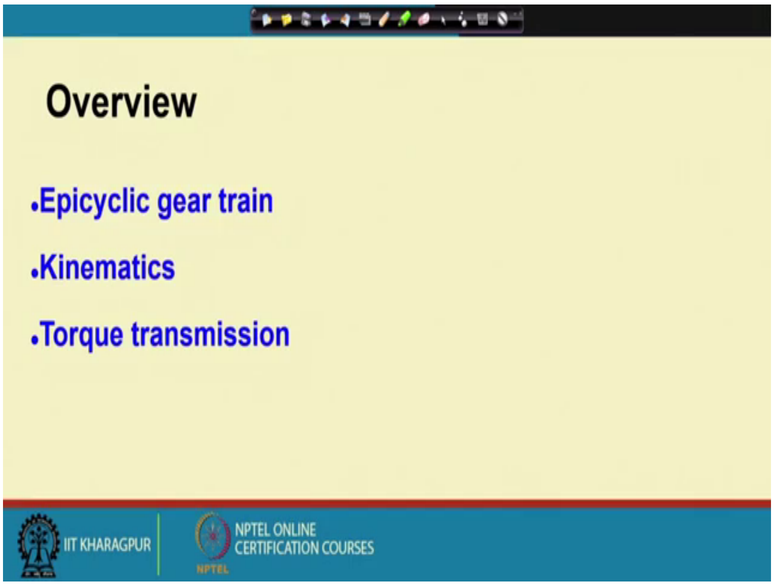
Kinematics of Mechanisms and Machines
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 39
Gear Trains – II

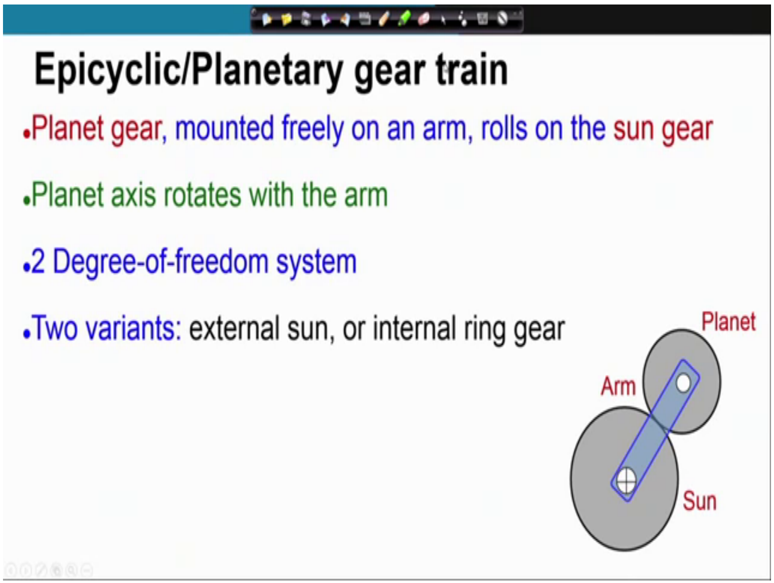
We will continue our discussion on Gear Trains.

(Refer Slide Time: 00:23)



In this lecture, I am going to discuss about epicyclic gear train; their kinematics and torque transmission.

(Refer Slide Time: 00:29)



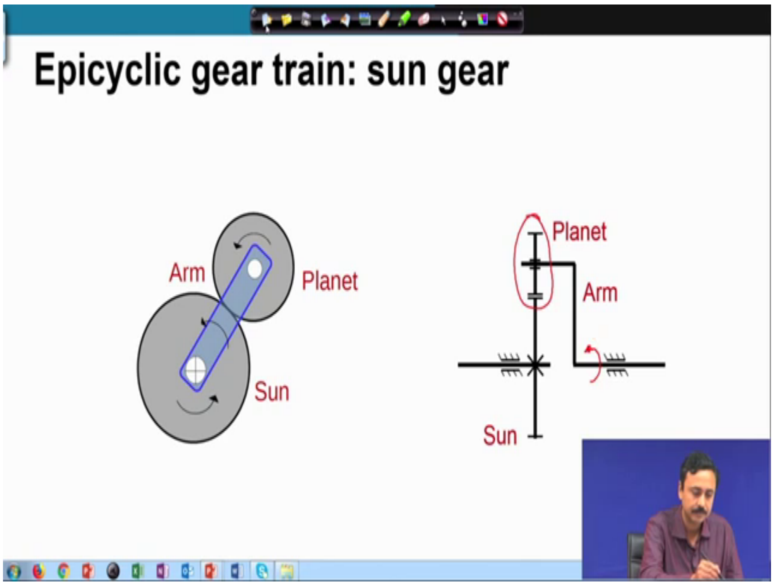
Then, epicyclic gear train as I have mentioned that there are gears who centres can move or rotate in space. This is unlike the simple and the compound gear trains where their centres are fixed. The epicyclic gear train has these components, it has got planet gear which is mounted freely on an arm and it rolls on what is known as the sun gear. This is the analogy is from our solar system.

So, here I have sun gear on which there is this planet gear which measures with the sun gear. The sun gear axis is fixed which is indicated by this cross at the centre; while the planet can roll freely on the sun gear. In order to keep the planet in place we have the arm. So, that keeps the planet in its place as it rolls on the sun gear. So, the planet axis rotates with the arm as it is shown here. This system has 2 degrees-of-freedom, let us understand this.

If I want to fix the configuration of the sun planet system, sun planet and arm system; then, suppose I fix the angle of the arm; I fix the angle of the arm. So, that is 1 degree-of-freedom, I can arbitrarily choose the angle of the arm. Suppose, I fix this; then still the sun and the planet can rotate with respect to one another; therefore, if I fix the sun at a particular angle then the planet also gets fixed. Therefore, I can fix the arm and the angle of the sun to fix the whole system. So, this has 2 degrees-of-freedom since we require two variables to fix up the configuration.

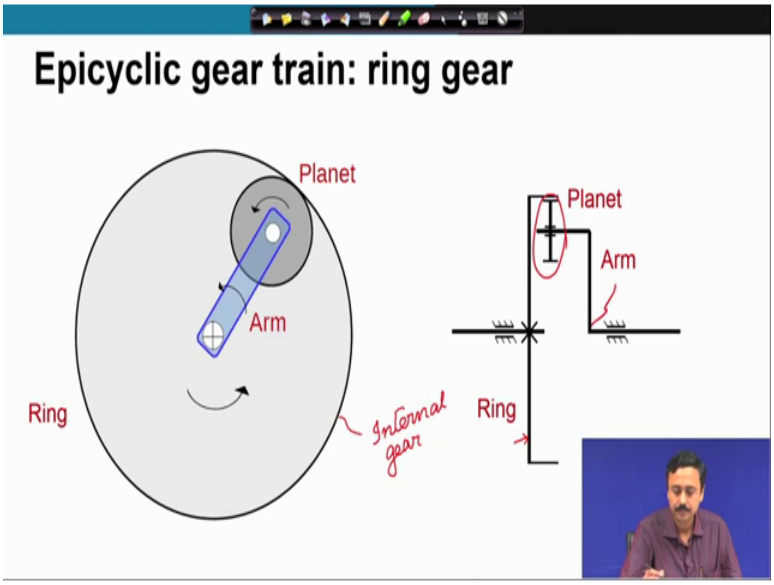
Now, there are two variants as we will soon see, we have this external sun external gear which is the Sun or an internal gear which is a ring gear.

(Refer Slide Time: 03:18)



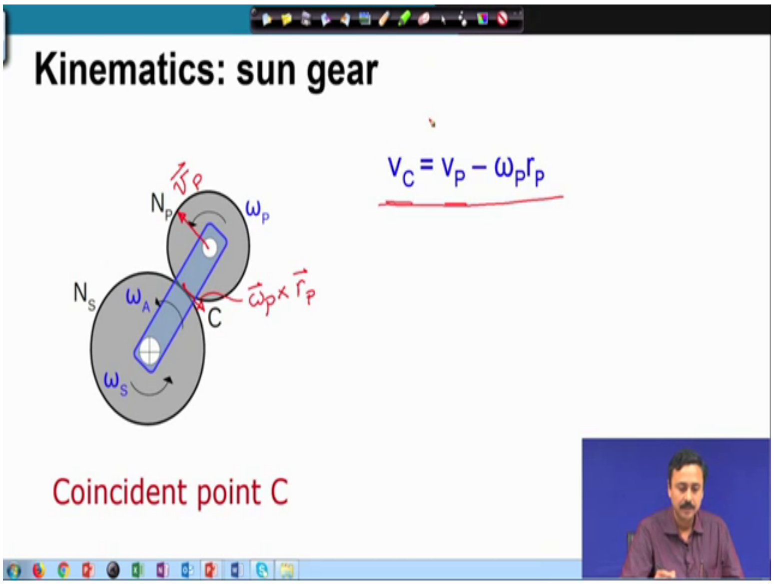
Here, is an epicyclic gear train with the sun gear, if you look at the representation; a schematic representation of this, the Sun gear axis is fixed where as the planet is mounted freely on the arm, the arm can rotate. So, therefore, the arm carries the planet as the planet measures on roles on the Sun.

(Refer Slide Time: 03:59)



This is the second type in which we have a ring here. So, this is an epicyclic gear train with a ring gear this is an internal gear, so, the ring gear is an internal gear. If you look at the schematic representation, this is the ring gear and we have the planet mounted freely on the arm.

(Refer Slide Time: 05:13)

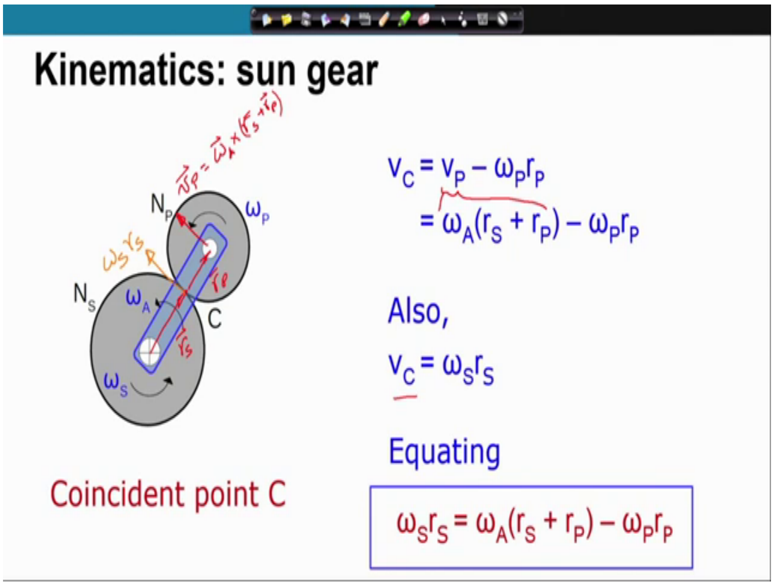


Let us now look at the kinematics of the epicyclic gear train. In order to derive the relation between the input and out now, we know that there are 2 inputs and 1 output just 2 degrees-of-freedom. Therefore, we consider this contact point C of the pitch circles of the sun and the planet gears and trying to do a velocity matching at the contact; this is for pure rolling.

Now, if you look at the velocity of the point C as seen from the planet gear side, this can be written as velocity of the contact on the planet is equals to velocity of the centre of the planet and plus omega cross omega planet cross the radius vector. Let us look at these vectors one by one. V P which is the velocity of the planet centre, this is the velocity of the planet because of the rotation of the arm. So, this velocity V P is because of the rotation of the arm.

Then we have because of the rotation of the planet itself about its centre, we have this is omega planet cross the radius vector. And if you do this calculation and finally, write out this vector equation, then it turns out that because these two vectors are in a position. So, velocity of the centre of the contact point velocity of the contact point C of the planet is equal to velocity of the centre of the planet minus omega P times r P.

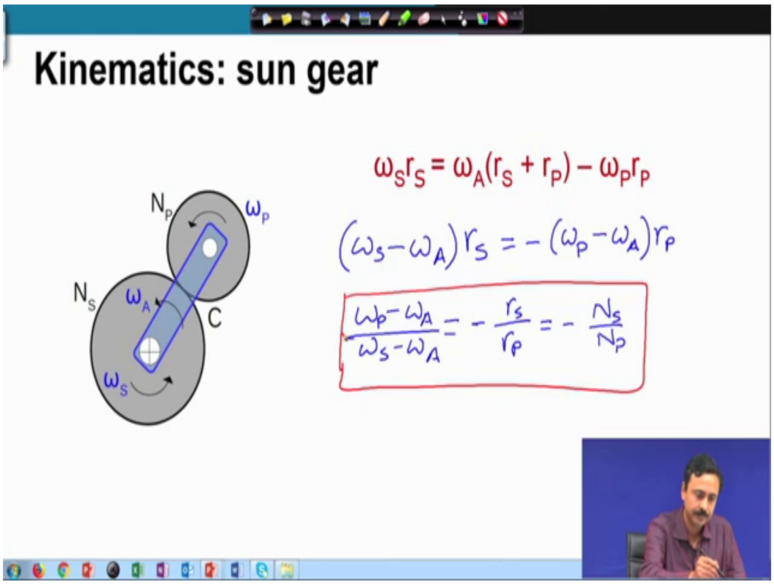
(Refer Slide Time: 08:01)



Now, velocity of the centre of the planet this vector V P which I had drawn this is V P. As I have mentioned that this is because of the rotation of the arm. So, this must be equal to, so, omega of the arm cross the length of the arm which is summation of these two vectors; one is radius of the radius vector of the sun and this vector the radius of the planet.

Therefore, if you express velocity of the centre of the planet, it will be omega A times sum of the radii of the sun and the planet and minus omega P r P. If you look from the sun side, then velocity of the contact from the sun side is equal to omega of the sun times the radius of the sun. So, this is omega of the sun times radius of the sun. So, that is the velocity of the contact when you calculate it from the sun gear side. Now, these two must match because these two pitch circles are in pure roll.

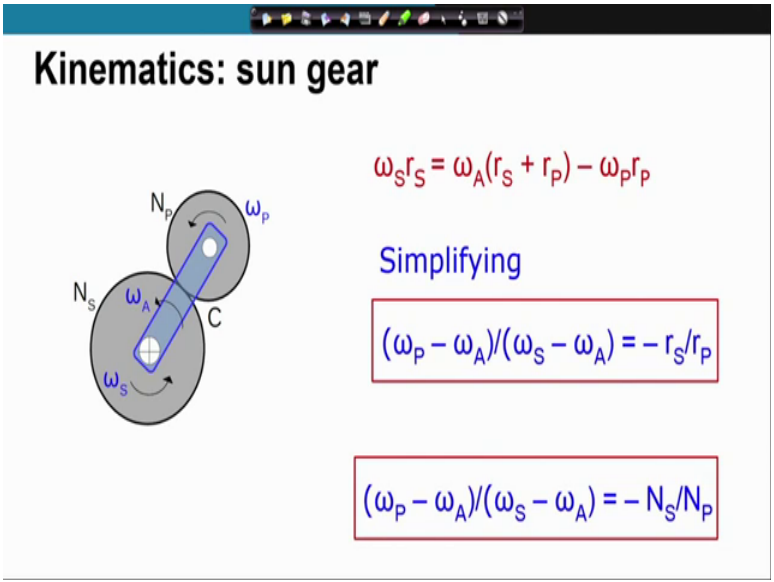
(Refer Slide Time: 10:30)



If you do that, then omega S r S must be equal to omega A r S plus r P minus omega P r P. Now, if you simplify this expression, you can write this as omega S omega S minus omega A times r S is equal to minus of omega P minus omega A r P. So, therefore, now I can take the ratio let us say omega P minus omega A by omega S minus omega A that is equal to minus of r S by r P and this you also know is equal to minus of N S by N P.

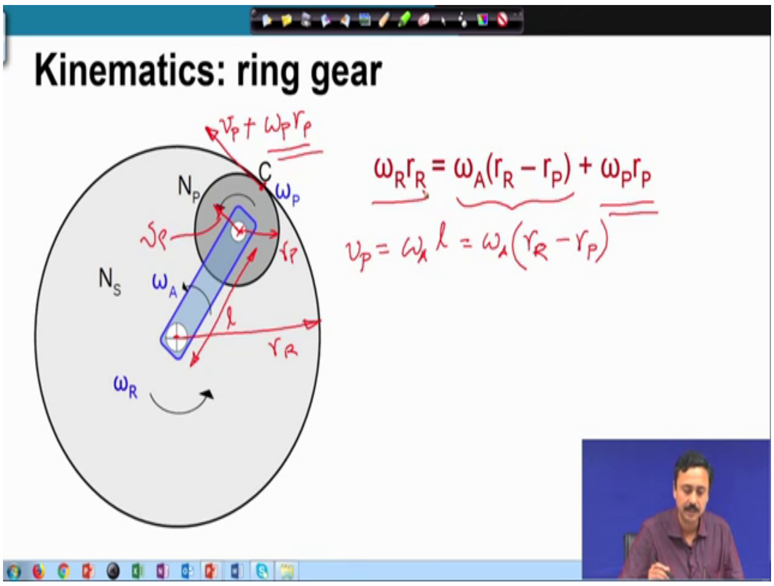
So, this relation that we have obtained relates omega of the planet omega of the sun and omega of the arm in terms of the number of teeth on the sun and the number of teeth on the planet.

(Refer Slide Time: 12:08)



So, this I write out for you. So, this is the final expression of the motion relation. From here given let us say omega of the arm and omega of the sun, you can find out omega of the planet or given the omega of the planet and omega of the sun, you can find out omega of the arm.

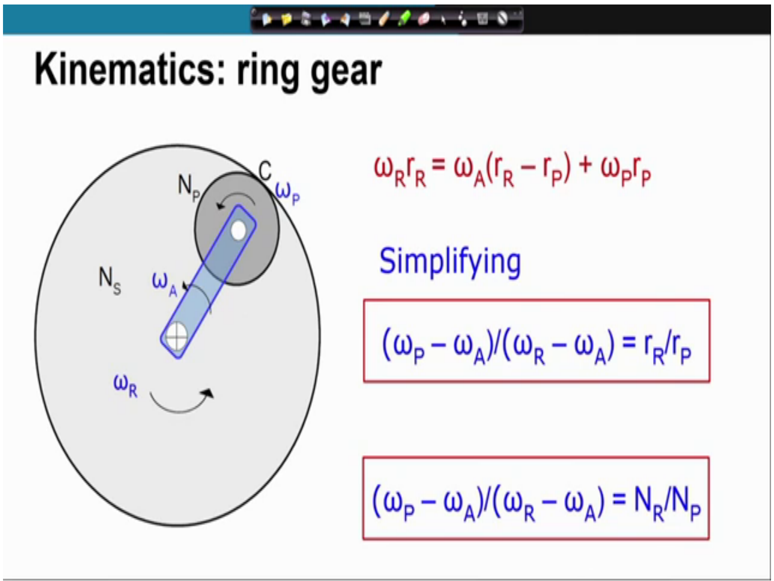
(Refer Slide Time: 12:35)



Let us now look at the other case where you have a ring gear. So, here you have this relation from similar considerations that omega of the contact as seen from the planet side, this must be equal to velocity of the centre of the planet. Now, plus omega planet times radius of the planet, so, from the planet side if you calculate, velocity of C is equal to velocity of the centre of the planet plus omega P times r P. Velocity of the planet is nothing but omega arm times this length, this is the velocity of the planet, this vector is V P.

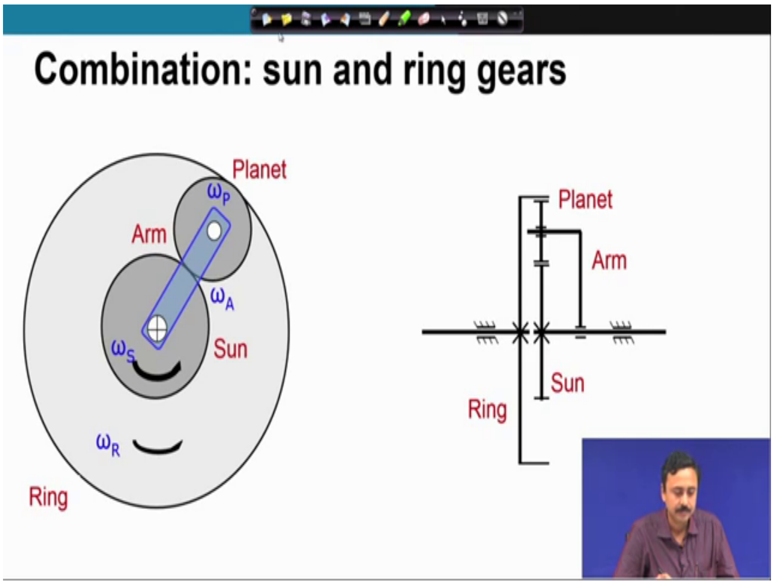
What is V P? Now, V P is equal to if I call this l, V P is equal to omega arm times this length l. Now this length l can be written as radius of the ring gear minus the radius of the planet gear. Radius of the ring gear minus radius of the planet gear and that has been substituted here, this term comes from here and we match the velocity of the contact from the ring gear side which is omega R times radius of the ring gear.

(Refer Slide Time: 15:20)



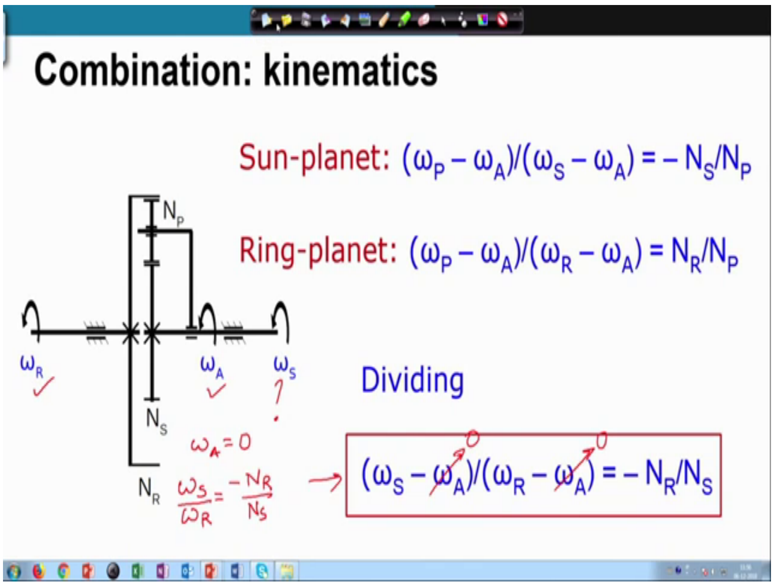
If you simplify this expression, we have this relation and you can write this ratio of the on the right hand side r R by r P as N R by N P. Here you should notice that because this ring gear being internal gear, the sign here is positive. This is unlike the relation we obtained when we had a sun gear. So, that completes the kinematics of the epicyclic gear train.

(Refer Slide Time: 16:00)



Let us now look at the combination of both sun gear and a ring gear with a planet. Now this is the fundamental unit that is used in automatic transmission as we will discuss. This is the schematic representation of the gear train with this combination of ring and the sun.

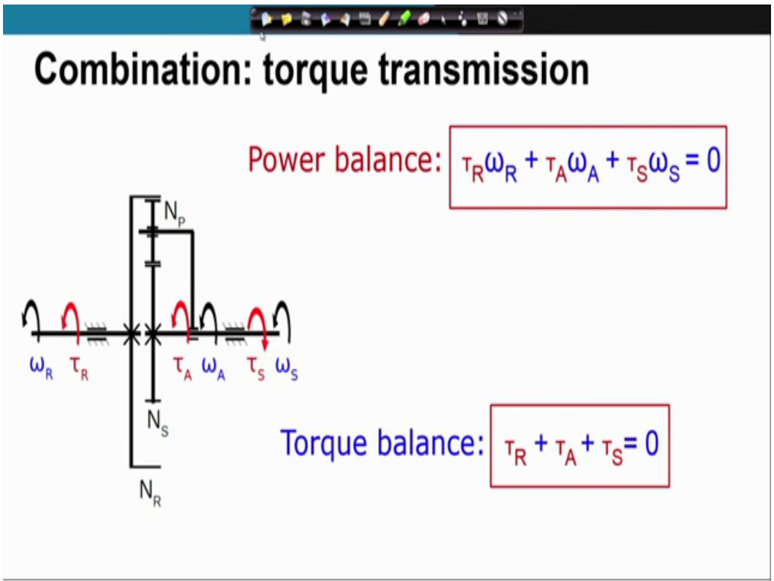
(Refer Slide Time: 16:31)



Let us now look at this kinematics relations and go in pairs we will discuss we will look at these this combination in pairs. First let us look at the sun planet pair which means we are looking at this system in which we have the sun-planet and the arm. We have derived the kinematic relation which is of this form. Next we look at the ring-planet combination; this is the ring-planet combination.

So, we had the ring gear, the planet gear and the arm and the kinematics for this, we have derived is written out here for you. You can clearly note the difference in these two relations. Now if you divide one by the other, then you can find out this relation between omega sun, omega arm, omega ring in terms of the teeth number of teeth on the ring and the sun. You will notice that this does not involve the planet anywhere. So, we have a relation between the angular speed of the sun the ring and the arm in terms of the number of teeth on the ring and the number of teeth on the sun.

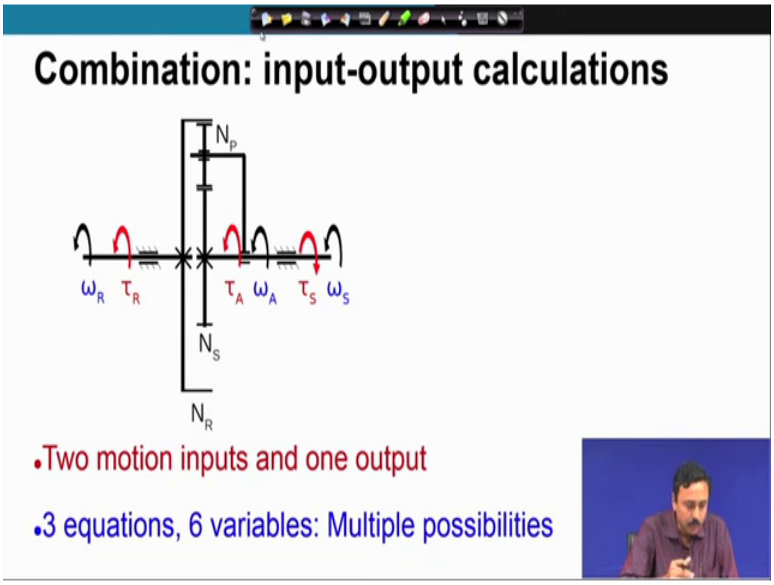
(Refer Slide Time: 18:20)



Now, let us look at torque transmission, here I have drawn out the torques that are acting tau R is the torque on the ring gear; tau A is the torque on the arm and tau S is the torque on the sun. If you do a power balance, then this is the relation we have been looking at this relation; if you do a power balance, then the net rate of work done on the system must vanish. There are 3 inputs, so, to say you have the power input at the ring, the power input at the arm and the power input at the sun and the total power input must add up to 0 that gives us the work power balance.

And we also have the torque balance relation because the whole thing is in equilibrium we are assuming that whole thing is in equilibrium; there is no dynamic forces acting, so, everything is in steady state.

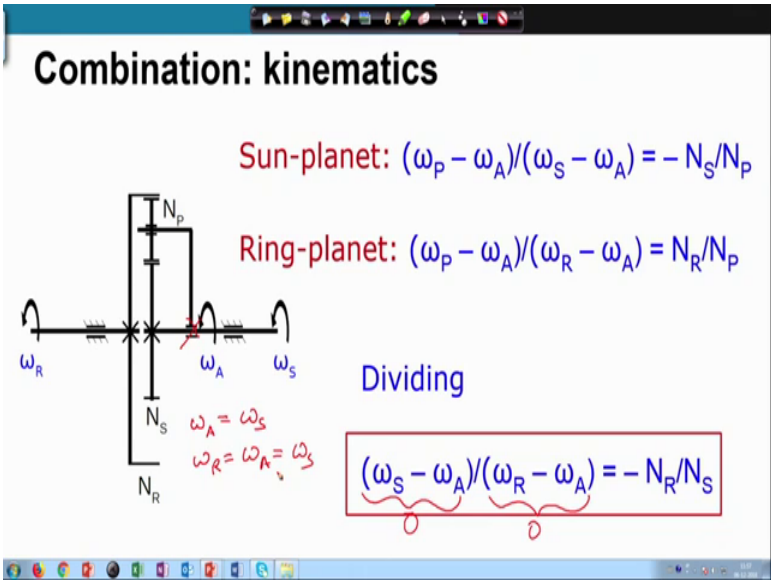
(Refer Slide Time: 19:46)



So, what do we have finally? We have 2 motion inputs and 1 output and we have obtained 3 equations in 6 variables. So, what are these three equations? One equation was the kinematic relation between the omega sun, omega ring and omega arm in terms of the number of teeth on the sun and the ring. The other equation was of power balance. So, the power input at the sun, power input at the arm and the power input at the ring. They must add up to 0 and the torque balance. These are the three equations we have in 6 variables, the 6 variables are shown in this figure; omega R, tau R, tau A, omega A and tau S, omega S. Now if we are given 3 of these variables and we can solve for the other 3.

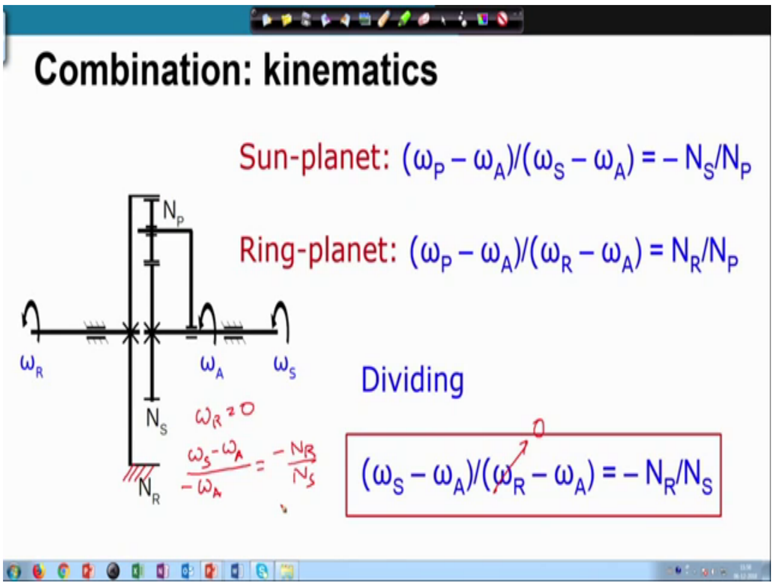
If you quickly go back and look at this motion relation let us say, if we are given the omega of the ring and omega of the arm; then, from this relation we can solve for omega of the sun, so, we can solve the omega of the sun. Let us look at some very special cases. Let us say that I fix up omega of the arm as 0. Let me take some special cases; one special case can be omega arm is 0, which means my arm is fixed, it is not rotating. If omega arm is 0, then we have this relation omega S by omega R is minus of N R by N S which means its a simple transmission between the sun and the ring, I mean what we have for two external gears. On the other hand, if you let us say fix the arm with one of the gears.

(Refer Slide Time: 22:40)



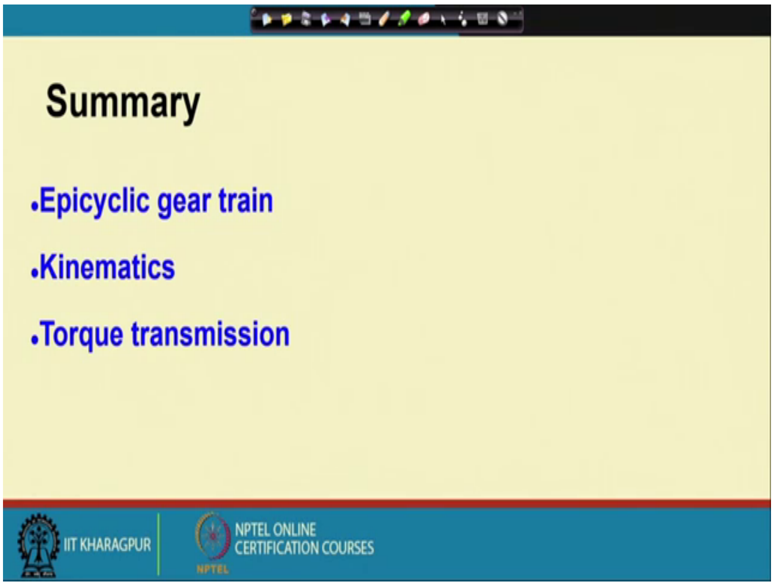
Let us say omega arm is equal to omega sun. Suppose I fix the arm on to the sun which means I fix this, then omega arm and omega sun, they are equal. If that be so, then this whole thing vanishes, this becomes 0. Therefore, in order to have this relation valid this also must be 0 which means that the omega of the ring gear must be equal to omega of the arm and that is equal to omega of the sun, therefore, all of them are match. Therefore, if I fix the arm on to let say the sun, all the shafts move on at the same angular speed.

(Refer Slide Time: 24:01)



So, omega all the gears move at the same angular speed omega R is equal to omega A is equal to omega S. If I let us I fix the ring gear; let us say if I fix the ring gear which means I do not allow this to rotate, then I have this relation omega sun minus omega arm by minus omega a is minus N R by N S. So, now various combinations you can calculate the relation between the gears the angular speeds of the gears.

(Refer Slide Time: 24:46)



So, let me conclude by summarising what we have discussed. We have looked at the epicyclic gear train their kinematics considering both a sun-planet gear and a planet-ring gear and a combination of these two. You have calculated the motion transmission and the torque transmission ratios. So, with that I will close this lecture.