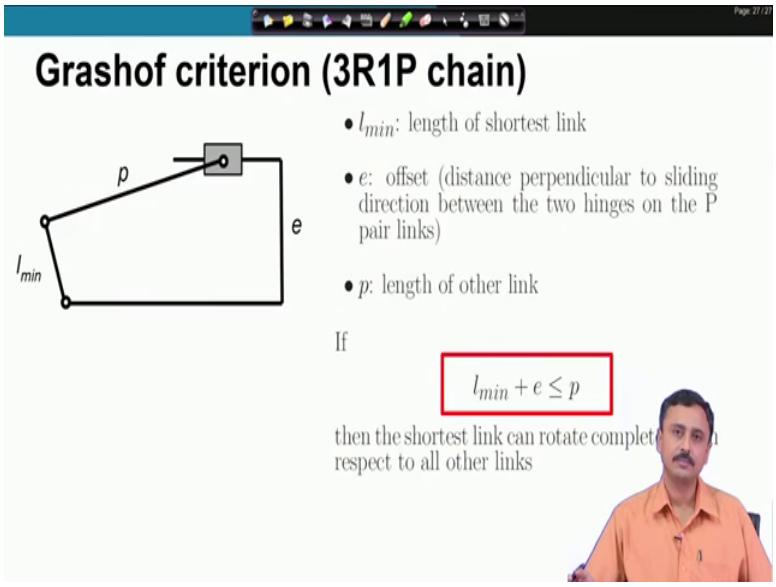

So, it can rotate completely with respect to all other links for a 3 R 1 P chain, the Grashof criterion says that, if the shortest link plus the offset is less than or equal to the other link, then the shortest link is a crank.

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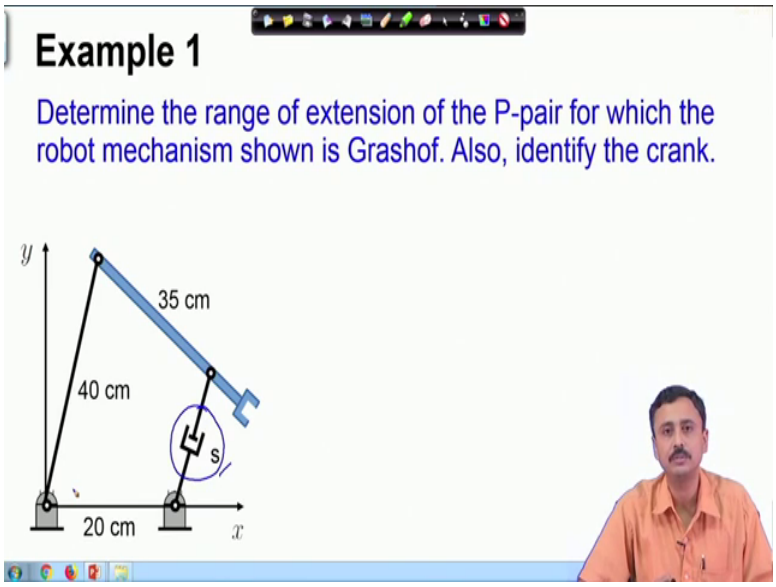

Now, let us come to this problem, this problem says determine the range of extension of the P pair for which the robot mechanism shown is Grashof, also identify the crank. So, first let me show you this P pair is here, s is the length of the P pair so, s is the link length. The other link lengths are fixed and are specified, you can easily check that this is a robot this has got 2 degrees of freedom. So, you can you have to specify s and let us say one angle to specify the configuration of this robot.

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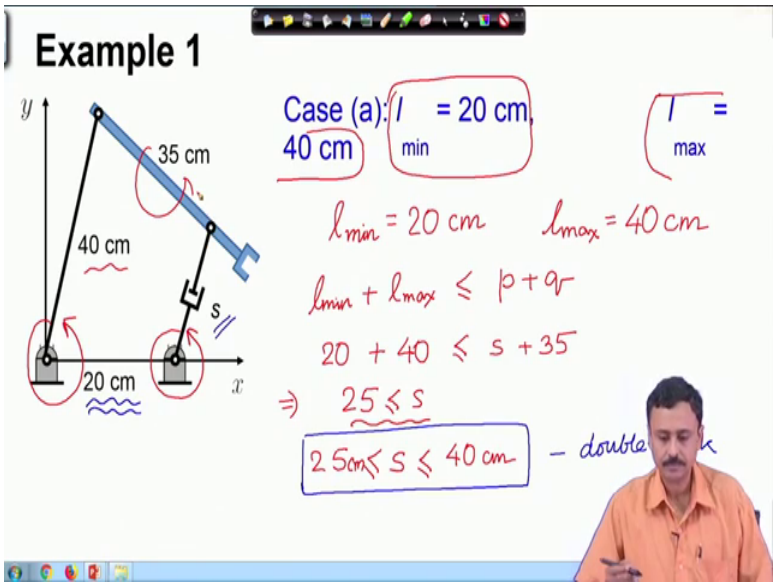

Now, there can be a various cases of this a Grashof criterion, because we have this s at our disposal, we want to put this or set this value of s and want to find out, whether it is Grashof or not for that value of s.

Now, there can be a possibility in which this 20 centimeter link which is the ground link here, that is the shortest link, this is what I have mentioned. So, l min is 20 centimeter. Now, once I have l min of 20 centimeter l max can have two values: one is this 40 centimeter or s, to begin with in case a we are considering l max to be 40 centimeter.

Therefore, l max is 40 centimeter. So, let us now apply Grashof criterion for to this to this situation. So, Grashof criterion tells us that l min plus l max should be less than equal to p plus q. Therefore, 20 plus 40 should be less than the other two links are s plus 35, this implies s should be greater than equal to 25 in centimeter.

Now, I have considered that s is an intermediate link, l max is 40 l min is 20, s it says it should be less than 25 centimeter, it should be greater than 25 centimeter this tells us that s should be greater than 25 centimeter, but s cannot exceed l max. Therefore, our range of s is 25 centimeter 40 centimeter. So, from this case a where we have chosen l min is 20 centimeter and l max is 40 centimeter, the range of s for which this chain is Grashof is 20 s is between 25 centimeter and 40 centimeter.

In this case the ground link is the shortest link. Therefore, as we have discussed before because, ground link is the shortest link it can rotate continue completely with respect to all other links therefore, this in this case we have a double crank. If s lies between these two values 25 centimeter and 40 centimeter, then the mechanism is a double crank mechanism which means that both these links can rotate completely and because it is a double crank this can also rotate completely the coupling link can also rotate completely.

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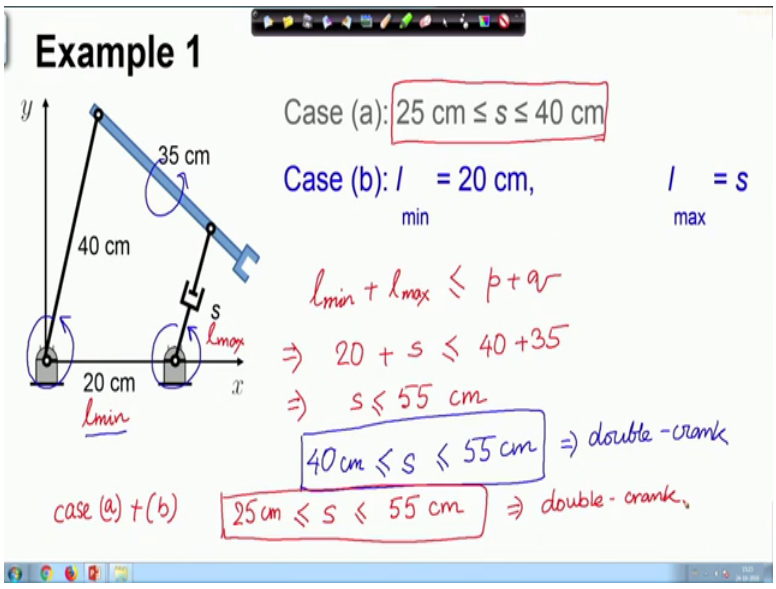

So, this is case a here I have put that case a in background. So, 25 to 40 centimeter, then we go to case b in which again l min is this 20 centimeter. And now l max is s therefore, our Grashof criterion gives us s to be less than equal to 55 centimeter. Now, s is l max therefore, s has to be greater than 40 centimeter; s has to be greater than 40 centimeter, because s is l max. Therefore, that complete range s so, s is greater than 40 centimeter and less than 55 centimeter, this case also gives us a Grashof chain. And since the ground

link is l min once again this ground link and rotate completely with respect to all other links therefore, this is also a double crank.

So, this can rotate completely this can also rotate completely and the coupler can also rotate completely. Now, if you look at these two ranges for case one and for case a and case b then for s lying between 25 and 55 centimeter the mechanism is Grashof and its a double crank. Therefore, the complete range for case a and case b taken together that complete range is this. So, this takes care of both these cases a and b and the mechanism is a double crank mechanism for this range of s.

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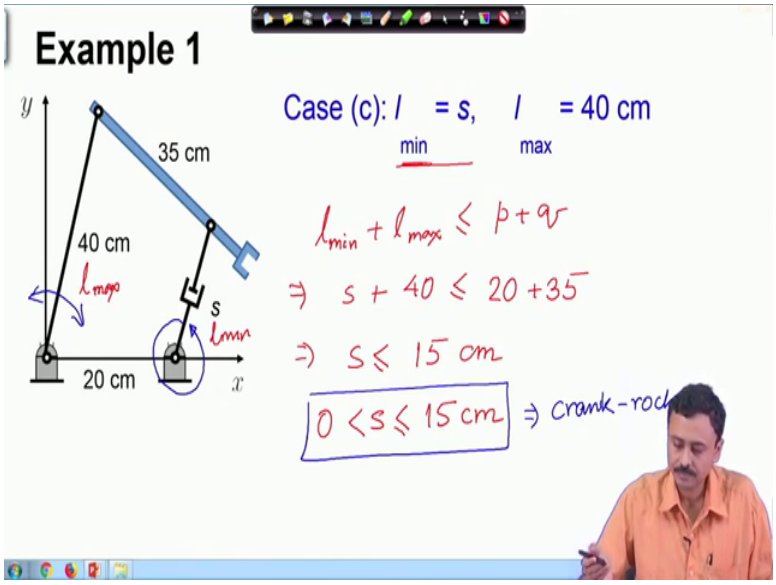

Now, we come to case c. In case c we have s as the shortest link, s is the shortest link. And if s is the shortest link, then this 40 centimeter link must be the longest link. Therefore, our Grashof criterion, tells us that s plus 40 should be less than or equal to 20 plus 35 that implies s should be less than equal to 15 centimeter. Now, s is l min but then s cannot be 0 therefore our range. So, this case is s should be greater than 0 and less than or equal to 15 centimeter. In this case the prismatic link the link having the prismatic pair is the shortest link. So, s is the shortest link.

And in a Grashof chain the shortest link can rotate completely with respect to all other links therefore, in this case this can rotate completely with respect to the ground and the other links will only oscillate. Therefore, this mechanism is now a crank rocker, this is a crank rocker mechanism with the link with the prismatic pair as the crank and the other

link connected to the ground is as the rocker. So, this completes analysis of this robot mechanism.

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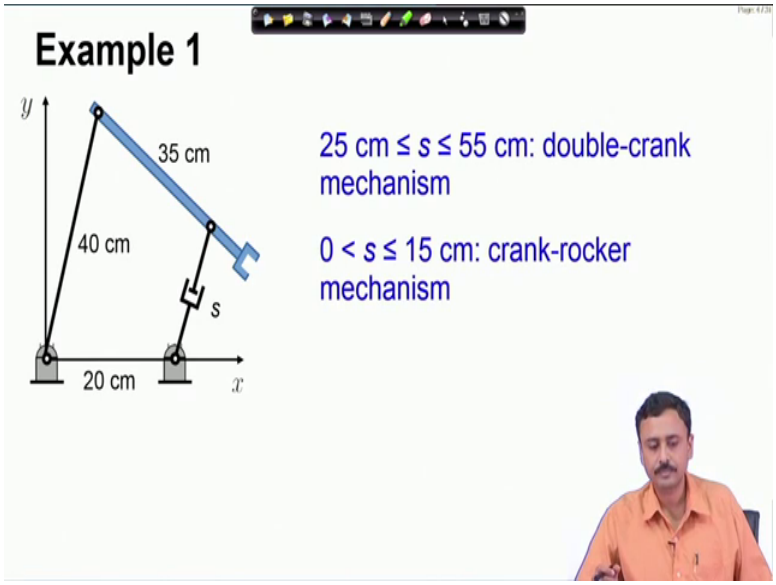

So, here I have recapitulated in the results. So, when s is lies between 25 centimeter and 55 centimeter, we have a double crank mechanism and if s lies between 0 and 15 centimeter then it is a crank rocker mechanism.

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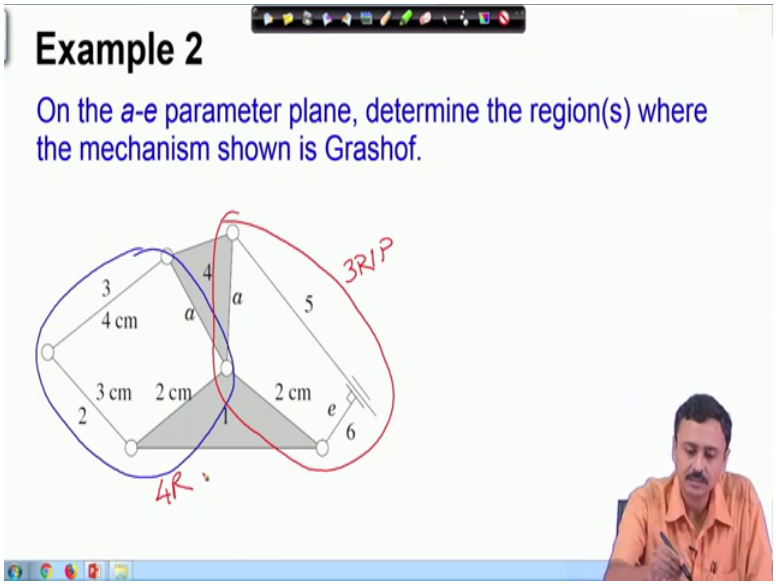

Let us move to this next example here, I have a mechanism that involves the 4 r. So, this is a 4 R chain coupled with a 3 R 1 P chain. So, we have a coupled 4 R and 3 R 1 P chains in this example the problem says on the a e parameter plane.

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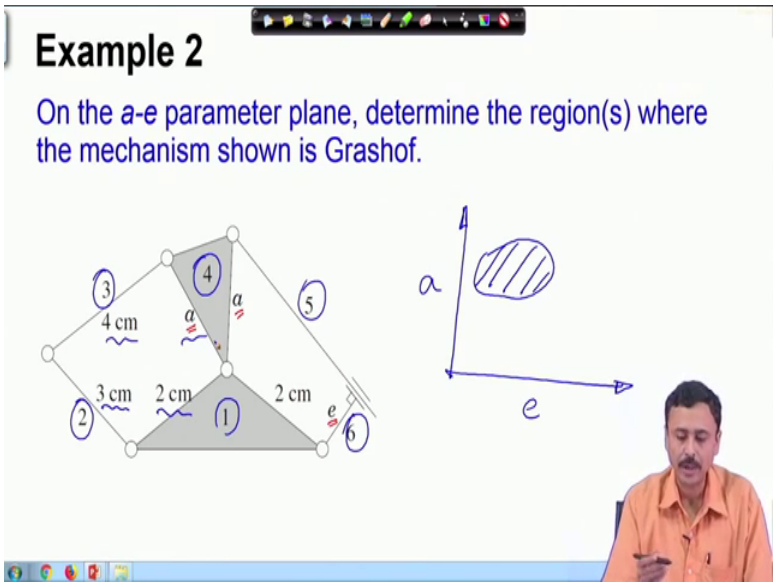

Now, here we have a as the parameter and e as the parameter. So, a is the link length of link 4 the ternary link and e is the offset for this 3 R 1 P chain. So, the problem says on the a e parameter plane determine the region or regions, where the mechanism shown is Grashof. If you think of this parameter plane these a and e are the unknown link lengths, on this parameter plane there can be regions, if you choose a and e within this region then the mechanism is Grashof outside this region it is non-Grashof.

We would like to know these regions find out these regions, if you note carefully in this chain I have numbered the links. So, there are 6 links and I have given their dimensions as well. Now, if you consider the link two for example, this it's length is 3 centimeter and there are link lengths of 2 centimeter and 4 centimeter. So, definitely this 3 centimeter is an intermediate link the first thing that I am trying to do here is determining which can be the shortest link. So, definitely link two cannot be shortest, that leaves us with link one this 2 centimeter or a links 3 also cannot be the shortest. Of course, therefore, we can have the shortest link as this 2 centimeter which is linked 1 or the shortest link will come from this ternary link 4 which is a for this 4 R chain.

The longest link can be this 4 centimeter or a. So, there are various cases that are possible let us look at them 1 by 1.

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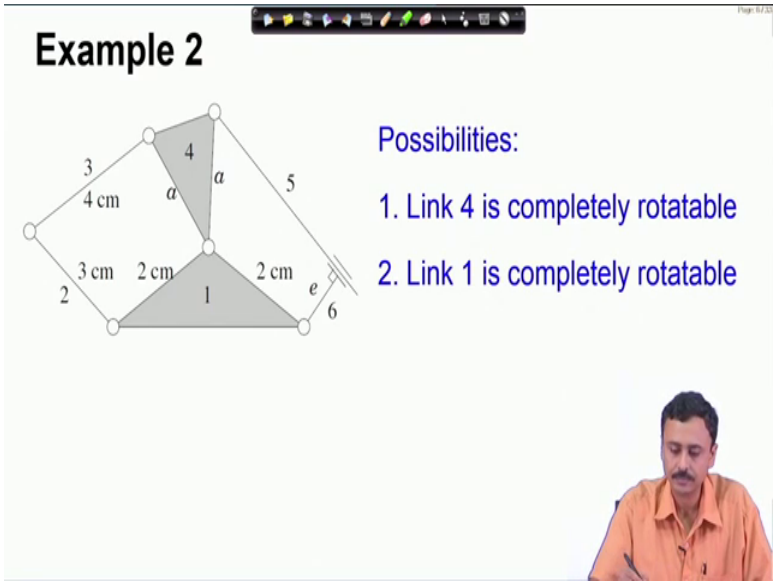

The first case that I am considering is link 4 is completely rotatable which means that a is the shortest link so, this is one possibility the other possibility is link 1 is completely rotatable. So, which means this 2 centimeter link is the shortest link.

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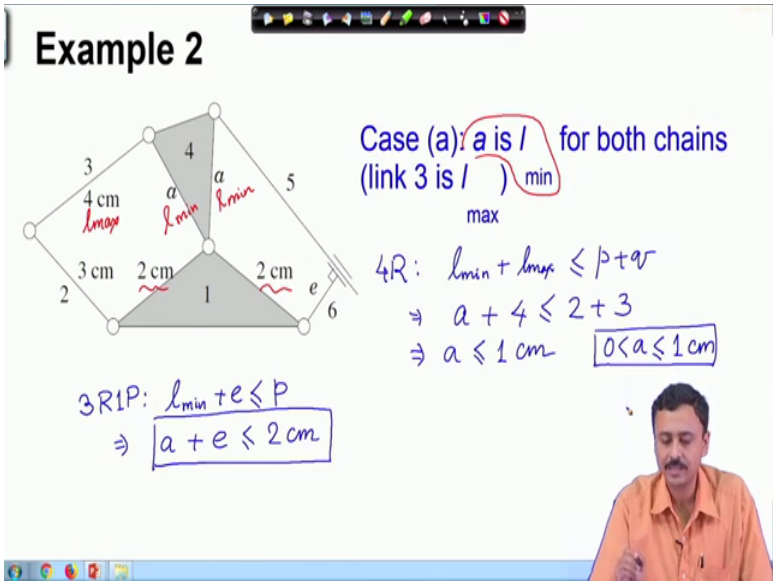

Now, let us consider this cases under these two possibilities, the first case this case a and considering that a is l min, a is the shortest link and if this is the shortest link, then it

must be less than 2 centimeter and here on the 3 R 1 P chain side we also have 2 centimeter. So, A will be the shortest link on both sides. So, A will be the shortest link for both the 4 R chain as well as the 3 R 1 P chain. Now if a is the shortest link, then you can very easily identify that this link 3 which is 4 centimeter that is the longest link. Let us now consider the 4 R chain side.

So, from the 4 R chain side the Grashof criterion tells us that l min plus l max should be less than equal to p plus q. So, that implies a plus 4 should be less than equal to 2 plus 3, which implies a should be less than equal to 1 centimeter. Now, a definitely cannot be 0 so, it must be greater than 0 on less than 1 centimeter.

So, the complete range for a should be greater than 0 and less than or equal to 1 centimeter. So, this is from the 4 R chain side. Now, we consider the 3 R 1 P chain, for this the Grashof criterion says l min plus e should be less than or equal to P, you realize that if link 4 has to let it completely if a is the shortest link, then it must rotate completely with respect to all other links. If a has to rotate completely or link 4 the ternary link 4 has to rotate completely, then it should rotate completely not only from the 4 R s chain side, but also from the 3 R 1 P chain side, both sides it should be the shortest link and should be satisfying the Grashof criterion.

Now, the Grashof criteria for the 3 R 1 P chain is this implies a plus e should be less than equal to P here is 2 centimeter, now this is one region and earlier we have found out. The range of a from the 4 R chain side so, a should lie between 0 and 1 centimeter and here from the 3 R 1 P chain side, it says that a plus e should be less than equal to 2 centimeters. So, this is a straight line in the a e parameter plane, this is case a.

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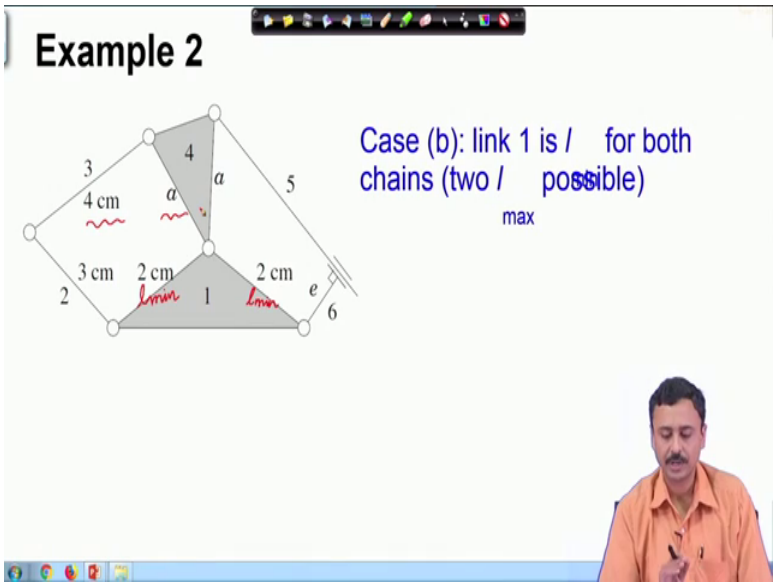

Let us now go to the next case this is case b in which link 1 is l min which means this is l min, this is l min from the 4 R chain side. Now, if this is l min from the 4 R chain side then if the chain the 4 R chain is to be Grashof, or if we choose if we can choose a such that this 4 R chain is Grashof, then a should be rotating completely with respect to link one if that be so, from the 3 R 1 P chain side therefore, this ground must be l min. Therefore, link 1 is l min for both chains, now for l max there are two possibilities from the 4 R chain side.

Since this is l min now, you can have this as l max the 4 centimeter a link as l max or we can have a as l max, will consider these two cases separately.

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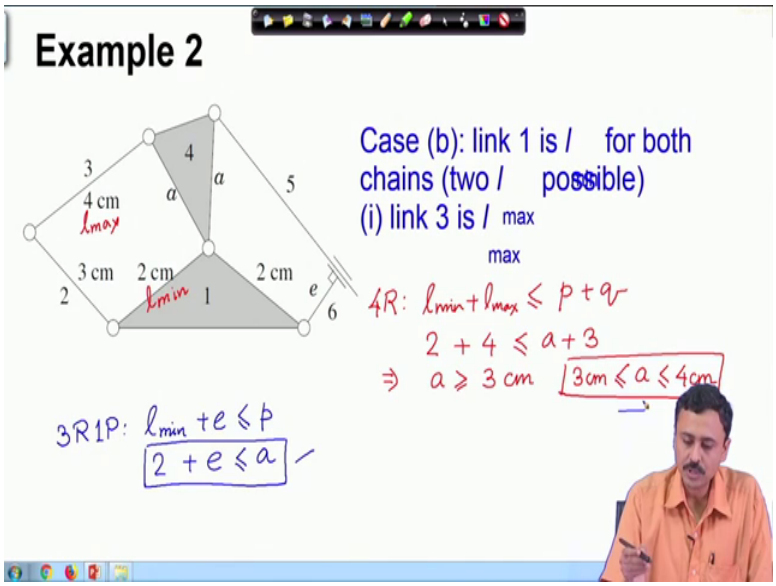

The first case is link 3 is l max therefore, this is l min and this is l max the 4 centimeter long link is l max. So, from the 4 R chain side l min plus l max should be less than equal to p plus q. So, l min is 2 centimeter plus l max is 4 centimeter should be less than equal to a plus 3. This implies a should be greater than equal to 3 centimeter, but definitely a cannot exceed 4 centimeters therefore, the complete range for a is this so, a must lie between 3 centimeter 4 centimeter.

Now, we go over to the 3 R 1 P chain side here Grashof criterion says l min plus e should be less than equal to p therefore, we have 2 plus e should be less than equal to a. In that case l min which is the 2 centimeter long link that is that can rotate completely that is the crank. And which satisfies our condition that link 4 will be able to rotate completely with respect to the link 1 therefore, we have these two conditions again they mark out certain regions in the a e parameter space, this condition is a straight line and this gives us region bounded by 2 straight lines. So, this was the sub case 1 under case b in which link 3 was taken to be l max.

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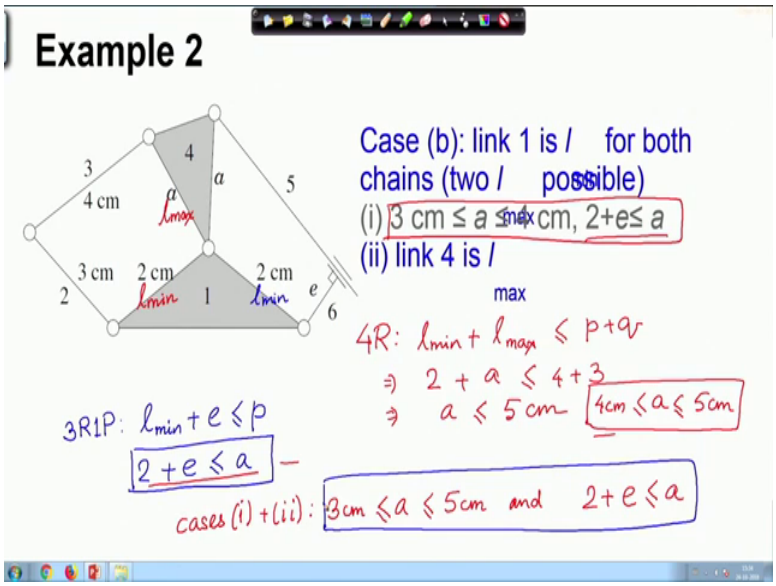

Now, suppose I consider the case that link 4 is l max. Therefore, this is l min and this is l max, if this is l max then a must definitely be greater than 4 centimeter. So, from the 4 R chain side l min is 2 plus l max is a should be less than or equal to 7 that gives us a should be less than a or equal to 5 centimeter. Now, here a is l max so, therefore, it has to be greater than 4 centimeter. So, that complete range for a is given by this; so, a must lie between 4 centimeter and 5 centimeter.

Now, we go over to the 3 R 1 P chain side so, the Grashof criterion is l min plus e should be less than or equal to P so, this is our l min for 2 plus e should be less than or equal to a, this is again a straight line in the a e parameter space. Now, if you look at the regions under sub case 1 and the region under sub case 2, which is given here you can combine in sub case 1 a was to lie between 3 and 4 centimeter in sub case 2 a has to lie between 4 and 5 centimeter. Therefore, combining sub cases 1 and 2 we must have a lying between 3 centimeter and 5 centimeter. And 2 plus e should be less than equal to a this remains common in both. Therefore, the range for k is b is this that a should lie between 3 and 5 centimeter and 2 plus e should be less than equal to a.

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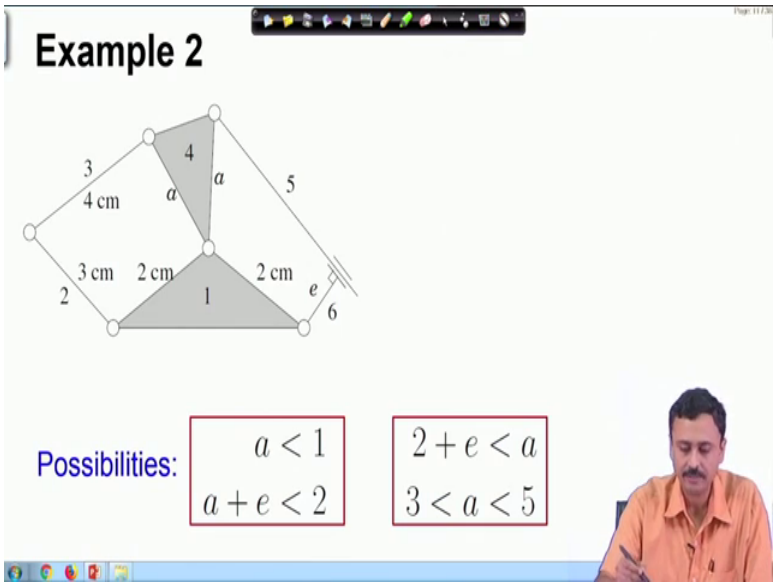

Now, we need to combine you recall that in case a we had this condition. So, here 0 is less than equal to this is so, a must lie between 0 and 1 centimeter and a plus e should be less than equal to 2 and in case b we had 2 plus e should be less than equal to a and a must lie between 3 and 5 centimeter, you can easily draw these regions. So, I have drawn these regions out for you.

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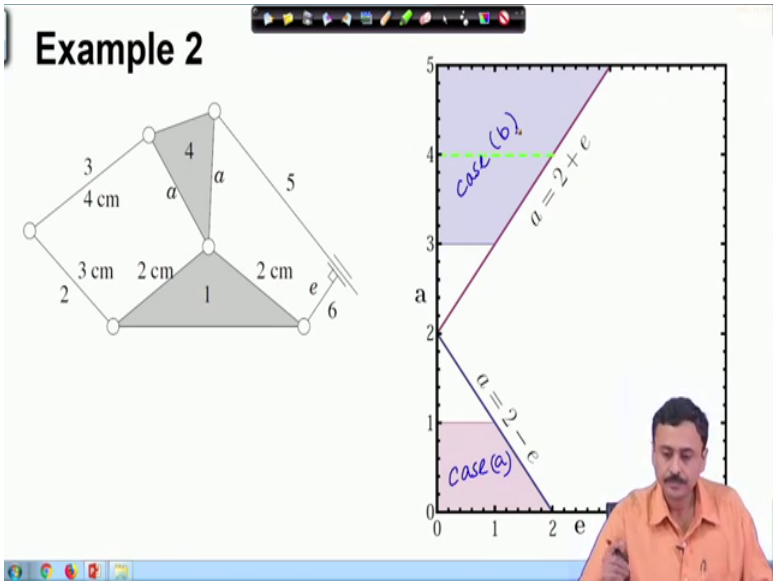

This is for this is the region this is the region for case a and this is the region for case b. So, the shaded region is these are the regions, where if you choose values of a and e, then the kinematic chain will be a Grashof chain.

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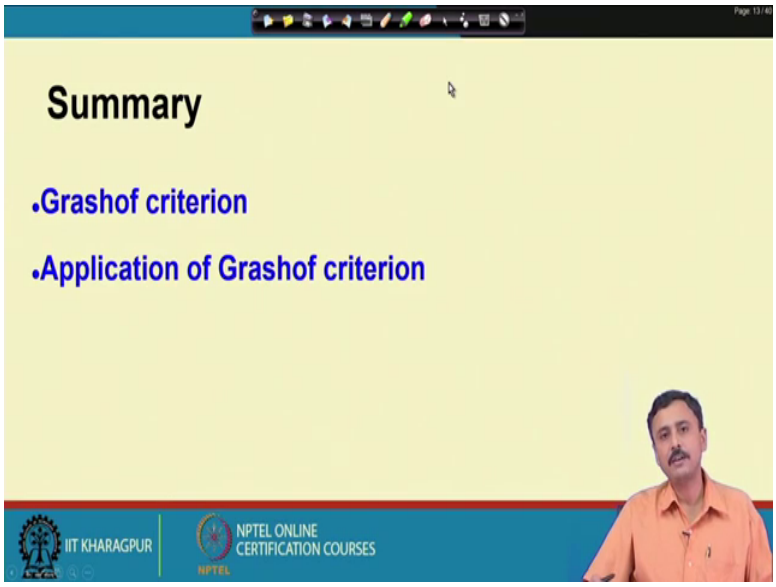

So, in this lecture we have discussed we have first recapitulated the Grashof criterion, which tells us the presence of crank in a kinematic chain. And, then through two examples I have demonstrated the application of Grashof criterion, for a determining a crank in a kinematic chain with that I will close this lecture.