Kinematics of Mechanisms and Machines
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Lecture – 04
Degree of Freedom

In this lecture we are going to discuss a very important concept in kinematics, which is the concept of Degree of Freedom. You all must have some idea about what is degree of freedom.

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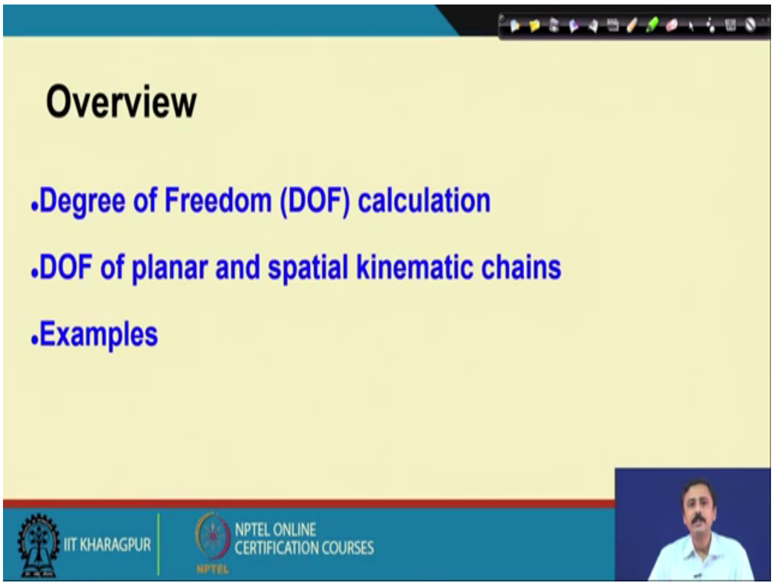

Let me give you an overview of what we are going to discuss today. We are going to discuss the calculation, how do we calculate the degree of freedom of a given kinematic chain or a mechanism, I will consider planar and spatial kinematic chains and demonstrate this calculation through examples.

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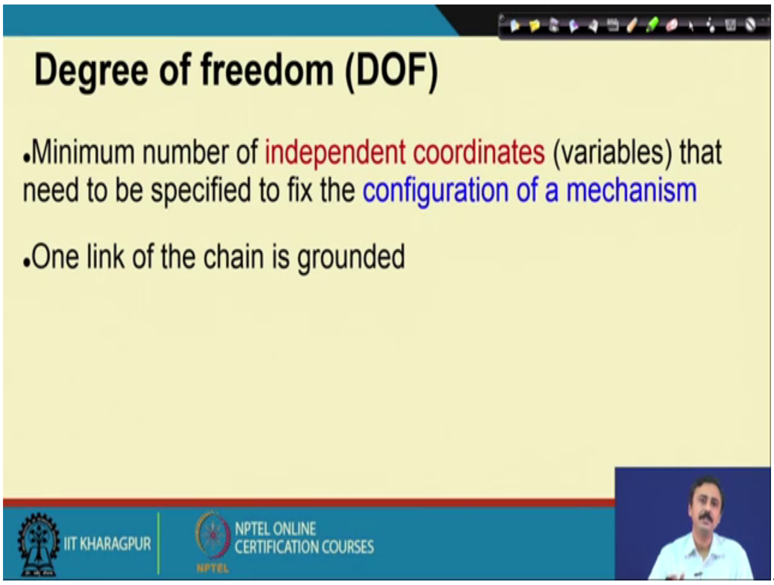

So, what is degree of freedom? The definition is the minimum number of independent coordinates, or variables that need to be specified to fix the configuration of a mechanism, here there are some key words.

The minimum number of independent coordinates and configuration of a mechanism. For example, if you have a point in a plane, it has got two degrees of freedom, as you know a point can be specified, its configuration its location can be specified by specifying the x and y coordinates of the point. So, a point has two degrees of freedom.

Similarly, we can go to rigid bodies in plane or in space, then we will go to connected rigid bodies and I will show you how the calculation of degree of freedom proceeds. Now, when we discuss the degree of freedom of a mechanism or kinematic chain one of the links of the chain will be grounded; that means, it will be immobilized. The reason is suppose you have a plier, now a plier can be taken anywhere in space, but when you talk about degree of freedom of the plier it is the relative motion of the links. Therefore, it is with respect to one link or one of the links that we will discuss the concept of degree of freedom of the plier.

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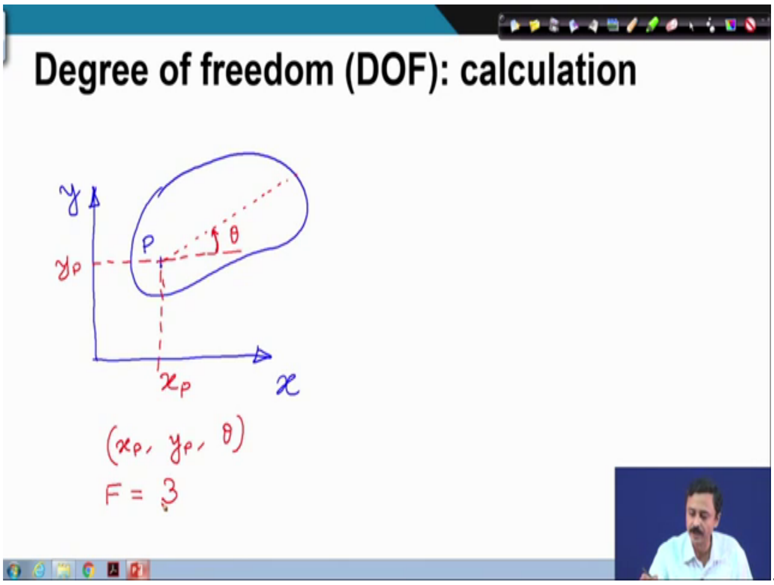

Let me now go to this calculation of degree of freedom, I have already discussed about a point, let us start with rigid body in a plane consider this is a rigid body, in the plane of the screen and x y denotes the Cartesian coordinates in which we will fix the location of this body. Now, how do I fix this body in this space. Suppose I say that there is a point P on the rigid body. So, this is a material point of the rigid body, whose coordinates I will specify.

Once I specify the coordinates of this point P just like we do for a point, the point P is fixed, but because this is a rigid body it has got an orientation, which is of importance. Therefore, it is not enough to fix only this point P, we also need to fix some line belonging to the rigid body, let us say this is a line belonging to the rigid body I can specify, let us say this angle let me call it theta.

Now, then if I specify x p y p and theta, then you will notice that this rigid body has been immobilized in the Cartesian space two dimensional Cartesian space which is shown. Therefore, a rigid body in a plane has 3 degrees of freedom, if you specify these 3 variables or coordinates as we call them, if you specify these 3 then the body gets immobilized this configuration is fixed.

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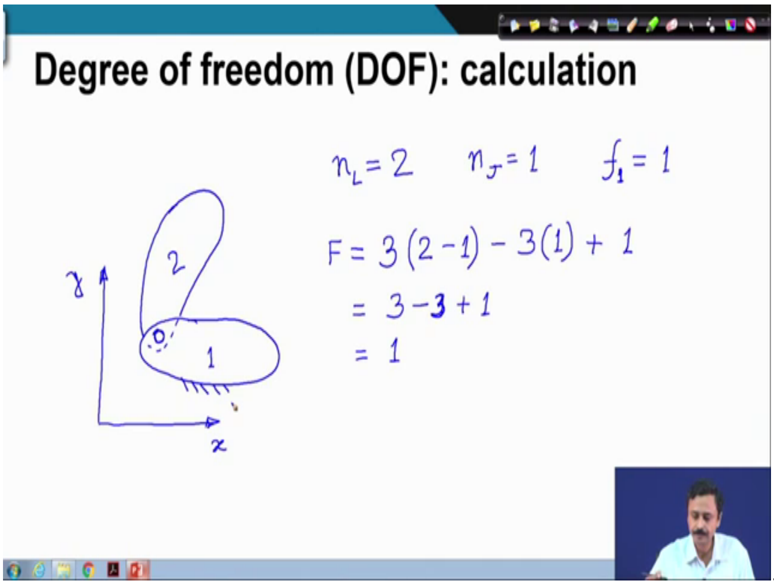

Now, let us enlarge this, consider two bodies which are connected. So, there is a kinematic pair connecting these two rigid bodies. Now this is a chain, let me fix one of the links of this chain therefore, I will fix one of this, one of the two links of this chain this link let us say, let me now number these two links this ground is numbered one, and this is the other link number two and we have a kinematic pair.

If I now want ask this question what is the degree of freedom of this system, how many coordinates will be required to fix the configuration of this chain. There are two rigid bodies, now two rigid bodies again this is of course, in a plane. We are discussing in the planar case first, there are two rigid bodies therefore, there should be 2 into 3 because each rigid body has 3 degrees of freedom so, 2 into 3 6 degrees of freedom, but one of the rigid bodies is already fixed.

Therefore if I have as in this case number of links is 2, the degree of freedom calculation, as it starts as we have discussed it has only 3 degrees of freedom as of now, because one body is already mobilized, it has got 1 kinematic pair will call this or denote this as n J, it has got one joint or 1 kinematic pair. Let us assume to begin with that this kinematic pair has 1 degree of freedom. So, pair variable we have discussed pair variables. So, this has 1 pair variable so, it has got 1 degree of freedom.

I will note this as f i or f 1 there is since there is only 1 kinematic pair. So, f 1 the degree of freedom of that kinematic pair is 1. Now, the degree of freedom calculation therefore, now proceeds like this f is equal to 3 times, now one body is already immobilized. Therefore, of the two bodies I must subtract 1 and multiply it with 3 to give me the degree of freedom of the rigid bodies involved.

Now, we come to the kinematic pair, we will follow this logic that each kinematic pair initially takes away all these 3 degrees of freedom accorded to a rigid body in a plane. Here there is one kinematic pair so we say that this kinematic pair will take away 3 degrees of freedom. Therefore, I subtract 3 times the number of kinematic pairs involved here it is only 1. So, minus 3 times 1, but it really does not take away all the degrees of freedom, it actually gives or has 1 degree of freedom this kinematic pair has 1 degree of freedom. So, we add it back into this calculation.

Therefore the degree of freedom calculation becomes 3 minus 3 plus 1 which is 1. So, this chain as I have shown you has 1 degree of freedom.

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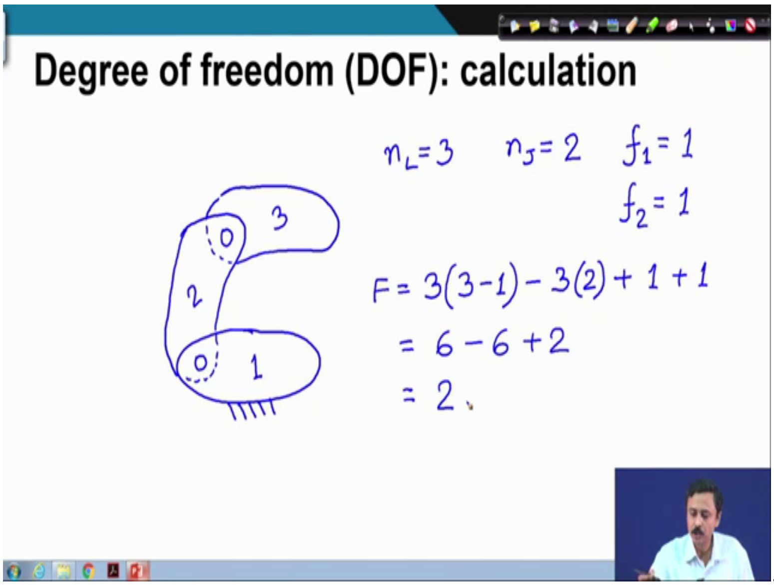

If you now, consider little more extended chains, consider there are 3 rigid bodies, they are connected by kinematic pairs, there are 2 kinematic pairs, as before we will fix 1 and number these rigid bodies ground is 1 2 and 3.

Let me start writing the number of links is 3 number of joints, we have 2 kinematic pairs. So, number of joints is 2 the degree of freedom of the first joint let us consider that is a again a simple hinge is 1 and the degree of freedom of the second kinematic pair, let me also assume that also is a simple hinge. So, that also has 1 degree of freedom.

Therefore the degree of freedom calculation for this chain, becomes 3 times number of links minus 1, because 1 is grounded minus the 2 kinematic pairs will now take away 3 times 2 it will take away 6 degrees of freedom, but it will give back 1 plus 1, because it allows each kinematic pair allows 1 degree of freedom.

Therefore this becomes 6 minus 6 plus 2 so, that is 2 so this kinematic chain has 2 degrees of freedom. Now, this can continue, you can extend it in the planar case in the spatial case. So, let me show you how.

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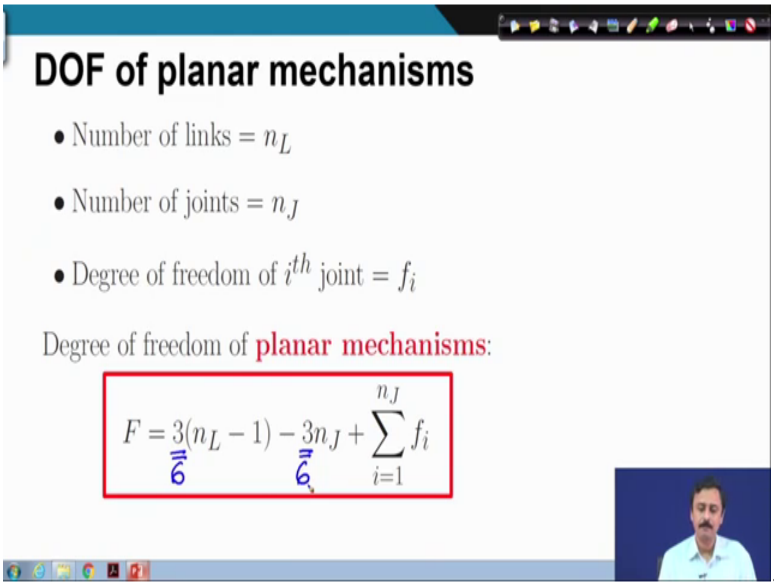

So, this is the formal way we have derived the degree of freedom calculation. So, if the number of links is n L number of joints is n J. And the degree of freedom of the ith joint is f i, then the degree of freedom of a planar mechanism is 3 times n L minus 1 minus 3 times n J plus summation of degree of freedom of individual kinematic pair.

Now, what happens when we go to spatial kinematic chains, in spatial in space a rigid body has 6 degrees of freedom as you know a rigid body in space has 6 degrees of freedom, 3 positional degrees of freedom and 3 orientational degrees of freedom.

Therefore, the calculation becomes exactly the same except that this 3 get is replaced by 6. And similarly this 3 also get is replaced by 6, other things will remain the same.

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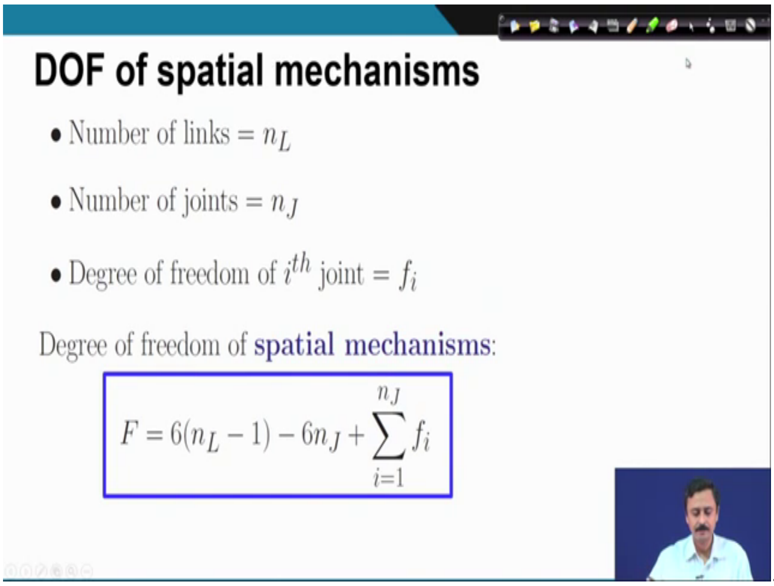

So, formally in a spatial kinematic chain, we have 6 times n L minus 1 minus 6 times n J plus summation of degree of freedom of individual kinematic pairs.

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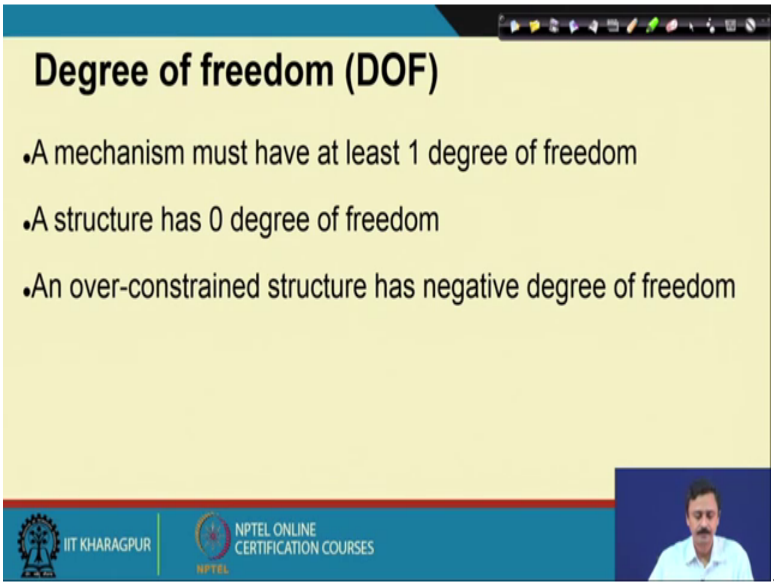

Now, when do we call a chain a mechanism? So, the criterion for naming a chain as a mechanism, is when it has at least 1 degree of freedom. So, it must have at least 1 degree of freedom, then we call it a mechanism.

If it has 0 degrees of freedom, then it is a structure and if it has negative degrees of freedom it can happen in the calculation you come up with negative numbers as degree of freedom, then it is an over constrained structure.

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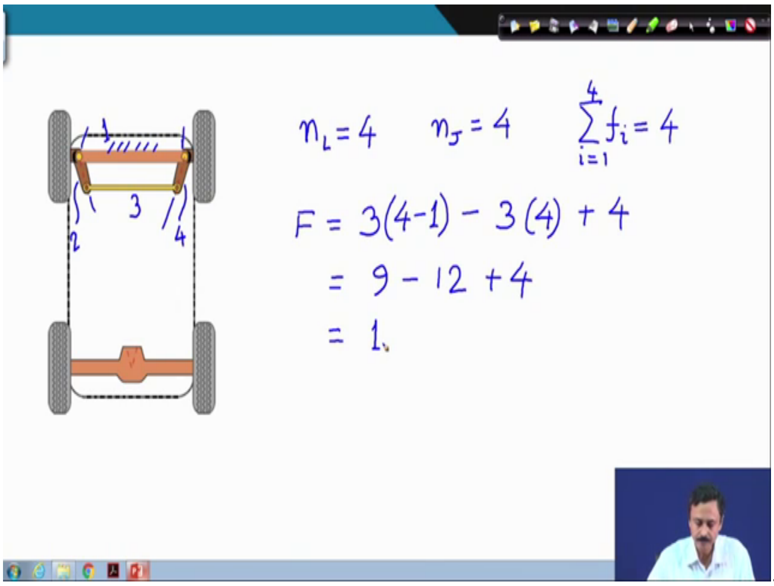

Let us now go to examples, I will show you how to calculate this example is of the steering wheel mechanism, this link is fixed to the car body. So, I will number this as 1, the grounded link as 1, this link is 2 the coupling link as 3 and this is 4.

So, 2 and 4 are connected directly to the wheels. In this case therefore, number of links is 4, number of joints or number of kinematic pairs, here we have 1 kinematic pair, here we have 1 kinematic pair, this is the 3rd kinematic pair and this is the 4th kinematic pair. So, we have 4th kinematic pairs 4 joints.

Now, degree of freedom of individual kinematic pairs, these are all revolute joints as you can see. Therefore, each kinematic pair has 1 degree of freedom, in other words the summation of degree of freedom of all these kinematic pairs is 4, because each kinematic pair has 1 degree of freedom and there are 4 kinematic pairs so, 4.

Therefore degree of freedom is 3 times 4 minus 1, because 1 is already one link is already grounded minus 3 times 4 plus 4. So, this gives us 9 minus 12 plus 4 and that is 1 degree of freedom. And as we know this 1 degree of freedom is controlled by the driver who steers the vehicle.

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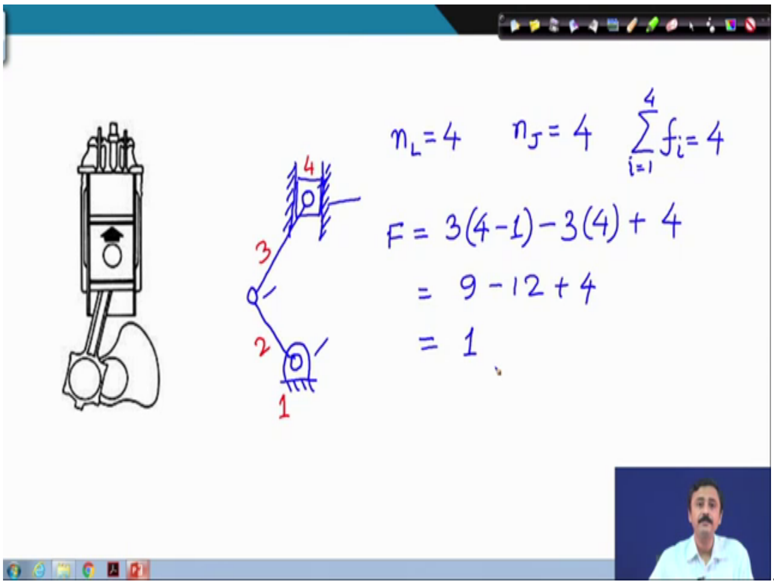

This is an IC engine, let me start by drawing the kinematic diagram of this. So, there is a coupling link the crank, this is the ground I will number the links first ground is 1 2 3 and the piston is 4. Therefore, the number of links is 4, number of kinematic pairs number of joints we have revolute revolute revolute and we have a prismatic. So, there are 4 kinematic pairs 3 revolutes and 1 prismatic.

Therefore summation of degree of freedom of all these kinematic pairs turns out to be 4, because revolute has 1 degree of freedom prismatic also has 1 degree of freedom. Therefore, degree of freedom is 3 times 4 minus 1 minus 3 times number of joints, which is 4 plus summation of degree of freedom of all joints that gives us a degree of freedom 1. And as we know that IC engine has 1 degree of freedom. So, the piston drives the crank.

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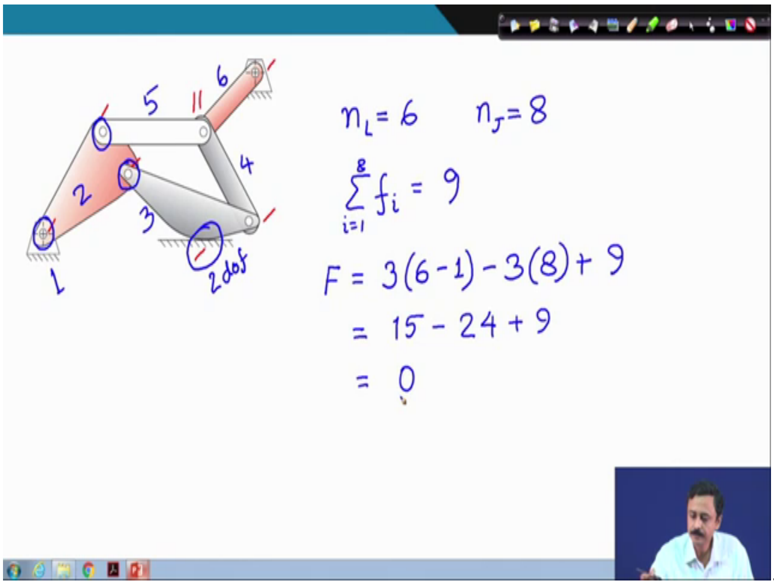

Now, this is another kinematic chain, let us number the links ground is 1 2 3 4 5 6 in this chain therefore, there are 6 links n L equal to 6 number of kinematic pairs, here there is 1 revolute 2 3, this is the contact between link 3 and the ground so, there is a third kinematic pair 4.

Now, here there are 3 links connected as you can see there are 3 rigid bodies connected 4 5 and 6 therefore, there are two kinematic pairs here there are two kinematic pairs here and we have another kinematic pair with the ground. Therefore, total number of kinematic pairs 1 2 3 4 5 6 7 and 8.

So, total number of joints is 8, now we have to count the degree of freedom of individual kinematic pairs and sum them up. This kinematic pair has 1 degree of freedom 2 3, now this kinematic pair this is a higher pair as you can see there is a point contact. Therefore, in this point contact it can allow both sliding as well as rolling, this kinematic pair will allow sliding as well as rolling without losing contact of course.

Therefore this kinematic pair this higher pair has two degrees of freedom. So, let us count once again the summation of degree of freedom of individual kinematic pairs 1 2 3 4 5 6 7 8 9. Therefore, the summation of degree of freedom of all the kinematic pairs taken together is 9. Therefore, degree of freedom of the whole chain is 3 times number of links minus 1 minus 3 times number of joints plus summation of degree of freedom of all joints this gives us 15 minus 24 plus 9 and that turns out to be 0.

Therefore this assortment of links even though it looks like it can move possibly, but it cannot it is a structure.

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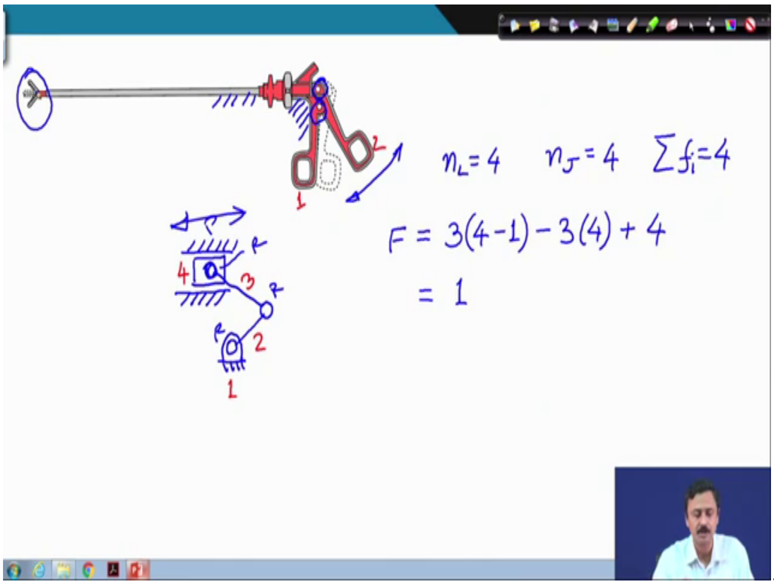

This is the surgical tool which I have shown you before we will start by grounding one of the links, I will ground this finger grip which also actually grounds the barrel. Let me first draw the kinematic diagram for this. So, the ground link, there is a ground hinge. So, this hinge is the ground hinge to which the other finger grip is connected and that is connected to a revolute pair here, which goes and connects to a slider inside the barrel.

So, this ground is the barrel, let me then number the links ground is 1, 2 is the this finger grip, 1 is this finger grip and this is link 3 and inside the barrel, there is a slider which is number 4. Therefore, in this case the number of links is 4, number of joints is revolute revolute revolute and prismatic.

So, 4 and summation of degree of freedom of all these joints is also 4 because each has 1 degree of freedom. Therefore, the degree of freedom is 3 times number of links minus 1 minus 3 times number of joints plus degree of freedom of all joints and this gives us degree of freedom 1, which is controlled by the finger the surgeons fingers.

So, by moving this finger grip the surgeon controls the motion of the barrel of the slider inside the barrel, this slider inside the barrel is extended right up to the end effector of this tool, which controls the scissors or the clamp.

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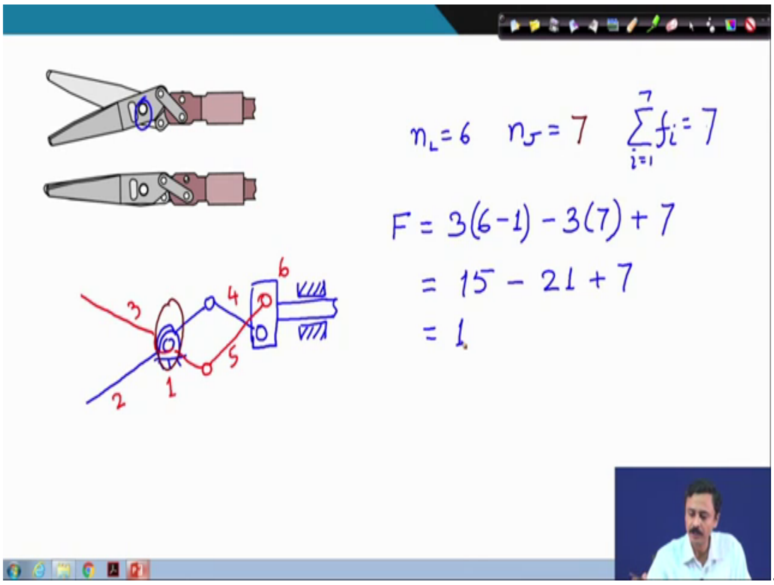

Now, this is the end effector, let me draw the free body diagram. So, this is the ground hinge, this is the lower blade connected to a coupler to a slider which goes in into the barrel. And the upper blade also connects to the slider in this manner, let me count the number of links ground is 1 2 3 4 5 6. So, number of links is 6 number of kinematic pairs. Now here there are 2 kinematic pairs, 2 revolutes a 2 3 4 5 6 and 7. So, total number of kinematic pairs is 7 and summation of degree of freedom of these kinematic pairs is also 7 as you can see that, they are all revolute or prismatic pairs they have all 1 degree of freedom.

Therefore degree of freedom of this end effector this scissors is 3 times 6 minus 1 minus 3 times 7 plus 7. So, 15 minus 21 plus 7 that is 1. Therefore, by the surgeon's finger movement this scissors can be uniquely controlled.

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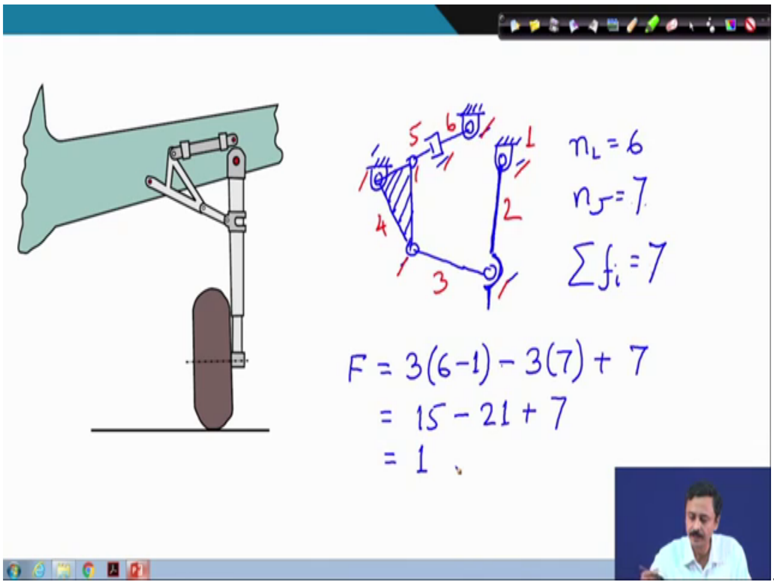

This is the landing gear of the aircraft, there are 3 ground hinges, the ternary link connected to the actuator and this is the wheel.

Let me count the number of links ground is 1 2 3 4 5 6. So, number of links is 6 number of joints is 1 2 3 4 5 6 and summation of degree of freedom of these kinematic pairs, they are all revolute or prismatic therefore, it is 6 this the number of kinematic pairs, let us count once again 1 2 3 4 5 6 7.

So, number of kinematic pairs is 7 and each one has 1 degree of freedom therefore, total number of degrees of freedom is also 7. So, degree of freedom is 3 times number of links minus 1 minus 3 times number of kinematic pairs plus summation of degree of freedom, that gives us 1 degree of freedom for this aircraft landing gear.

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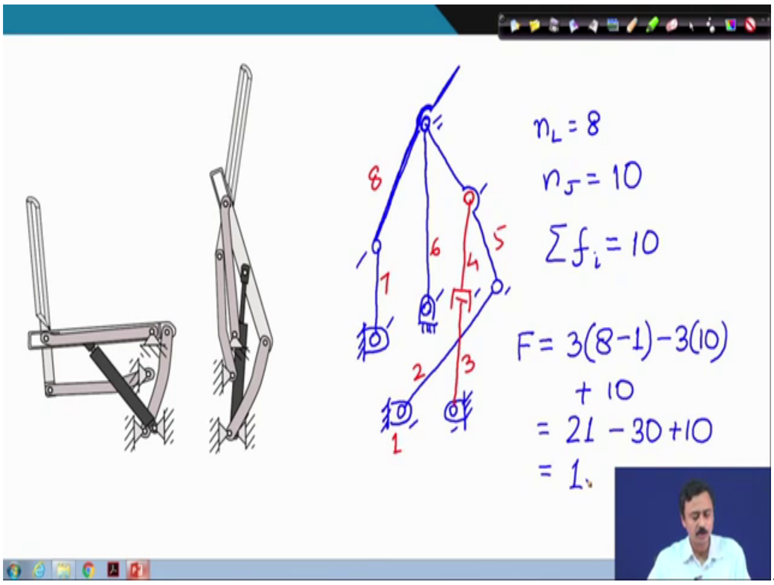

This is the transfer aid there are 4 ground hinges, here we have 1 link, this is the actuator, this is the backrest this is a single link.

Let me count ground is 1 2 3 4 5 6 7 8. So, there are 8 links number of joints 1 2 3 4 5 6 7 8 9 10, there are two kinematic pairs here. So, 10 summation of degree of freedom of all these kinematic pairs, they are all revolute or prismatic therefore 10, so degree of freedom is 3 times number of links minus 1 minus 3 times number of joints plus summation of degree of freedom of all joints. So, 21 minus 30 plus 10 that gives us 1 degree of freedom and this is controlled by this prismatic actuator.

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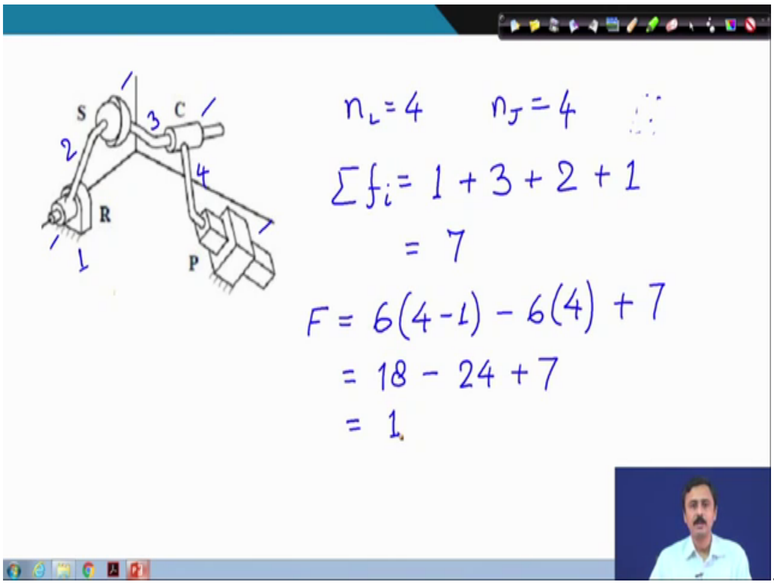

Let us discuss some examples of spatial kinematic chains, let me count the links ground is 1 2 3 and 4. So, number of links is 4 number of joints we have a revolute here a spheric 2, a cylindric 3 and a prismatic 4. So, number of joints is 4 and summation of degree of freedom of all joints, let me do this calculation here.

So, for the revolute it is 1 for the spheric it is 3, for the cylindric we have 2 for the prismatic we have 1. Therefore, we have total 7 degrees of freedom for the kinematic pairs. So, therefore, degree of freedom now because this is a spatial kinematic chain, we have 6 times number of links minus 1 minus 6 times number of joints plus summation of degree of freedom of individual joint, this gives us 18 minus 24 plus 7 and that gives us 1. Therefore, this spatial kinematic chain has one degree of freedom. And this is how we calculate degrees of freedom of spatial kinematic chains.

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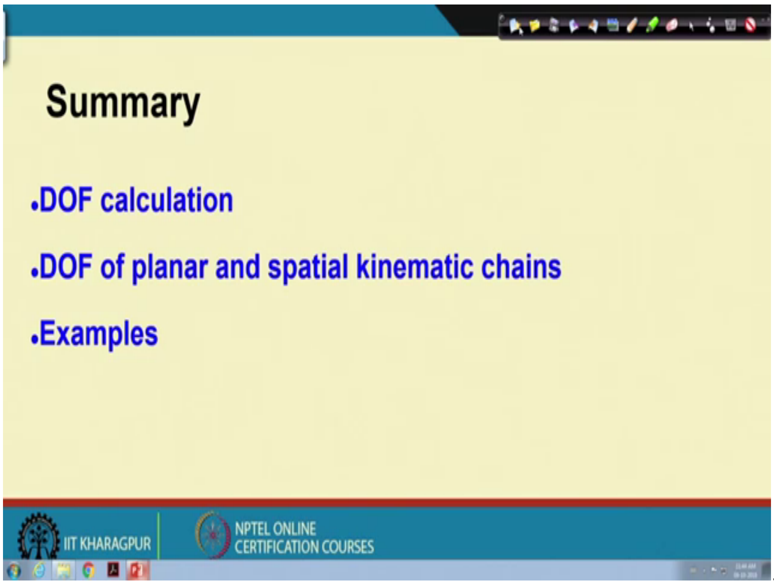

So, let me summarize what we have discussed, we have calculated the degree of freedom of a kinematic chain. So, I have shown you how this calculation is done and I have taken some examples of planar and spatial chains and demonstrated this calculation with that I will close this lecture.