Kinematics of Mechanisms and Machines
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 06
Failure of DOF Calculation

In this lecture, we are going to discuss the relook at the Degree of Freedom calculation and I will show you that the calculation that we have done for the degree of freedom of a mechanism for calculating the degree of freedom of a mechanism is not foolproof. There are examples where it does not work. Therefore, I call this the failure of degree of freedom calculation.

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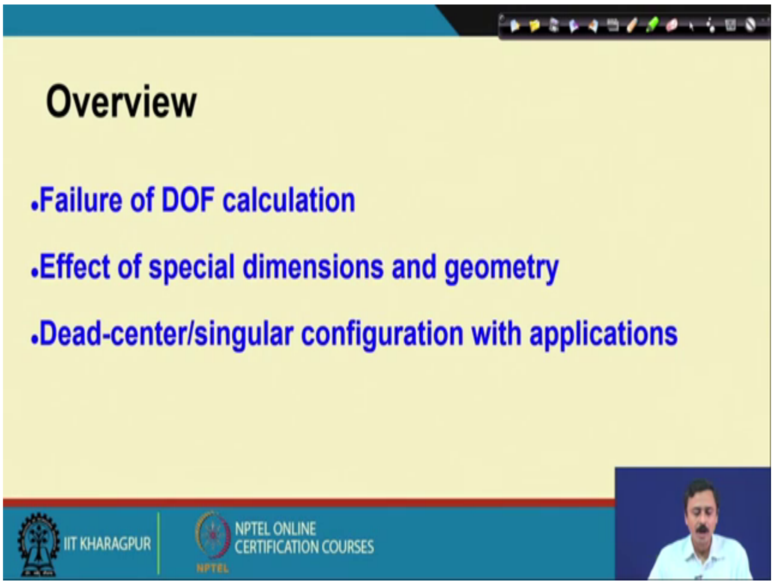

So, I will show you how special dimensions and geometry can lead to failure of this degree of freedom calculation. Then I will touch upon the concept of dead center or similar configurations with some applications.

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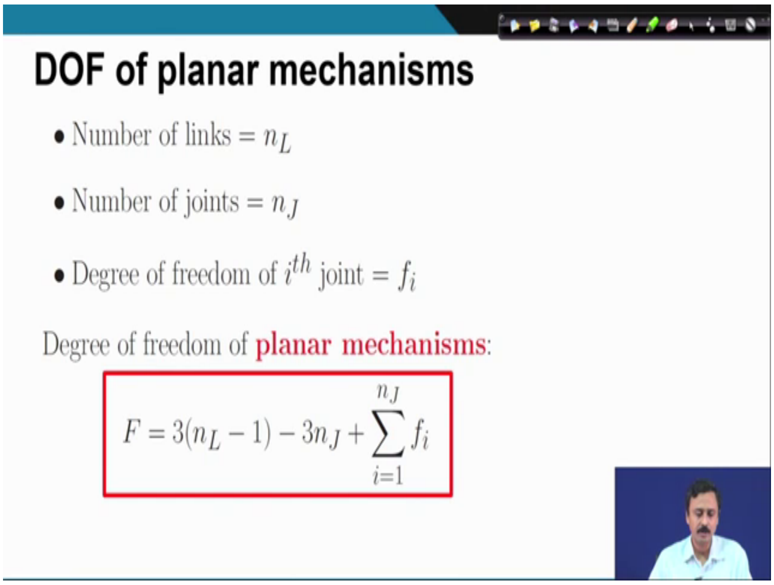

Just to recapitulate, we have done this calculation. The degree of freedom of a spatial of a planar kinematic chain is given by 3 times number of links minus 1 minus 3 times number of joints plus summation of degree of freedom of individual joints.

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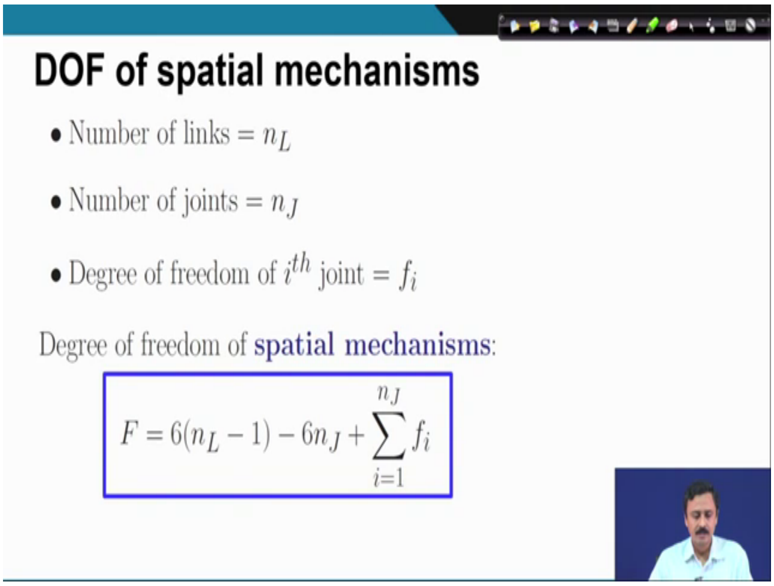

Similarly, for spatial kinetic chains or mechanisms, we have 6 times number of link minus 1 minus 6 times number of joints plus summation of degree of freedom of all the joints.

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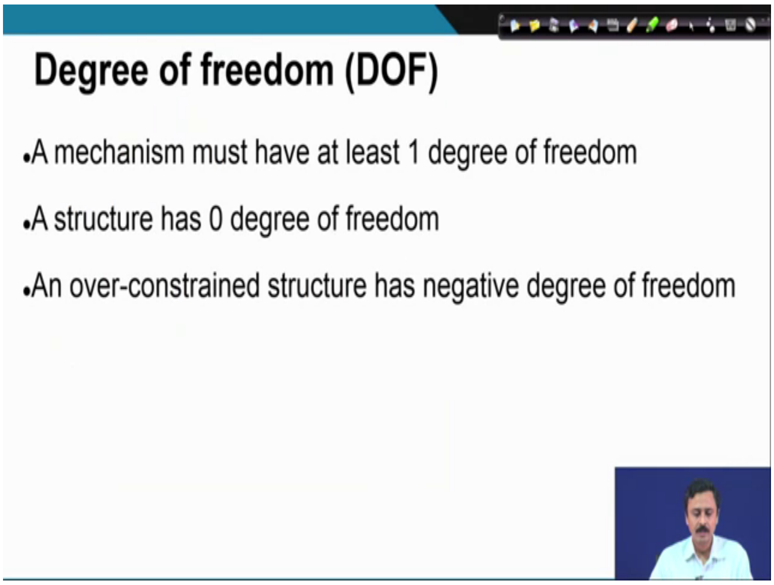

And I have also told you that a mechanism must have at least one degree of freedom, structure has 0 degrees of freedom and an over constrained structure has negative degrees of freedom.

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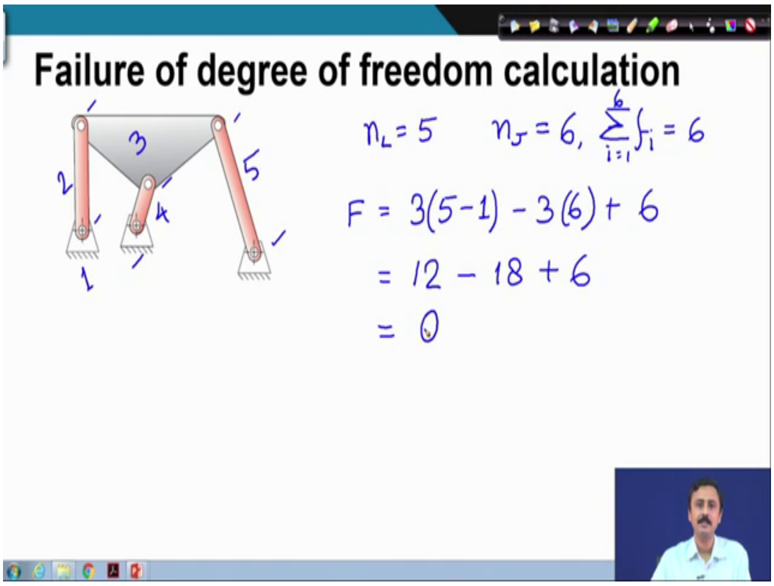

Now, we will go and look at the failure conditions. Here I have shown you a mechanism a kinematic chain. Let me calculate the degree of freedom. I will number the links first. So, ground is 1 2 3 4 5. So, number of links is 5, number of joints 1 2 3 4 5 6. So, there are 6 joints. There are all revolute pairs. Therefore, summation of degree of freedom of these joints is 6. Therefore, the degree of freedom of this mechanism or this chain is 3 times number of links minus 1 minus 3 times number of joints the summation of degree of freedom of all joints. This gives us 12 minus 18 plus 6 and that turns out to be 0. Therefore, even though this chain looks like a mechanism will I mean it looks like it can move, but it cannot. The degree of freedom calculation tells us that it is a structure.

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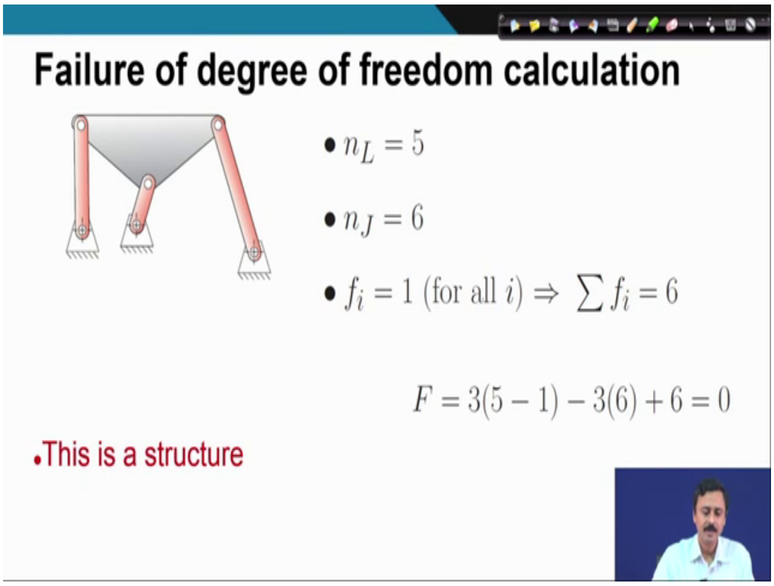

So, this is our calculation. Now this is a structure.

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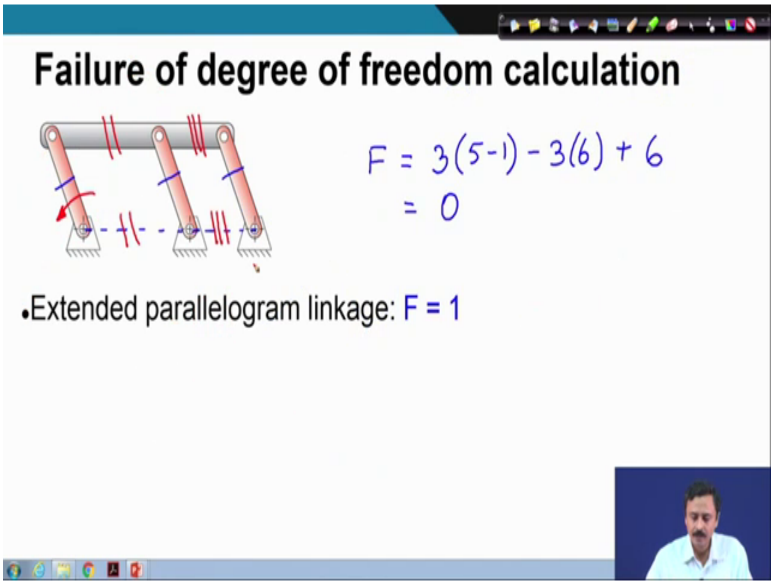

But what about this? Here also the calculation remains the same because I have 1 2 3 4 5. So, number of links is 5, number of joints is 6 and the previous calculation holds. Therefore, this must also be a structure. According to our degree of freedom calculation; according to our degree of freedom calculation, this has 0 degrees of freedom, but actually this has one degree of freedom and that is because the dimensions are very special. This length is equal to this length is equal to this length, this length is equal to this length and this length is equal to this length. Therefore, there are 2 coupled parallelograms in this.

Now, what happens due to that? As the link tries to move the parallelogram structure is retained. Therefore, the equal link lengths they all remain parallel to one another. Therefore, this is actually a mechanism. This five link chain is actually a mechanism, but your degree of freedom calculation tells us this has 0 degrees of freedom. So, this is one case where the degree of freedom calculation has failed.

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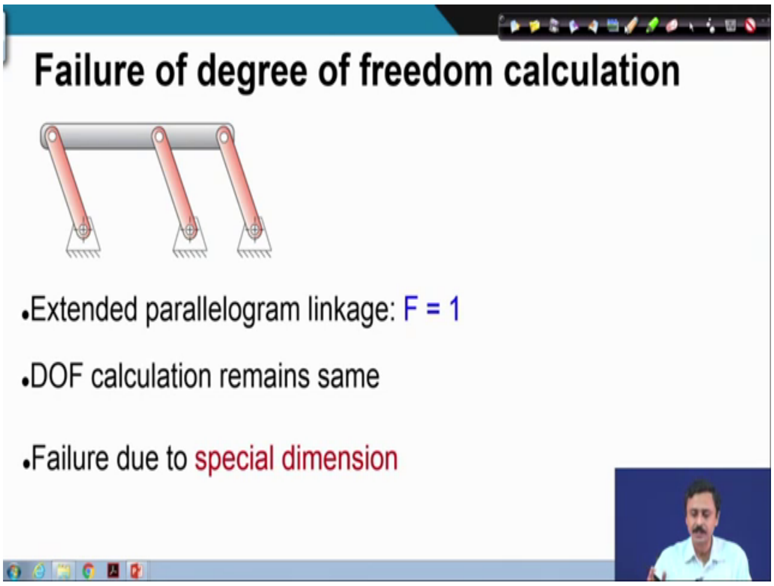

The failure is because of the special dimensions as I have mentioned. Therefore, due to special dimensions the degree of freedom calculation can fail. Now this is an important consideration; therefore, because our degree of freedom calculation never takes care of dimensions of the links. Therefore, it can fail because of very special dimensions as you can see in this case.

Now, even though this fails the degree of freedom calculation actually is a very generic feature it gives us a very generic feature. Now what do I mean by a generic feature? A generic feature is one which is not changed under perturbations. In this parallelogram chain or extended parallelogram linkage as you see in this figure, if I slightly change one of the dimensions or this link length let us say suppose I change I reduce one of the link lengths, immediately this is going to become a structure. It cannot move because the parallelogram nature is lost. The property the parallelogram property is lost. Therefore, a small change in dimension is going to make it a structure, but when you see this as a structure a small change in dimensions is not going to change it to a mechanism if you look at the figure here If you make small changes in it is dimensions, it is still going to be a structure it is it is invariant under perturbation.

So, therefore, the degree of freedom calculation is invariant under small perturbations. This is a generic feature, this I call a generic feature of the degree of freedom calculation.

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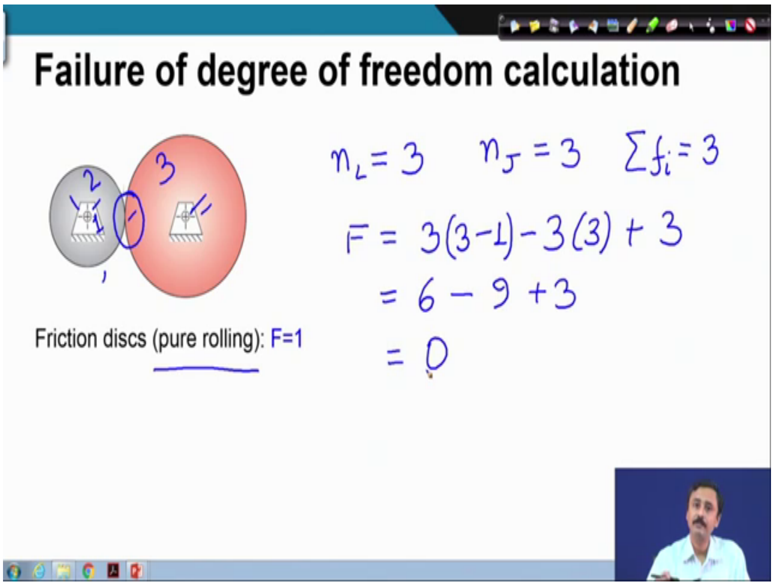

Let us look at another example. There are two friction discs just like in gear say two gears and in mesh. We can consider the pit circles as two friction discs and we know that two gears in mesh can rotate. So, they have one degree of freedom, let us now do this calculation for these two friction discs. So, the ground is 1, this disc is 2 and this disc is 3. So, a number of links is 3, number of joints is 1 2 3 and summation of degree of freedom of individual kinematic pairs. We have a revolute here that is one degree of freedom. We have another revolute here that is another degree of freedoms or 2 degrees of freedom.

Now, here we have a higher contact pair. There is a higher pair. Now this higher pair; however, has only one degree of freedom, because there is pure rolling. It does not allow sliding. This is our assumption that is it is like 2 gears. Therefore, this only allows pure rolling. So, this has also one degree of freedom. Therefore, total number of degree of freedom of the kinematic pairs is 3. Therefore, degree of freedom of the whole chain becomes 3 times number of links minus 1 minus 3 times number of joints plus summation of degree of freedom of individual kinematic pair. So, this becomes 6 minus 9 plus 3 and that tells us 0, but we definitely know that this has one degree of freedom. Two gears can rotate, two friction discs like this will rotate. Therefore, our degree of freedom calculation has failed yet again.

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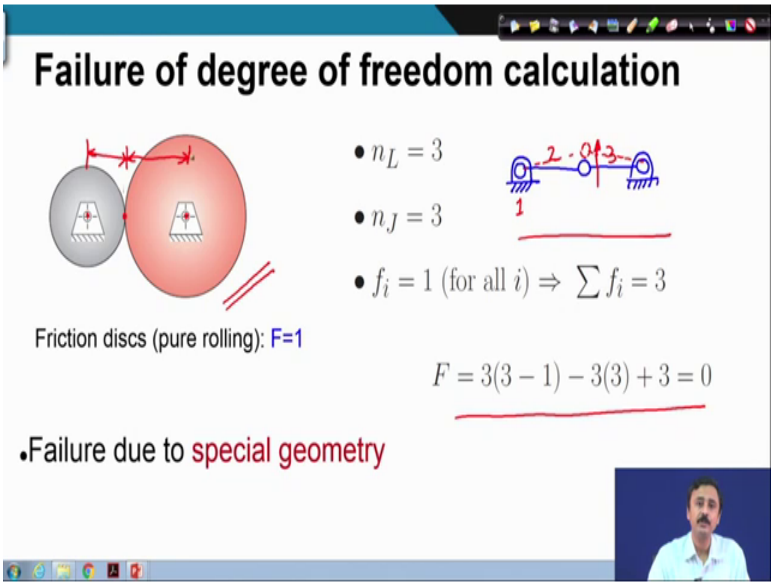

Now, this failure is due to very special geometry; the circular geometry. So, we have two circles which are rolling over one another. Because of this, the distances between the centers they are maintained. Let us look at this chain. Here, ground is 1, 2, 3. So, here also we have 3 links and 3 kinematic pairs and degree of freedom of individual is one; therefore, for summation of degree of freedom is 3 and degree of freedom calculation for this chain is also 0 gives us 0. And we know that this is a structure; why, because any small motion any small tendency of motion is going to change the distance between these 2 round hinges. These bodies must extend to accommodate any motion. Therefore, motion is not possible.

Therefore in this particular case, it is true that it is 0, but because of very special geometry which is we have these two circles; which maintain the distance as they move; therefore, the chain here has one degree of freedom. The circles will maintain the distance between the contact and the ground hinges. This distance and this distance is always being maintained by the circular geometry so, because of this therefore, the special geometry; because of special geometry, we have a failure in the degree of freedom calculation.

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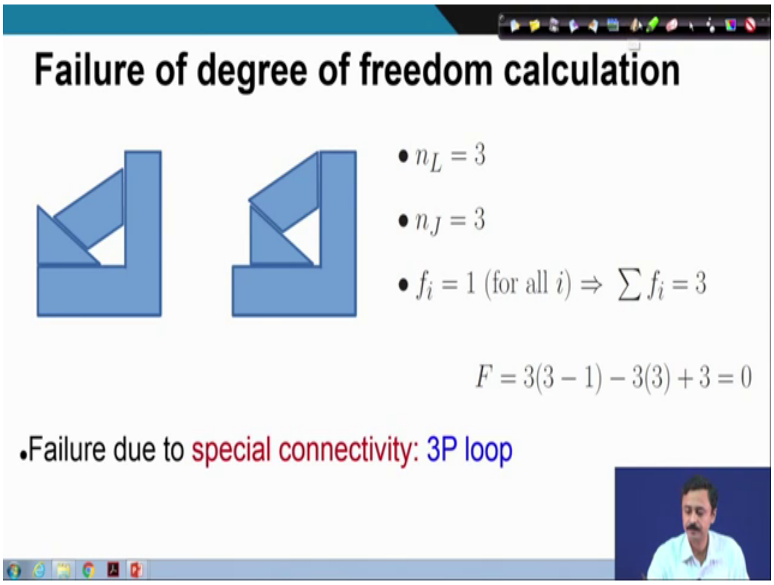

We can have failure of degree of freedom calculation because of another reason. And this is the third situation where it fails. In this example, we have three bodies; 1, 2, 3. Let me ground this one. There are sliding pairs between these rigid bodies. The number of links is 3; number of joints is also 3. There are 3 prismatic pairs actually. Individually, they have one degree of freedom. So, summation of degree of freedom is 3. Therefore, the degree of freedom of this chain comes out to be 0, but this can definitely move. Let us see how. If you push this block, this can move in and this moves out; this can definitely move. Now here, therefore, the failure is because of special connectivity. There is a 3 P loop. There are 3 prismatic pairs in a loop. Had this mean 3 revolute pairs, this would have been 0 degree of freedom; as we have just now seen the previous example. If there are 3 revolute pairs, a 3 R loop has 0 degree of freedom, a 3 P loop has one degree of freedom because of special connectivity therefore. Because the degree of freedom calculation does not distinguish between the kinematic pair whether it is a prismatic or a revolute; therefore, because of this special connectivity 3 P loop, we have a failure n of degree of freedom therefore, whenever you have 3 P loops you must add one degree of freedom to the total degree of freedom calculation.

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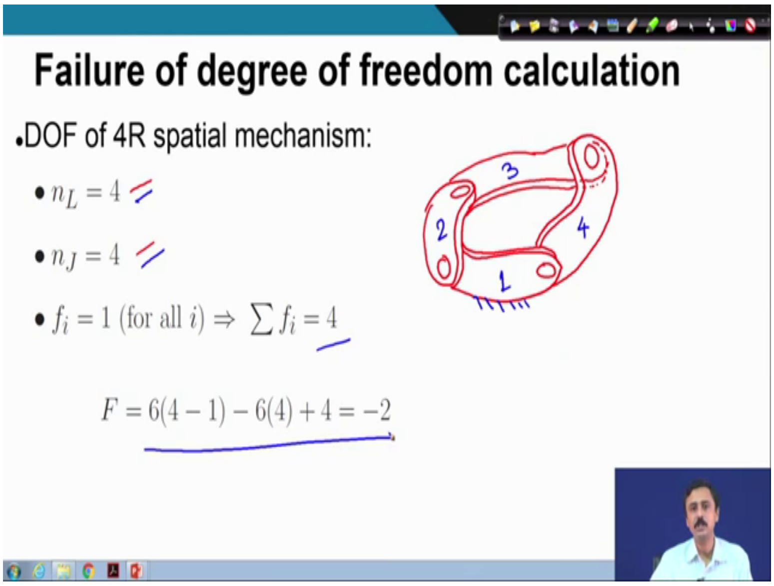

Now, let us look at a spatial the kinematic chain. If you have a spatial kinematic chain special 4 R chain, let us look at the degree of freedom calculation. So, number of links is 4, number of joints is also 4; let me try to draw such a chain. So, this roughly is a 4 R special kinematic chain, 1 2 3 4 let me ground link one. So, therefore, number of links is 4, number of kinematic pairs is also 4, now, degree of freedom of each kinematic pair because they are revolute pairs. So, summation of degree of freedom is 4; therefore, if you do a calculation of degree of freedom of this chain. It turns out to be minus 2 which means it is an over constraint structure.

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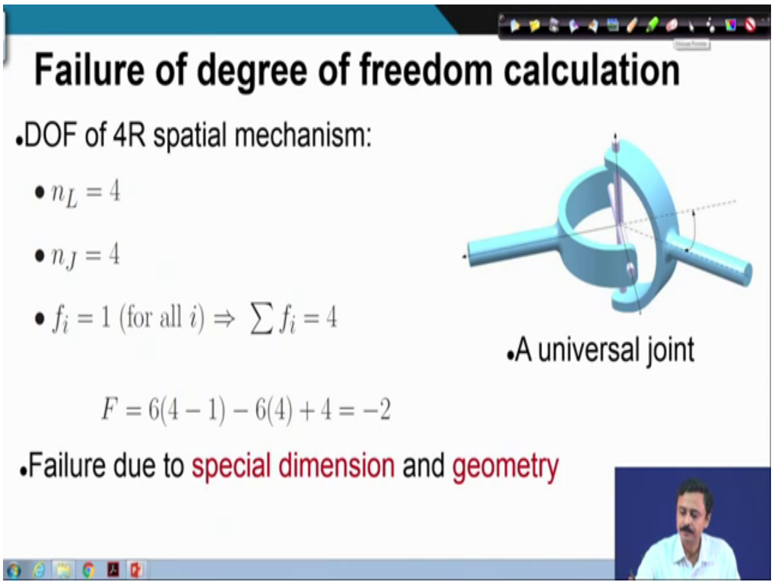

But let us look at this interesting example, a universal joint. This is a 4 R special kinematic chain. Let me show you how; this is one revolute pair, this is the second revolute pair, this is the third revolute pair and this is the fourth revolute pair. Therefore, this is 4 R spatial kinematic chain and you know that a universal joint does transmit motion. Therefore, it can move. So, it does not have it is not an over constraint structure as this calculation tells.

Now, why do we have failure then? Because of very special dimensions and geometry; now remember that there are very special angles here for example, between this, these 2 axes of the 2 revolute it is 90 degree; between these 2 axes it is again 90 degree and between these 2 axis it is also 90 degree. And because of very special dimensions and geometry because all the revolute pairs as you can see, the axis meets at a point. Because of this very special geometry and dimensions, this is a mechanism and it actually has one degree of freedom. Therefore, our failure of degree of freedom calculation; I have once again written them down because of special dimensions, because of special geometry and because of special connectivity. And it can also have a combination of two of them or of all of these.

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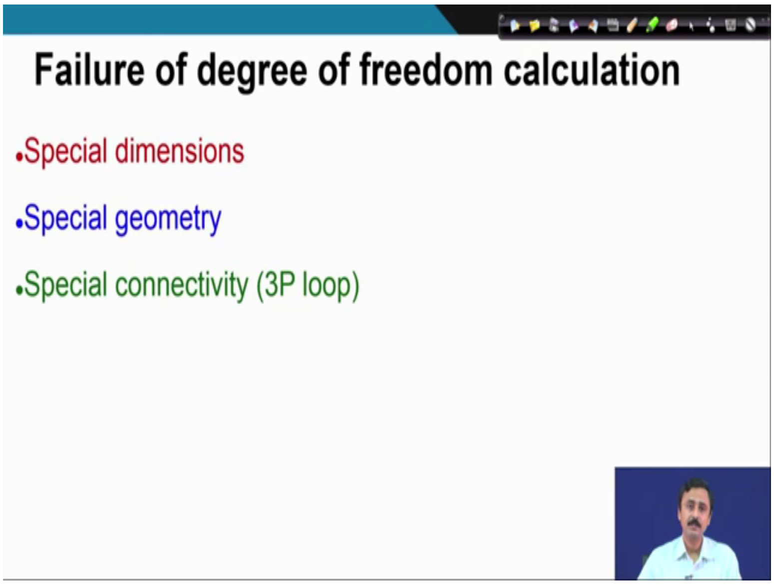

Therefore, under a very special conditions the degree of freedom calculation can fail and you have to take care of such pathological cases separately. For example, we have discussed the 3 P loop presence of 3 P loop; if you have 3 P loops in a kinematic chain, then you must add by hand one degree of freedom.

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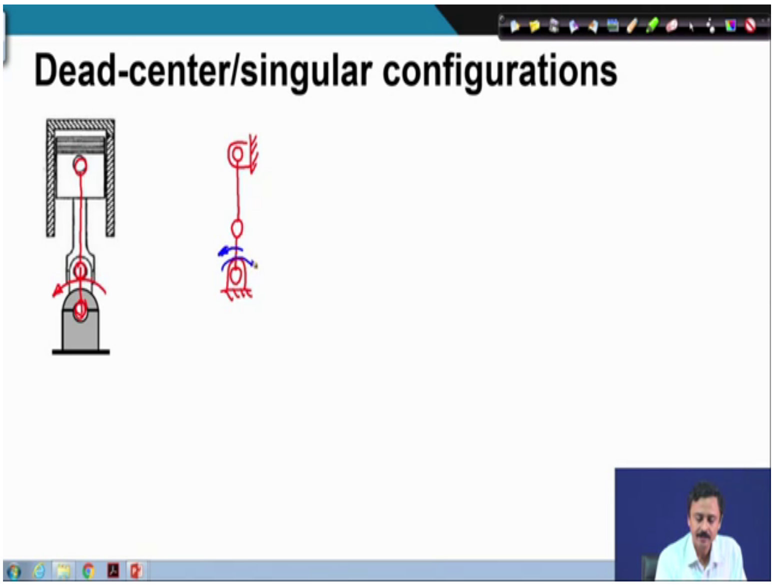

So, per 3 P loop the number if there are more than one 3 P loops, then you must add. So, many degrees of freedom so, every 3 P loop has one degree of freedom. Now, let us discuss briefly introduce, let me introduce this topic of Dead-center or singular configurations. Here, I have shown you an IC engine and you already know this terminology top dead-center and bottom dead-center of an IC engine. Here, the kinematic chain is shown in the top dead-center configuration.

Now, what is this dead-center configuration? At this configuration, one of the links is immobile. You may think that therefore, the mechanism has lost his degree of freedom, but actually it does not. Here, the piston in it is top dead-center comes to a stop; a momentary stop. As if the mechanism has cease to move, this link is not moving. The piston at this configuration actually has 0 velocities; it cannot move, but that does not mean that the other parts of mechanism cannot move. Now this is the difference between a structure which actually has 0 degrees of freedom and a mechanism at it is dead-center configuration.

Now this is a very special configuration at which one of the links one of the links which was moving might lose motion as in this case. Yet, this has one degree of freedom because this crank can still rotate. This is aligned; these two links are the crank and the connecting rod, they are now aligned and the piston has come to momentary rest. So, at this instant if I draw something like an equivalent kinematic chain, it looks like this is a 3 R as it is seen here, but this has one degree of freedom exactly at this configuration; not if it moves out of this configuration even slightly. Now, how do I understand this? At this configuration, the crank can either move this way or it can move this way. As the piston comes down, as the mechanism moves out of it is dead-center configuration, the crank actually has as a mechanism. It actually has two choices it can move in the counterclockwise manner or it can go in the clockwise direction.

Now, that choice gives us actually a degree of freedom, this choice is the degree of freedom of this kinematic chain at it is dead-center configuration therefore, even though the piston has come to momentary rest the mechanism actually has not lost it is degree of freedom.

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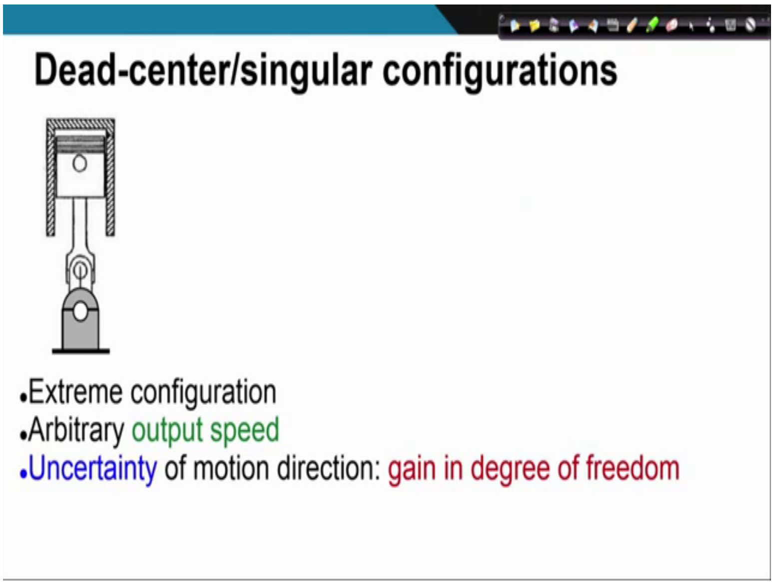

So, this is an Extreme configuration. Now because of this, you can have arbitrary output speed. You see the engine can actually be, the crank shaft can actually be rotating at an arbitrary speed. Even though at this momentary instant, the piston is immobile and there is uncertainty as I have told you, uncertainty in motion direction that translates to a gain in degree of freedom, because you have a choice.

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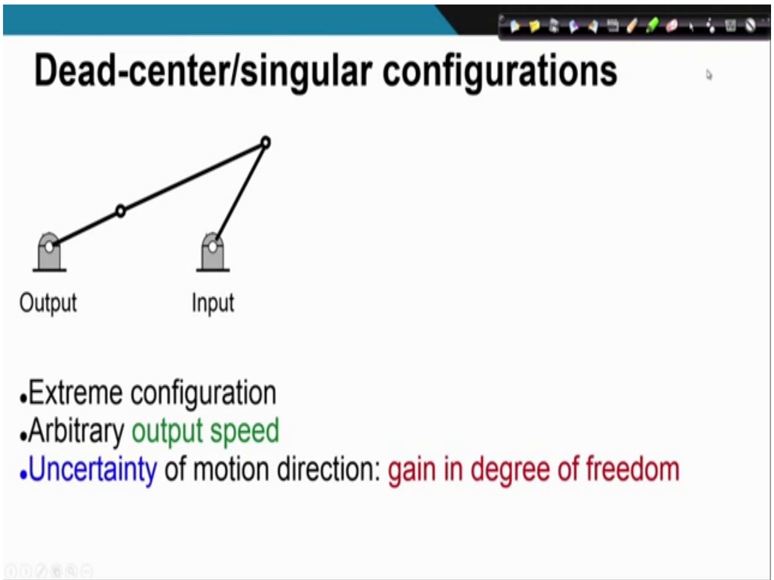

This is another example of a dead-center configuration for a 4 R kinematic chain. At this configuration, as you will realize this link which I call the input, let us say this has become immobile. It cannot move any further in this direction.

It has to go back. It must go back. It cannot move further in the clockwise direction. It has to go back. Now, as it goes back the output has two possibilities; either it can rotate in the counterclockwise sense or in the clockwise sense as far as the mechanism is concerned. Therefore, you have a choice and that gives us a degree of freedom even though the input length has lost its degree of freedom, it has come to a momentary stop. So, again this is an extreme configuration and the output speed can be arbitrary and again uncertainty in the direction of motion of the output.

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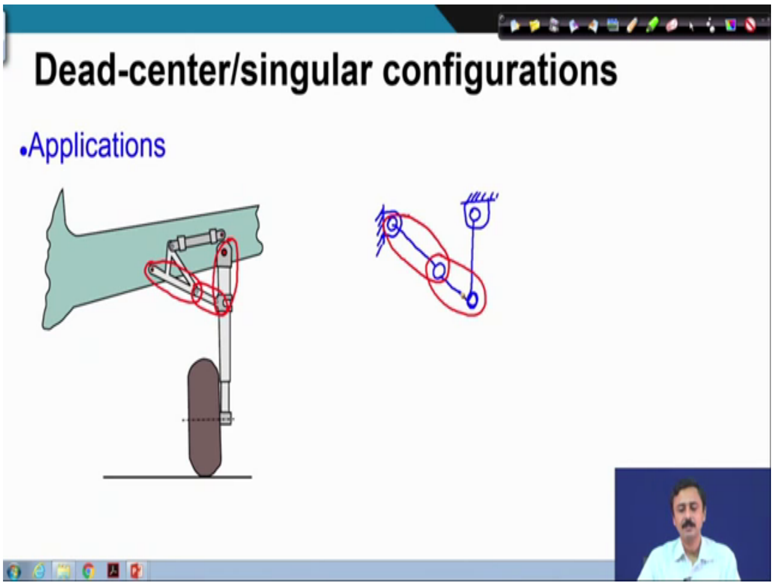

Now, this has some interesting applications. This shows the aircraft landing here, you can see very easily that this link and this link, if you consider these two links as a part of this 4 R chain. Let me draw this 4 R chain. So, this is the 4 R chain in which this link and this link, they are now collinear. Therefore, this mechanism is now in it is dead-center configuration or similar configuration. Now, this helps this mechanism to actually take huge loads and this calculation, we will see a little later in this course of lectures.

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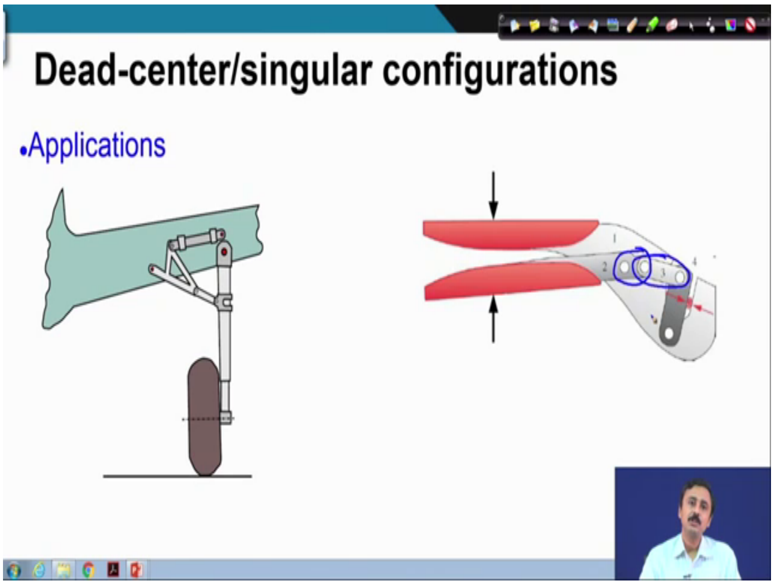

This is another example, the crimping tool. Here also, if you notice link 2 and link 3 of this 4 bar chain or 4 R chain they are almost collinear.

So, this is also very close to the dead-center configuration. These are applications of dead-center configuration which we are going to discuss and I will show you through calculations; how this configuration is actually very useful in force multiplication.

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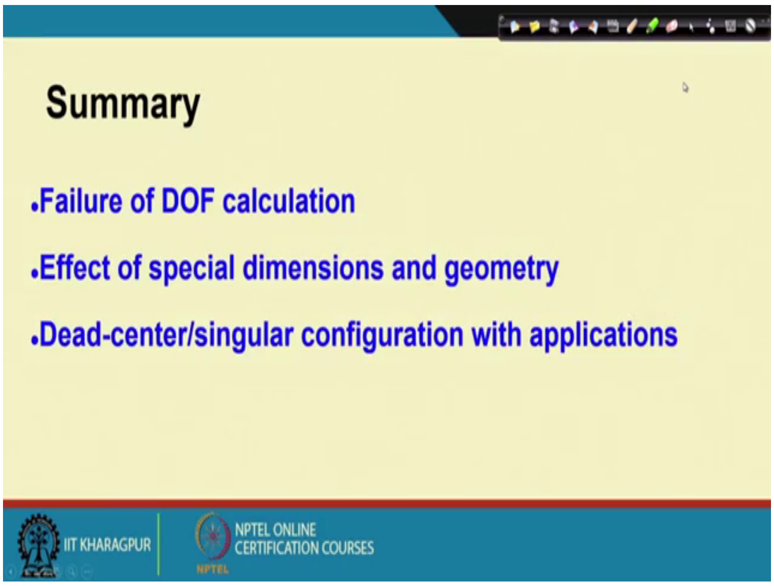

So, let me summarize. We have looked at the failure conditions for the degree of freedom calculation. I have shown you how special dimensions, special geometry, special connectivity can lead to failure of degree of freedom calculation. I have also introduced the topic of dead-center or similar configurations and I have shown you some applications which we will do detailed calculations about a little later in this course of lectures. With that I will close this lecture.